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AND

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PREFACE.

THIS book, as its title implies, contains all subjects of arithmetic considered essential by the most progressive educators of to-day. The subject matter is so arranged that the more difficult, and sometimes the less important, subjects of a chapter are placed last in the chapter as a supplement, and may be omitted by teachers desiring a shorter course.

The most important features of the book are:

1. Numerous problems of a practical and original character, carefully graded, and free from all ambiguous phraseology and puzzling conditions.

2. Solutions of prominent types of problems, which set forth in correct form the logical steps leading to results, and which avoid the many inaccurate yet common expressions used by students.

3. The treatment of Negotiable Papers, which contains photographic forms of notes and drafts, together with explanations of their purposes, their uses in actual business transactions, and the relations of the parties involved.

4. Definitions that are concise and yet define, sufficiently illustrated to emphasize their meaning.

5. The distinction made and maintained between numbers and amounts. This distinction gives the student a philosophical conception of number and a clear comprehension of the meaning of the definitions and principles of arithmetic, as well as of their application.

6. The separation of the processes of finding one of the equal parts of a number, and of finding how many times one number contains another as a unit of measure, into two distinct elementary operations, called Division and Mensuration. This separation leads early to the important distinction between the quotient and the ratio of two numbers, concerning the meaning of which terms there is often much confusion in the mind of the student.

DAVID M. SENSENIG.
ROBERT F. ANDERSON.

STATE NORMAL SCHOOL,
WEST CHESTER, PA.
March, 1902.

CONTENTS

INTRODUCTION.

PAGE
NUMBERS 1–12
Arabic Notation 1
Roman System of Notation 9
General Definitions 11

CHAPTER I.

WHOLE NUMBERS AND AMOUNTS. FUNDAMENTAL OPERATIONS 13–75
Addition 13
Subtraction 20
Multiplication 28
Division 38
Mensuration 50
Arithmetical Expressions 55
Factors, Exact Divisors, and Exact Measures 57
Common Divisors and Common Measures 61
Common Multiples 65
Cancellation 68
Supplement 72
Highest Common Divisor of Numbers not Readily Factored 72
Least Common Multiple of Numbers not Readily Factored 74

CHAPTER II.

FRACTIONS 76–114
General Definitions 76
General Principles 78
Reduction of Fractions 80

FRACTIONS — *continued.* PAGE
Addition of Fractions 83
Subtraction of Fractions 86
Multiplication of Fractions 89
Division of Fractions 95
Mensuration of Fractions 101
Complex Fractions 105
Miscellaneous Problems 107

CHAPTER III.

DECIMALS 115–132
General Definitions 115
Numeration of Decimals 116
Notation of Decimals 117
Reduction of Decimals 118
Addition and Subtraction of Decimals 120
Multiplication of Decimals 122
Division of Decimals 125
Mensuration of Decimals 128
Miscellaneous Problems 130

CHAPTER IV.

DENOMINATE AMOUNTS 133–197
United States Money 133
Bills and Accounts 135
Measures of Weight 138
" Length 141
" Surfaces 143
" Solids 146
" Capacity 149
" Time 151
Angular Measure 153
Circular Measure 154
Foreign Money 156
Compound Amounts 158
Metric System 161

DENOMINATE AMOUNTS—*continued.* PAGE
Practical Measurements 169
Carpeting 169
Papering 171
Plastering, Painting, and Paving 172
Lumber Measure 173
Masonry and Brick Work 175
The Circle 178
The Cylinder 179
Miscellaneous Problems 181
Supplement 185
Addition and Subtraction of Compound Amounts . . 185
Multiplication, Division, and Mensuration of Compound Amounts 187
Latitude and Longitude 190
Longitude and Time 192

CHAPTER V.

PERCENTAGE 198–235
Commercial Discount 209
Profit and Loss 212
Commission 217
Simple Interest 220
Stocks and Bonds 228
Supplement 233
Compound Interest 233

CHAPTER VI.

NEGOTIABLE PAPERS 236–260
Promissory Notes 236
Drafts 242
Bank Discount 249
Domestic Exchange 251
Supplement 254
Foreign Exchange 254
Partial Payments 257

CHAPTER VII.

PAGE
RATIO, PROPORTION, PARTNERSHIP, AND AVERAGE . . 261–277
Ratio 261
Simple Proportion 263
Compound Proportion 269
Proportional Parts 271
Partnership 273
Average 276

CHAPTER VIII.

POWERS, ROOTS, AND MENSURATION 278–319
Squaring and Cubing of Numbers 278
Square Root and Cube Root 279
Mensuration of Surfaces 287
Lines 287
Surfaces 287
Angles 287
Parallel and Perpendicular Lines 288
Plane Figures 289
Triangles 289
Quadrilaterals 294
Parallelograms 295
Trapezoid 297
Trapezium 298
Regular Polygons and Circles 301
Mensuration of Solids 303
Right Prisms 304
Pyramids and Cones 306
Frustums 309
Spheres 311
Similar Figures 312

SUPPLEMENT
Timber Measure 320

MISCELLANEOUS PROBLEMS 323

Essentials of Arithmetic.

INTRODUCTION.

NUMBERS.

1. Arithmetic begins with the study of the simple numbers.

2. The **simple numbers** are *one, two, three, four, five, six, seven, eight,* and *nine. One* is the **unit of number.** The other simple numbers are composed of ones, or units.

Arabic Notation.

3. The nine simple numbers are expressed by nine **figures** having the same names as the numbers for which they stand. Thus:

Figures:	1	2	3	4	5	6	7	8	9
Names:	*one,*	*two,*	*three,*	*four,*	*five,*	*six,*	*seven,*	*eight,*	*nine.*

4. Nine and one make *ten.* When ten are thought of as a single group, such a group is called *a ten.* Two tens make *twenty;* three tens, *thirty;* four tens, *forty;* five tens, *fifty;* six tens, *sixty;* seven tens, *seventy;* eight tens, *eighty;* nine tens, *ninety.* These numbers are called *the tens.*

5. The tens are expressed by writing the nine figures in order before 0.

Note. The character 0 is called *naught, cipher,* or *zero.*

Forms:	10	20	30	40	50	60	70	80	90
Names:	*ten,*	*twenty,*	*thirty,*	*forty,*	*fifty,*	*sixty,*	*seventy,*	*eighty,*	*ninety.*

6. If the simple numbers in order are united with ten, the numbers *eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen,* and *nineteen* are formed; if united with twenty, the numbers *twenty-one, twenty-two,* to *twenty-nine* are formed; if united with thirty, the numbers *thirty-one, thirty-two,* to *thirty-nine* are formed; and so on until the numbers *ninety-one, ninety-two,* to *ninety-nine* are formed.

7. A number consisting of tens and units is expressed by two figures. The right-hand figure expresses the number of units and the left-hand figure, the number of tens.

Thus, *fifty-six* is expressed 56.

8. Ten tens make *one hundred.* One hundred thought of as a single group is called *a hundred.* Hundreds, like units and tens, are numbered from one to nine inclusive, forming the numbers *one hundred, two hundred, three hundred,* and so on, to *nine hundred.*

9. The hundreds are expressed by writing the nine figures in order before 00.

Thus, 100, 200, 300, 400, 500, 600, 700, 800, 900.

10. If each number less than one hundred is united with one hundred, the numbers *one hundred one, one hundred two,* to *one hundred ninety-nine* are formed; if united with two hundred, the numbers *two hundred one, two hundred two,* to *two hundred ninety-nine* are formed; and so on until the numbers *nine hundred one, nine hundred two,* to *nine hundred ninety-nine* are formed.

11. A number consisting of hundreds, tens, and units is expressed by three figures. The first, counting from the right, expresses the number of units; the second, the number of tens; the third, the number of hundreds.

When there are no tens, a cipher is written in tens' place; when no units, a cipher in units' place.

Thus, 374 expresses *three hundred seventy-four;* 204, *two hundred four;* 240, *two hundred forty.*

EXERCISE I.

Write in figures the numbers:

1. From one to ten.
2. From ten to twenty.
3. From twenty to thirty.
4. From thirty to forty.
5. From forty to fifty.
6. From fifty to sixty.
7. From sixty to seventy.
8. From seventy to eighty.
9. From eighty to ninety.
10. From ninety to one hundred.

Write in figures the numbers:

11. From one hundred to one hundred ten.
12. From four hundred to four hundred ten.
13. The tens between two hundred and three hundred.
14. The tens between seven hundred and eight hundred.
15. Four hundred eighty-three. Five hundred six.
16. Seven hundred forty-six. Nine hundred four.
17. Seven hundred ten. Four hundred fifty.

Write in words:

18. 310.
19. 470.
20. 206.
21. 403.
22. 508.
23. 609.
24. 770.
25. 540.
26. 527.
27. 399.
28. 888.
29. 999.

12. Ten hundred make *one thousand*, or *a thousand*. Thousands are numbered from one to nine hundred ninety-nine inclusive. Ten thousand make *a ten-thousand;* ten ten-thousand make *a hundred-thousand.*

13. Units, tens, hundreds, thousands, ten-thousands, and hundred-thousands are called respectively units of the *first, second, third fourth, fifth,* and *sixth order.*

14. Thousands are written in the fourth, fifth, and sixth places from the right on the line of writing, called the *line of notation; unit-thousands* in the fourth place; *ten-thousands* in the fifth place, and *hundred-thousands* in the sixth place. Thousands are followed by three ciphers, occupying the hundreds', tens', and units' places.

Thus, 5,000 represents *five thousand;* 54,000, *fifty-four thousand;* 496,000, *four hundred ninety-six thousand.*

15. If each number less than one thousand is united with one thousand, the numbers from *one thousand* to *one thousand nine hundred ninety-nine* are formed; if united with two thousand, the numbers from *two thousand* to *two thousand nine hundred ninety-nine* are formed; and so on until the numbers from *nine hundred ninety-nine thousand* to *nine hundred ninety-nine thousand nine hundred ninety-nine* are formed.

16. A number consisting of thousands, hundreds, tens, and units is expressed by writing after the number of thousands the number of hundreds, tens, and units. If units of any order are wanting, a cipher must be written in that order's place.

Thus, 375,845 expresses *three hundred seventy-five thousand eight hundred forty-five*, and 40,701, *forty thousand seven hundred one.*

EXERCISE 2.

Read, and write in words:

1. 1234.
2. 5073.
3. 3406.
4. 7644.
5. 1076.
6. 4009.
7. 2109.
8. 5732.
9. 9001.
10. 9009.
11. 9019.
12. 9090.
13. 13472.
14. 26045.
15. 54060.
16. 70007.
17. 60600.
18. 50040.
19. 136579.
20. 208440.
21. 650292.
22. 452025.
23. 300470.
24. 806053.
25. 720008.
26. 999999.
27. 800800

EXERCISE 3.

Write in figures:

1. The thousands from one thousand to nine thousand

2. The ten-thousands from ten thousand to ninety thousand.

3. The hundred-thousands from one hundred thousand to nine hundred thousand.

4. The numbers between one thousand and one thousand ten.

5. The numbers between five thousand and five thousand ten.

6. The numbers between two thousand ten and two thousand twenty.

7. The numbers between six thousand thirty and six thousand forty.

8. The numbers between nine thousand eighty and nine thousand ninety.

9. Three thousand four hundred sixty-three.

10. Two thousand seven hundred ninety.

11. Four thousand sixty-three. Nine thousand nine hundred ninety-nine.

12. Seven thousand thirty. Eight thousand eight.

13. Twelve thousand four hundred fifty-six.

14. Fifty thousand three hundred sixty-two. Seventy thousand seven.

15. Forty-eight thousand forty-five. Fifty thousand twenty-five.

16. Four hundred fifty-six thousand three hundred seventy-five.

17. Five hundred six thousand twenty-five.

18. Six hundred thousand six hundred.

19. Nine hundred thousand nine.

20. Three hundred one thousand forty-six.

17. One thousand thousand make *a million;* one thousand million, *a billion;* and one thousand billion, *a trillion.*

18. Millions, billions, and trillions, like thousands, are numbered from one to nine hundred ninety-nine inclusive. Millions are written in the seventh, eighth, and ninth places on the line of notation and are followed by six ciphers; billions are written in the tenth, eleventh, and twelfth places and are followed by nine ciphers; and trillions are written in the thirteenth, fourteenth, and fifteenth places, and are followed by twelve ciphers.

Thus, 104,000,000 represents *one hundred four million;* 326,000,000,000 represents *three hundred twenty-six billion;* 196,000,000,000,000 represents *one hundred ninety-six trillion.*

Note. Any number less than one thousand may be regarded as a number of units.

Thus, 325 may be regarded as 325 units.

19. If each number less than one million is united with one million, the numbers between *one million* and *two million* are formed; if united with two million, the numbers between *two million* and *three million* are formed; and so on until the numbers between *nine hundred ninety-nine million* and *one billion* are formed. In a similar manner, if each number less than one billion is united with each of the billions, the numbers between the billions to a trillion are formed.

20. A number may be expressed in figures by separating it into units, thousands, millions, billions, etc., and writing these in succession from the highest to the lowest, allowing three places for each after the highest.

Thus, *six billion four hundred twenty million thirty thousand six* is written 6,420,030,006.

Remark. The method of forming and naming numbers is **Denomination.** The method of writing numbers by characters is **Notation.** The method of reading numbers expressed by characters is **Numeration.**

21. TABLE OF NOTATION.*

Hundred-trillions.	Ten-trillions.	Trillions.	Hundred-billions.	Ten-billions.	Billions.	Hundred-millions.	Ten-millions.	Millions.	Hundred-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.
5	2	7	2	4	8	0	2	9	8	6	7	0	3	4
Trillions. 5th period.			Billions. 4th period.			Millions. 3d period.			Thousands. 2d period.			Units. 1st period.		

EXERCISE 4.

1. Read, and write in words 4576047204.

FORM. 4,576,047,204.

SOLUTION. Beginning at the right and pointing off the number in periods of three places each, we find 4 in the fourth, or billions' period;

---

* According to the English method of numbering, it requires a million million to make a billion, a million billion to make a trillion, and so on. [See New Complete Arithmetic, p. 5.]

576 in the third, or millions' period; 047 in the second, or thousands' period; and 204 in the first, or units' period. Therefore the number is *four billion five hundred seventy-six million forty-seven thousand two hundred four.*

Read, and write in words:

2. 1000000.
3. 20000000.
4. 200000000.
5. 400400000.
6. 602000200.
7. 62400400.
8. 100100100.
9. 200200002.
10. 4278345,
11. 78300458.
12. 800432184.
13. 842040203.
14. 2000000000.
15. 30303030480.
16. 400704004001.
17. 876432895843.
18. 340000356834.
19. 7000586000012.

EXERCISE 5.

1. Write in figures twenty-four billion sixteen million three hundred forty thousand seven hundred.

Form. 24,016,340,700.

Solution. There being *twenty-four* billion, *sixteen* million, *three hundred forty* thousand, *seven hundred* units, we write 24 in the fourth, or billions' period; 016 in the third, or millions' period; 340 in the second, or thousands' period; and 700 in the first, or units' period.

Write in figures:

2. Three million.

3. Forty million.

4. Six hundred million.

5. Five million six hundred.

6. Sixty million sixty.

7. Four hundred million four hundred.

8. Seventy-eight million six hundred seven.

9. Forty-eight million three hundred thousand ninety-nine.

10. Seventy million one hundred sixty-five thousand nine hundred.

11. Fifty-five million five hundred sixty-seven thousand eighty-four.

12. Eight hundred ninety million seventy thousand four hundred six.

13. Nine hundred nine million nine hundred thousand.

14. Two billion twenty million five hundred thousand.

15. Fifty-six billion five hundred million.

16. Forty billion forty-seven million forty-five thousand.

17. One hundred sixty-six billion two hundred million fifty-six thousand.

Roman System of Notation.

22. The Romans used seven letters and combinations of them to represent numbers. These letters are I representing *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

23. The simple numbers, the tens, the hundreds, and the thousands are represented by combinations formed according to the following principles:

1. *If a letter is repeated its value is repeated.*
2. *When a letter or a combination of letters of less value is written after a letter or combination of greater value, the sum of the values is expressed.*
3. *When a letter of less value is written before one of greater value the difference of their values is expressed.*
4. *A dash placed over a letter or a combination of letters makes the letter or combination represent thousands.*

24. TABLE.

Units.		Tens.		Hundreds.		Thousands.	
I	= 1	X	= 10	C	= 100	M	= 1000
II	= 2	XX	= 20	CC	= 200	MM	= 2000
III	= 3	XXX	= 30	CCC	= 300	MMM	= 3000
IV	= 4	XL	= 40	CD	= 400	$\overline{IV}$	= 4000
V	= 5	L	= 50	D	= 500	$\overline{V}$	= 5000
VI	= 6	LX	= 60	DC	= 600	$\overline{VI}$	= 6000
VII	= 7	LXX	= 70	DCC	= 700	$\overline{VII}$	= 7000
VIII	= 8	LXXX	= 80	DCCC	= 800	$\overline{VIII}$	= 8000
IX	= 9	XC	= 90	CM	= 900	$\overline{IX}$	= 9000

Note. The symbol = is read *equals.*

25. In writing numbers between those in the above table, hundreds are written after thousands, tens after hundreds, and units after tens.

Thus, 1567 is written MDLXVII; 2053, MMLIII; 4508, I̅V̅DVIII; 3600, MMMDC.

Note 1. IIII = 4; XXXX = 40; CCCC = 400, are also in use.

Note 2. The use of the Roman system of notation is confined chiefly to the numbering of chapters and sections of books, pages of prefaces and introductions, medical prescriptions, public documents, divisions on clock dials, etc.

EXERCISE 6.

Write by the Roman method:

1. The nine simple numbers.
2. The nine tens.
3. The nine hundreds.
4. The first nine thousands.

The numbers between:

5. 10 and 20.
6. 20 and 30.
7. 30 and 40.
8. 40 and 50.
9. 50 and 60.
10. 60 and 70.
11. 70 and 80.
12. 80 and 90.
13. 90 and 100.

The following numbers:

14. 150.
15. 235.
16. 304.
17. 440.
18. 564.
19. 601.
20. 746.
21. 808.
22. 990.
23. 1856.
24. 1905.
25. 20020.

Write in figures the following:

26. XIX.
27. XXV.
28. LXXVII.
29. XLIV.
30. XCIX.
31. XXXIX.
32. LXXXIX.
33. XLVIII.
34. XCIV.
35. CXIV.
36. CCXIX.
37 CDXLIV.
38. CMXCVI.
39. DCCCIV.
40. MCMIX.
41. MDCLXVI.
42. X̅DLIII.
43. MCCCXXXV.
44. MCMIV.
45. MDCCCXCIX.
46. MDCCCC.

General Definitions.

26. Quantity is anything that can be measured; as, *length, weight, value, land, grain, live-stock.*

27. A **Unit of Quantity** is any definite part of quantity or any single thing taken as a standard of measure; as, *a yard, a pound, a dollar, an acre, a bushel, a head.*

28. An **Amount** is an expression denoting the number and the kind of units in a quantity; as, *20 yards, 2 pounds, 5 dollars, 4 acres, 6 bushels, 8 head of stock.*

Note. Some writers call an amount a **Concrete Number.**

29. Similar Amounts are such as are composed of the same kind of units of quantity; as, *4 yards* and *3 yards.*

30. Arithmetic is the science of numbers and amounts and the methods of computing with them.

Note. A number answers the question, *How many?* An amount answers the question, *How much?*

EXERCISE 7. ORAL.

In the following amounts name at sight the unit and the number of units:

1. 10 cents. *Answer.* The unit is 1 cent; the number of units, 10.

2. 3 yards.
3. 5 pounds.
4. 6 bushels.
5. 8 days.
6. 6 hours.
7. 2 acres.
8. 5 horses.
9. 12 men.
10. 18 trees.
11. 16 cows.
12. 12 houses.
13. 4 flocks.
14. 9 baskets.
15. 8 chairs.
16. 5 dozen.
17. 6 regiments.
18. 4 trains of cars.
19. 7 droves.

20. A man drove 5 miles.

1. How many miles did he drive? *Ans.* 5.
2. How far did he drive? *Ans.* 5 miles.

21. A laborer earned $5.

1. How many dollars did he earn?
2. How much money did he earn?

22. A hunter shot 10 birds.

1. How many birds did he shoot?
2. How much game did he shoot?

23. A farmer employed 5 hands.

1. How much help did he employ?
2. How many hands did he employ?

24. A dealer bought a cow for $40.

1. What did he pay for her?
2. How many dollars did he pay for her?

25. A haystack was 20 feet long.

1. What was the length of the haystack?
2. How many feet long was it?

Ask and answer two arithmetical questions from each of the following statements:

26. A man bought 10 oranges.

27. A farmer cut 8 acres of wheat.

28. A bushel of wheat weighs 60 pounds.

29. A man broke 10 cartloads of stone.

30. A dealer sold 24 silver spoons.

CHAPTER I.

WHOLE NUMBERS AND AMOUNTS—FUNDAMENTAL OPERATIONS.

Addition.

31. Addition is the process of uniting two or more numbers or similar amounts into one number or amount called their **sum.**

Note. The symbol of addition is +, read *plus.*

EXERCISE 8. ORAL.

1 3 6 9 4 8 2 7 5

Add at sight each of the above numbers:

1. To 11.
2. To 22.
3. To 33.
4. To 44.
5. To 55.
6. To 66.
7. To 77.
8. To 88.
9. To 99.

Add at sight the following columns:

10.

$$\begin{array}{l}\left.\begin{array}{r}2\\3\\3\end{array}\right\}\\\left.\begin{array}{r}4\\6\end{array}\right\}\\\left.\begin{array}{r}6\\2\\1\end{array}\right\}\\\hline \mathbf{27}\end{array}$$

Solution. So group the numbers that the sum of each group does not exceed 11. Then, beginning at the foot of the column,

Think 1 + 2 + 6 is 9; then 9 + 10 is 19; then 19 + 8 is 27. Say 9, 19, 27.

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**
2	6	3	5	6	3	8	4	8
3	4	2	4	4	3	6	4	4
4	3	1	5	7	2	6	7	3
5	4	8	4	3	2	5	2	7
3	7	4	5	5	4	2	3	4
2	9	7	4	5	4	8	1	2
6	6	3	5	4	3	3	2	6
4	3	5	4	5	5	7	5	4
3	2	4	5	1	6	5	3	3
6	5	5	4	9	2	5	7	3

Find at sight the sum of the following:

20. $\overline{2+3+4}+\overline{3+5}+\overline{4+6}+\overline{2+6}$.
21. $\overline{3+7}+\overline{4+6}+\overline{5+3+2}+\overline{2+8}$.
22. 6 + 8 + 5 + 7 + 6 + 8 + 3 + 5 + 4.
23. 7 + 9 + 6 + 8 + 3 + 4 + 2 + 3 + 9.
24. 8 + 1 + 7 + 2 + 3 + 2 + 4 + 2.
25. 6 + 3 + 4 + 5 + 3 + 2 + 7 + 8 + 9.
26. 3 + 4 + 6 + 8 + 4 + 3 + 8 + 7 + 2.
27. 9 + 8 + 7 + 6 + 5 + 4 + 2 + 1.
28. 7 + 8 + 6 + 3 + 5 + 4 + 3 + 8 + 7.
29. 8 + 3 + 5 + 4 + 3 + 7 + 6 + 4 + 2.

EXERCISE 9. ORAL.

Find at sight the value of the following:

1. 40 + 30; 46 + 30; 45 + 38.

Solution. (1) 40 and 30 are 70.
(2) 46 and 30 are 76.
(3) 45 and 30 are 75 and 8 are 83.

2. 19 + 10.
3. 28 + 20.
4. 37 + 30.
5. 46 + 40.
6. 55 + 50.
7. 64 + 60.
8. 73 + 70.
9. 82 + 80.
10. 91 + 90.
11. 21 + 13.
12. 41 + 15.
13. 61 + 17.
14. 81 + 19.
15. 32 + 24.
16. 52 + 26.
17. 72 + 28.
18. 93 + 24.
19. 53 + 36.
20. 73 + 38.
21. 94 + 35.
22. 64 + 47.
23. 84 + 49.
24. 65 + 57.
25. 85 + 59.
26. 76 + 68.
27. 97 + 68.
28. 98 + 79.

EXERCISE 10. ORAL.

1. A farmer has 5 cows in one stable and 4 in another; how many has he in both?

SOLUTION. Since the farmer has 5 cows in one stable and 4 in another, the number he has in both is 5 plus 4, or 9.

2. A pound of sugar cost 6 cents and a can of corn 8 cents; find the cost of both.

SOLUTION. Since a pound of sugar cost 6 cents and a can of corn 8 cents, they both cost 6 cents plus 8 cents, or 14 cents.

3. There are 7 quarts of berries in one bucket and 8 quarts in another; find the number of quarts in both.

4. James paid $6 of a debt and still owed $12; how much did he owe at first?

5. A farmer uses 8 hundred rails to build one fence and 12 hundred to build another; how many hundred does he use for both?

6. The goods for a dress cost $18 and the making of it $8; find the cost of the dress.

7. A laborer had $32 and earned $9 more; how much money had he then?

8. A contractor employed 12 Italians and 9 Hungarians; how many laborers did he employ?

9. A dealer paid $40 for a cow and $7 for a calf; what did he pay for both?

10. A huckster bought melons at 16 cents each and sold them at a gain of 9 cents each; what did he receive for each melon?

11. A trucker sold 23 baskets of peaches and 8 baskets of apples; how many baskets of fruit did he sell?

12. In a certain school there are 28 girls and 8 boys; how many pupils are there in the school?

13. A teacher was 22 years old when he began teaching and has taught 8 years; how old is he?

14. 36 horses and 7 mules are how many draft animals?

15. A boy bought 6 apples for 8 cents and 4 peaches for 5 cents; how many pieces of fruit did he buy and what did they cost him?

16. A bushel of wheat weighs 60 pounds and a bushel of oats 32 pounds; what is their combined weight?

17. In an orchard there are 24 apple trees and 18 more peach trees than apple trees; how many peach trees are there in the orchard?

18. 8 horses, 7 cows, and 6 sheep make how many head of stock?

19. 12 men, 9 women, and 5 children in a car make how many passengers?

20. A boy paid 12 cents for a tablet, 8 cents for a ruler, and 5 cents for a pencil; how much did they together cost?

21. A farmer sowed 60 bushels of wheat, 6 bushels of clover seed, and 3 bushels of timothy seed; how many bushels of seed did he sow in all?

22. A dealer bought a carriage for $48, paid $12 to have it repaired, and then sold it at a profit of $20. How much did he receive for it?

23. A convict was imprisoned 8 years ago; he was then 25 years old; if he has yet 6 years to serve, how old will he be when released?

24. A laborer had $8 left of his month's salary after paying $15 house rent and $17 store bill; find his salary for the month.

EXERCISE II.

1. Find the sum of 428, 356, 479, and 543.

FORM.

$$\begin{array}{r} 428 \\ 356 \\ 479 \\ 543 \\ \hline 1806 \end{array}$$

SOLUTION. Add first the units, then the tens, then the hundreds. The sum of the units is 26 units, or 2 tens and 6 units; write 6 in units' place and add 2 tens to the tens. The sum of the tens is 20 tens, or 2 hundreds; write 0 in tens' place and add 2 hundreds to the hundreds. The sum of the hundreds is 18 hundred, or 1 thousand and 8 hundred; write 8 in hundreds' place and 1 in thousands' place. The sum is 1806.

Note. To test the correctness of the result in addition:

1. *See if it agrees with the sum obtained when the columns are added downwards.*

2. *Separate each column into partial columns, and see if the sum of their sums agrees with the result.*

Find the sum of:

2.	**3.**	**4.**	**5.**
456	584	865	8869
342	679	452	7988
178	245	398	5697
263	374	573	8775

6.	**7.**	**8.**	**9.**
32646	41508	92785	499
48734	84002	10026	27
12348	25439	46437	7862
26782	32459	25078	15346
34523	50672	34872	5982
46345	45683	94633	793
21376	74660	4594	84578

Add the following without writing the numbers in columns:

10. 24 + 36 + 48 + 72 + 83 + 15.
11. 123 + 235 + 478 + 672 + 156 + 235.
12. 437 + 315 + 628 + 379 + 541 + 582.
13. 1246 + 3482 + 4643 + 1357 + 2005.
14. 3046 + 2705 + 3890 + 6478 + 8964.
15. 6646 + 8008 + 7800 + 8469 + 7998.
16. 324 + 4679 + 6457 + 389 + 64 + 4642.
17. 5784 + 315 + 8600 + 428 + 3468 + 18.
18. 243 + 28 + 6442 + 19 + 7009 + 628.

EXERCISE 12.

1. A farmer raised 438 bushels of wheat, 542 bushels of corn, 347 bushels of oats, 97 bushels of rye, and 164 bushels of barley; how many bushels of grain did he raise?

Solution.

438 = the number of bushels of wheat raised.
542 = " " " " " corn "
347 = " " " " " oats "
97 = " " " " " rye "
164 = " " " " " barley "
∴ 1588 = " " " " " grain "

Note. The symbol ∴ is read *therefore.*

2. A man bought 4 horses, paying $225 for the first, $196 for the second, $208 for the third, and $173 for the fourth; how much did he pay for all?

Solution.

$225 = the amount paid for the first horse.
$196 = " " " " " second horse.
$208 = " " " " " third "
$173 = " " " " " fourth "
∴ $802 = " " " " all.

3. A hardware dealer made four sales of bar iron amounting respectively to 238 pounds, 346 pounds, 429 pounds, and 763 pounds; how many pounds of bar iron did he sell?

4. In a certain library there are five shelves, the first containing 185 books, the second 246 books, the third 288 books, the fourth 312 books, and the fifth 328 books; how many books are there in the library?

5. Mr. Jones's land is worth $8456, his house $4684, his barn $2138, and his other buildings $645; how much is his property worth?

6. A farmer delivered to a miller four loads of wheat: the first weighed 2260 pounds, the second 2501 pounds, the third 2304 pounds, and the fourth 2417 pounds; what was the weight of the four loads?

7. A contractor built five houses, receiving for the first $3245, the second $4764, the third $2875, the fourth $6432, and the fifth $8467; how much did he receive for all?

8. In an election A received 8475 votes, B 1675 more than A, and C 756 more than B; how many did B and C each receive?

9. A cattle dealer sold cattle for five years: the first year he sold 1346 head; the second, 1868 head; the third, 2046 head; the fourth, 2874 head; and the fifth, 2643 head; how many did he sell in all?

10. A man having four sons gave to the first $2465; to the second, $564 more than to the first; to the third, $348 more than to the second; and to the fourth, $345 more than to the third; how much did he give to each and how much to all?

11. Fill in the totals in the following weekly report of a circulating library.

Class.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total.
Fiction	756	864	795	791	794	1219	
Fine Arts	427	487	498	489	490	521	
History	685	714	716	725	698	797	
Literature	427	450	461	418	429	548	
Philosophy	89	101	100	99	102	181	
Religion	125	150	170	167	192	307	
Science	347	355	361	355	379	409	
Sociology	214	225	229	237	244	318	
Total							

12. The following tabulated statement shows the yield of trout and white fish for the years 1890 and 1893 in the Great Lakes; fill in the totals.

Lake.	Trout.		White Fish.		Total.
	1890.	1893.	1890.	1893.	
Superior	2,613,378	4,342,122	3,213,176	2,769,088	
Michigan	8,364,167	8,216,920	5,455,079	4.833.691	
Huron	1,505,619	3,439,575	1,004,094	1,222,687	
Erie	121,420	203,132	2,341,451	1.292,410	
Ontario	41,010	6,024	148,771	45,380	
Total					

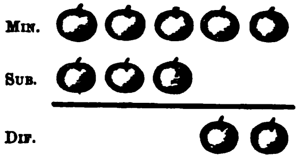
13. For the year ending June 5, 1899, the total number of pupils enrolled in four counties of Pennsylvania was as follows: Bucks county, males 7195 and females 6879; Chester county, males 9326 and females 8982; Delaware county, males 7605 and females 7558; Montgomery county, males 12360 and females 11875. How many pupils of each sex were enrolled, and how many in all?

Subtraction.

32. **Subtraction** is the process of taking one number or amount called the **Subtrahend** from another number or amount called the **Minuend.**

33. The **Difference** is the number or amount remaining after taking the subtrahend from the minuend.

ILLUSTRATION. If 3 apples are taken (subtracted) from 5 apples the difference is 2 apples.



34. It is readily seen from the above illustration that:

1. *The difference is such a number or amount as added to the subtrahend will make the minuend.*

2. *The difference is such a number or amount as taken from the minuend will leave the subtrahend.*

Hence it is seen that the difference of two numbers or amounts may be found in three ways:

1st. By finding what number or amount is left when the subtrahend is counted from the minuend.

2d. By finding what number or amount must be added to the subtrahend to make the minuend.

3d. By finding what number or amount must be counted from the minuend to make the subtrahend.

35. It is evident that in the subtraction of amounts the minuend, the subtrahend, and the difference are similar amounts.

36. The symbol of subtraction is —, read *minus*, and denotes, when written between two numbers or amounts, that the latter is to be subtracted from the former.

Thus, 8 — 5 = 3 is read *8 minus 5 equals 3*, and denotes that 5 subtracted from 8 leaves 3.

Note. The expression 8 + 4 — 3 + 6 — 2 denotes that 4 is to be added to 8, then 3 subtracted from the sum, then 6 added to the difference, then 2 subtracted from the sum. The numbers or amounts connected by + or — are called the **Terms** of an expression.

EXERCISE 13. ORAL.

2 5 3 7 6 4 8 10 9

Subtract at sight each of the above numbers:

1. From 11.
2. From 22.
3. From 33.
4. From 44.
5. From 55.
6. From 66.
7. From 77.
8. From 88.
9. From 99.

Starting with the number above the line, subtract at sight, in succession, the numbers below the line in the following examples:

10.	11.	12.	13.	14.	15.	16.	17.	18.	19.
57	78	84	91	65	92	63	96	79	70
2	9	2	9	8	1	4	3	8	7
9	2	4	1	1	9	9	9	1	6
4	7	6	7	6	2	6	2	7	9
7	4	8	5	3	8	7	8	2	8
6	5	1	3	4	3	8	5	5	1
5	6	3	8	5	7	5	7	3	3
8	3	5	2	2	4	3	4	4	2
3	1	7	6	7	6	1	6	6	5
1	8	9	4	9	5	2	1	9	4

EXERCISE 14. ORAL.

Solve at sight:

1. From 67 take 20.

SOLUTION. 67 minus 20 is 47.

2. From 75 take 39.

SOLUTION. 75 minus 30 is 45, minus 9 is 36.

3. From 28 take 10.
4. From 49 take 20.
5. From 65 take 30.
6. From 54 take 40.
7. From 21 take 13.
8. From 41 take 15.
9. From 61 take 17.
10. From 81 take 19.

Find the difference of:

11. 73 and 50.
12. 84 and 60.
13. 92 and 70.
14. 32 and 24.
15. 54 and 26.
16. 72 and 28.
17. 93 and 24.
18. 53 and 36.
19. 73 and 38.

Find the value of:

20. 106 — 80.
21. 115 — 90.
22. 73 — 38.
23. 94 — 35.
24. 64 — 47.
25. 84 — 49.
26. 65 — 57.
27. 85 — 59.
28. 76 — 68.
29. 97 — 68.
30. 98 — 79.
31. 109 — 99.

Find the value of:

32. $24 + 15 + 7$.
33. $36 - 18 + 10$.
34. $45 - 8 + 12$.
35. $68 - 23 - 18$.
36. $89 - 77 + 21$.
37. $92 - 68 + 27$.

EXERCISE 15. ORAL.

1. A boy had 7 apples and ate 4 of them; how many had he left?

SOLUTION. Since a boy had 7 apples and ate 4 of them, the number he had left was the difference between 7 and 4, or 3.

2. A man gained $5 by selling a cow for $30; how much did the cow cost him?

SOLUTION. Since a man gained $5 by selling a cow for $30, the cow cost him the difference between $30 and $5, or $25.

3. A merchant sold 3 cans of corn from a box containing 8 cans; how many cans remained?

4. I bought a bill of goods amounting to $9 and paid on it $6 cash; how much do I still owe?

5. The distance from my home to the station is 16 squares. In going from my home to the station I have already walked 9 squares; how many have I yet to walk?

6. A farmer bought a bill of goods amounting to $18, and gave in payment butter amounting to $6 and the balance in cash; how much cash did he pay?

7. A dealer sold 10 collars from a box containing 12; how many had he left?

8. A lady bought a bill of goods amounting to $12 and gave in payment a twenty-dollar bill; how much change should she receive?

9. A man had 25 chairs to carry to a room; if he has carried 18 of them, how many more has he to carry?

10. If a bin holds 48 bushels, and 12 bushels of grain are already in it, how many more bushels of grain will it take to fill it?

11. John was 50 miles from home, but has since traveled 15 miles toward home; how far is he still from home?

12. Mary buys an article for 16 cents and gives in payment a 50-cent piece; how much change should she receive?

13. William weighs 68 pounds and James 39 pounds; how much heavier is William than James?

14. A kite rose 78 feet from the ground and then fell 24 feet; how far was it still from the ground?

15. Twenty gallons of oil were drawn from a vessel containing 63 gallons of oil; how many gallons of oil remained in the vessel?

16. Frank is 48 years old and Amos 72; how much older is Amos than Frank?

17. A tree 94 feet high was broken off 36 feet from the top; how many feet remained standing?

18. A boy agreed to furnish a dealer 42 quarts of berries; he has gathered 27 quarts; how many more quarts must he gather?

19. At 6 o'clock the temperature was 36 degrees and at noon it was 64 degrees; how many degrees did the temperature rise?

20. A merchant had 6 hundred dollars in bank, then deposited 4 hundred dollars more and afterward drew out 5 hundred dollars; how many dollars had he then in bank?

21. A drover bought a cow for $45 and a calf for $5, and sold them both for $60; how much did he gain?

22. A farmer had 90 bushels of wheat and sold at one time 30 bushels and at another time 28 bushels; how many bushels had he remaining?

23. One man shot 40 times and killed 16 birds; another shot 20 times and killed 7 birds; how many times did they both miss?

24. A man had 20 cows and 12 horses; he sold 10 cows and bought 6 horses; how much stock had he then?

EXERCISE 16.

1. From 346 take 168.

FORM.

$$\begin{array}{r} 346 \\ \underline{168} \\ 178 \end{array}$$

SOLUTION. Write the subtrahend under the minuend as in the form and begin at the units' place to subtract. Thus, 8 units from 6 units we cannot take; we take 1 ten from the 4 tens (leaving 3 tens) and add it to the 6 units making 16 units. Now, 8 units from 16 units leaves 8 units. Again, 6 tens from 3 tens we cannot take; we take 1 hundred from the 3 hundreds (leaving 2 hundreds) and add it to the 3 tens making 13 tens. Now, 6 tens from 13 tens leaves 7 tens, and 1 hundred from 2 hundreds leaves 1 hundred.

Therefore, the difference is 178.

Note 1. In practice, think 8 from 16 is 8, 6 from 13 is 7, 1 from 2 is 1.

Note 2. To test the correctness of the result in subtraction:

1. See if the difference added to the subtrahend will produce the minuend.

2. See if the difference subtracted from the minuend will produce the subtrahend.

	2.	**3.**	**4.**	**5.**	**6.**
From	876	462	642	863	946
take	234	272	239	668	659

	7.	**8.**	**9.**	**10.**
From	2348	4092	2406	3791
take	1136	786	948	2069

Find the value of:

11. 2378 — 982.

12. 3106 — 1275.

13. 6372 — 4087.

14. 10836 — 8757.

15. 20020 — 15078.

16. 32006 — 24380.

17. 348 — 278 + 124.

18. 246 + 389 — 478.

19. 1024 — 832 + 508.

20. 9647 — 3245 — 5025.

21. 10416 — 6372 — 3415.

22. 9437 + 2349 — 8798.

23. 4318 — 2468 + 3754 — 4188.

24. 2409 — 876 — 478 + 325.

25. 4166 + 3274 — 5638 — 920.

EXERCISE 17.

1. The area of Pennsylvania is 45215 square miles and that of New Jersey 7815 square miles; what is the difference in their areas?

SOLUTION. 45215 square miles = the area of Pennsylvania.
7815 " " = " " " New Jersey.
∴ 37400 " " = the difference of their areas.

2. The number of pensioners in Pennsylvania, June 30, 1899, was 104681, and in New York, 87527; how many more pensioners had Pennsylvania than New York?

SOLUTION. 104681 = the number of pensioners in Pennsylvania.
87527 = " " " " " New York.
∴ 17154 = the number of pensioners more in Pennsylvania than in New York.

3. A man traveled 585 miles from home; how far must he return to be 275 miles from home?

4. A drover had 425 sheep and sold 286 of them; how many had he remaining?

5. A man has a thousand-mile ticket, but he has traveled 456 miles on it; how many miles may he yet travel on it?

6. Mr. Williams agreed to pay $2400 rent for a store; if he has paid $1865, how much remains unpaid?

7. Mr. Jones bought a house for $6245 and sold it for $7000; how much did he gain?

8. In a certain town of 8000 inhabitants 3578 are males; how many are females?

9. I bought a farm for $8765 and sold it for $6298; how much did I lose?

10. A man was born in 1858 and died in 1900; how old was he when he died?

11. Of two candidates for sheriff, one received 26132 votes and the other 35086 votes; what was the majority of the successful candidate?

12. A railroad hauled 12864 carloads of coal one year and 13872 the following year; what was the increase for the year?

13. A bought a house for $3245, then sold it for $4000, and bought it back again for $4800; how much had he then invested in the house?

14. In a certain Normal School there were 656 students; at the end of the year 146 of them graduated, and 138 others left not to return. The following year 341 new students entered; how many students were there then in the school?

15. Four adjoining farms contain respectively 160 acres, 248 acres, 178 acres, and 295 acres; how much do they together lack of 1000 acres?

16. A farmer had $475 to invest in horses; if he has bought one horse for $164 and another for $175, how much has he yet to invest?

17. A man bought a Government bond for $1015, then sold it for $965, and again bought it for $1000; how much had he then invested in it?

18. The Wilmington and Northern' railroad earned $380852 in 1889 and $454783 in 1899; how much more was earned in the latter year than in the former?

19. How many years are there between 365 years and 472 years? How many years from 365 years to 472 years (which includes the latter)? How many years from 365 years to 472 years inclusive (which includes both)?

20. The following table shows the value of our export and import trade with Porto Rico, Cuba, and the Philippines for the years 1897 and 1899; complete it.

Islands.	Exports.		Imports.	
	1899.	1897.	1899.	1897.
	Dollars.	Dollars.	Dollars.	Dollars.
Porto Rico	3,677,564	2,023,751	3,416,681	1,943,251
Cuba	24,861,261	9,308,515	29,619,759	16,238,456
Philippines	1,663,213	69,459	4,903,467	4,352,181
Total				

Islands.	Increase in exports in 1899 over 1897.	Increase in imports in 1899 over 1897.	Increase in exports above increase in imports.
	Dollars.	Dollars.	Dollars.
Porto Rico			
Cuba			
Philippines			
Total			

21. In a certain election the Republican candidates for Sheriff, Recorder, and Clerk of Courts received respectively 45534, 43802, and 38917 votes; and the Democratic candidates for these offices, 31553, 48431, and 33915 votes; what was the majority of each successful candidate, and to which party did he he belong?

Multiplication.

37. A number or an amount may be taken any number of times.

Thus, 3 may be taken once and the result is 3, and 3 may be taken 4 times and the result is 3 + 3 + 3 + 3, or 12.

38. **Multiplication** is the process of taking a number or an amount a number of times.

39. The **Multiplicand** is the number or the amount taken; the **Multiplier** is the number denoting how many times the multiplicand is taken; the **Product** is the result of multiplication.

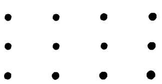
Remark. It is evident that the multiplier is always a number, and that, if the multiplicand is an amount, the product is a similar amount.

40. The symbol of multiplication is ×, read *times*. When it is used, the number written before it is the multiplier.

Thus, in the expression 3 × 4, read *3 times 4*, 3 is the multiplier. In 3 × 4 × 5 yards, 3 is the multiplier of 4 × 5 yards.

41. PRINCIPLE. *The product of two numbers is the same, whichever is made the multiplier.*

Thus, 3 rows of 4 dots each contain as many dots as 4 columns of 3 dots each, which shows that 3 × 4 = 4 × 3.



42. The following table, called the MULTIPLICATION TABLE, shows the results obtained by taking each of the numbers from 1 to 12, inclusive, from 1 to 12 times.

Note. The Multiplication Table may be extended beyond 12 × 12.

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

EXERCISE 18. ORAL.

20 30 40 50 60 70 80 90

Multiply at sight each of the above numbers:

1. By 2.
2. By 3.
3. By 4.
4. By 5.
5. By 6.
6. By 7.
7. By 8.
8. By 9.

Multiply at sight:

9. 35 by 4.

Solution. 4 times 30 equals 120, and 4 times 5 equals 20; 120 and 20 are 140.

10. 42 by 2.
11. 27 by 3.
12. 74 by 4.
13. 37 by 5.
14. 72 by 6.
15. 85 by 7.
16. 38 by 8.
17. 82 by 8.
18. 41 by 9.
19. 16 by 9.
20. 84 by 10.
21. 90 by 10.
22. 6 by 24.

Sug. 24 times 6 equals 6 times 24 [§ 41].

23. 4 by 36.
24. 5 by 45.
25. 6 by 16.
26. 8 by 42.
27. 4 by 16.
28. 5 by 25.

EXERCISE 19. ORAL.

1. A laborer worked 8 hours a day during the 6 working days of a week; how many hours did he work?

Solution. Since 8 is the number of hours the laborer worked in one day, then 6 times 8, or 48, equals the number of hours he worked in 6 days.

2. Find the cost of 3 melons at 10 cents each.

Solution. Since one melon cost 10 cents, 3 melons cost 3 times 10 cents, or 30 cents.

3. There are 7 days in one week; how many days are there in 8 weeks?

4. There are 12 in one dozen; how many eggs are there in 8 dozen of eggs?

5. Find the cost of 6 pounds of beef at 12 cents a pound.

6. How much will a clerk save in 12 months, or a year, if he saves $12 a month?

7. What do 7 dozen of eggs cost at 20 cents a dozen?

8. Find the cost of 4 pounds of Java coffee at 40 cents a pound.

9. There are 3 feet in one yard; how many feet are there in 25 yards?

10. If a lot is 30 yards long, how many feet long is it?

11. If it requires 2 bushels of wheat to sow one acre, how many bushels will it require to sow 50 acres?

12. If a train runs 32 miles an hour, how far will it run in 5 hours?

13. How many pounds of flour are there in 6 sacks, each containing 25 pounds?

14. A plasterer who earns 4 dollars a day earns how much in a month of 26 working days?

15. A lives 10 miles from a certain town; how many miles must he travel to make 6 round trips?

16. How many pounds of candy are there in 16 boxes, each containing 2 pounds?

17. How much money does a street-car conductor collect from 50 passengers, if each pays 5 cents?

18. Find the cost of 10 two-cent and 15 one-cent postage stamps.

19. Find the cost of 4 yards of muslin at 8 cents a yard, and 6 yards of flannel at 20 cents a yard.

20. A newsboy buys papers at 1 cent each and sells them at 2 cents each; how much does he gain on 42 papers?

21. A lady bought 4 dozen buttons at 8 cents a dozen and gave in payment 50 cents; how much change should she receive?

22. A dealer bought 20 lambs at $3 a head and sold them at a profit of $1 a head; how much did he receive for them?

23. A plasterer and his helper receive together $4 a day, the helper being paid $1 a day; how much does the plasterer earn in 6 days?

24. A grocer bought dried beef at 18 cents a pound and retailed it at 30 cents a pound; find his gain on a piece containing 6 pounds.

25. Twenty make a score; how many years old is a man who is 4 score and 6 years old?

26. A dealer bought 11 bushels of potatoes at 56 cents a bushel and sold them at 70 cents a bushel; how much did he gain?

27. A lady bought 5 yards of gingham at 12 cents a yard and received 15 cents change; how much money did she give the merchant?

EXERCISE 20.

1. Multiply 432 by 6.

FORM.

$$\begin{array}{r} 452 \\ 6 \\ \hline 2712 \end{array}$$

SOLUTION. 6 × 452 = 6 × 2 units + 6 × 5 tens + 6 × 4 hun. 6 × 2 units = 12 units, or 1 ten + 2 units; we write the 2 in units' place and add 1 ten to the next product. 6 × 5 tens = 30 tens, which added to 1 ten makes 31 tens, or 3 hun. + 1 ten; we write 1 in tens' place and add 3 hun. to the next product. 6 × 4 hun. = 24 hun., which added to 3 hun. makes 27 hun., or 2 thou. + 7 hun. We write 7 in hundreds' place and 2 in thousands' place, and find the entire product to be 2712.

Multiply:

2. 793 by 6.
3. 846 by 7.
4. 985 by 8.
5. 530 by 9.
6. 608 by 5.
7. 4352 by 2.
8. 6436 by 3.
9. 2057 by 4.
10. 6008 by 5.

11. There are 365 days in 1 year; how many days are there in 3 years?

SOLUTION. 365 = the number of days in 1 year.
∴ 3 × 365, or 1095 = the number of days in 3 years.

12. Find the cost of 375 tons of coal at 4 dollars a ton.

SOLUTION. $4 = the cost of 1 ton.
∴ 375 × $4, or $1500 = the cost of 375 tons.

13. A fly-wheel that makes 435 revolutions a minute, makes how many revolutions in 8 minutes?

14. A man built 7 houses that cost him $2750 apiece; how much did they all cost him?

15. A printing press that prints 295 copies of a paper in a minute, prints how many copies in 12 minutes?

16. How far does a vessel go in 8 days, if it runs on an average 475 miles a day?

17. A grain dealer received 9 carloads of wheat, each containing 900 bushels; how many bushels did he receive in all?

18. What are the daily wages paid to 950 men, if they each receive $3 per day?

19. A laborer earns $3 a day and his son $2 a day; how much do they together earn in 310 days? How much more does the father earn than the son?

20. There are 2240 pounds in a long ton and 2000 pounds in a short ton; how much more coal is there in 11 long tons than in 11 short tons?

EXERCISE 21.

1. Multiply 32 by 10.

Form.

$$\begin{array}{r} 32 \\ \underline{10} \\ 320 \end{array}$$

Solution. 10×2 units $= 20$ units, or 2 tens and 0 units; 10×3 tens $= 30$ tens, which added to 2 tens makes 32 tens, or 3 hun. + 2 tens. Therefore, $10 \times 32 = 320$.

From this we see:

43. Prin. *Annexing one cipher to a number multiplies it by 10.*

2. Multiply 32 by 100; by 1000.

Solution. (1) $100 = 10 \times 10$.

$\therefore 100 \times 32 = 10 \times 10 \times 32 = 10 \times 320$ [§ 43] $= 3200$ [§ 43].

(2) $1000 = 10 \times 100$.

$\therefore 1000 \times 32 = 10 \times 100 \times 32 = 10 \times 3200 = 32000$ [§ 43].

From this we see:

44. Prin. *Annexing two ciphers to a number multiplies it by 100; three ciphers by 1000; etc.*

3. Multiply 49 by 10; by 100; by 1000.

SOLUTION. $10 \times 49 = 490$ [§ 43].
$100 \times 49 = 4900$ [§ 44].
$1000 \times 49 = 49000$ [§ 44].

4. Multiply 75 by 500.

SOLUTION. $500 \times 75 = 5 \times 100 \times 75 = 5 \times 7500$ [§ 44] $= 37500$

Multiply:

5. 32 by 10; by 100; by 1000.
6. 275 by 20; by 200; by 2000.
7. 450 by 30; by 300; by 3000.
8. 789 by 40; by 400; by 4000.
9. 397 by 500. **10.** 865 by 600. **11.** 79 by 7000.
12. 98 by 8000. **13.** 900 by 900. **14.** 400 by 1000.

15. There are 100 cents in one dollar; how many cents are there in $200?

16. At $175 an acre, what will a farm of 200 acres cost?

17. How many feet long is a field that requires 200 panels to fence one side, if the panels are 10 feet long?

18. How much tuition is paid in a year by 625 students, if each pays $200?

19. Two cattle dealers bought 1000 head of Texas steers at $35 a head: one took 600 head and the other 400 head; how much more must the first man pay than the second?

20. A man bought 400 acres of woodland at $80 an acre: he afterward sold the wood at $90 an acre and the land at $20 an acre; find his gain.

EXERCISE 22.

1. How many tens are there in 10 times 7?

SOLUTION. $10 \times 7 = 70 = 7$ tens.

2. How many hundreds are there in 100 times 6?

SOLUTION. $100 \times 6 = 600 = 6$ hundreds.

3. How many thousands are there in 1000 times 5?

SOLUTION. $1000 \times 5 = 5000 = 5$ thousands.

4. How many ten-thousands are there in 10000 times 4?

Solution. $10000 \times 4 = 40000 = 4$ ten-thousands.

From these solutions we see:

45. Prin. *Ten times any number equals that number of tens; one hundred times any number, that number of hundreds; one thousand times any number, that number of thousands; ten thousand times any number, that number of ten-thousands; etc.*

5. Multiply 426 by 345.

Form.

$$\begin{array}{r} 426 \\ 345 \\ \hline 2130 \\ 1704 \\ 1278 \\ \hline 146970 \end{array}$$

Solution. $345 \times 426 = 300 \times 426 + 40 \times 426 + 5 \times 426$. $5 \times 426 = 2130$, which we write so that the right-hand figure stands in units' place. $40 \times 426 = 4 \times 10 \times 426 = 4 \times 426$ tens, or 1704 tens, which we write so that the right-hand figure stands in tens' place. $300 \times 426 = 3 \times 100 \times 426 = 3 \times 426$ hundreds, or 1278 hundreds, which we write so that the right-hand figure stands in hundreds' place. The entire product equals the sum of 2130, 1704 tens, and 1278 hundreds, or 146970.

6. Multiply 375 by 408.

Form.

$$\begin{array}{r} 375 \\ 408 \\ \hline 3000 \\ 1500 \\ \hline 153000 \end{array}$$

Solution. $408 \times 375 = 400 \times 375 + 8 \times 375$. $8 \times 375 = 3000$. $400 \times 375 = 4 \times 100 \times 375$, or 4×375 hundreds, or 1500 hundreds. The entire product equals the sum of 3000 and 1500 hundreds, or 153000.

Note. To test the correctness of the result in multiplication:

See if the multiplicand times the multiplier produces the same result.

Multiply:

7. 46 by 23.
8. 84 by 56.
9. 79 by 63.
10. 98 by 18.
11. 49 by 86.
12. 432 by 21.
13. 385 by 43.
14. 806 by 72.
15. 689 by 49.
16. 846 by 77.
17. 1345 by 26.
18. 3742 by 38.
19. 4062 by 45.
20. 6006 by 58.
21. 8162 by 47.

Find the value of:

22. 342 × 416.
23. 218 × 512.
24. 609 × 348.
25. 567 × 406.
26. 304 × 509.
27. 642 × 743.
28. 128 × 309.
29. 408 × 501.
30. 816 × 719.
31. 489 × 982.
32. 776 × 1509
33. 888 × 2666.
34. 707 × 2707.
35. 3918 × 819.
36. 2999 × 999.

Multiply 4200 by 130.

Form.

$$\begin{array}{r|l} 42 & 00 \\ 13 & 0 \\ \hline 126 & \\ 42 & \\ \hline \multicolumn{2}{l}{546000} \end{array}$$

Solution. 130 × 4200 = 10 × 13 × 100 × 42, or 10 × 100 × 13 × 42, or 1000 × 546, or 546000.

From the above solution we see that the product of two numbers ending in ciphers may be obtained by the following:

46. Rule. *Find the product of the numbers without the ciphers, and annex to it as many ciphers as there are at the right of both numbers.*

Find the value of:

37. 50 × 160.
38. 30 × 240.
39. 70 × 200.
40. 140 × 800.
41. 240 × 300.
42. 160 × 390.
43. 1400 × 340.
44. 750 × 340.
45. 1500 × 880.
46. 990 × 900.
47. 80 × 8080.
48. 1900 × 456.

Find the value of 24 × 36 × 75.

Solution. 36 × 75 = 2700.
∴ 24 × 36 × 75 = 24 × 2700 = 64800.

Find the value of:

49. 48 × 63 × 90.
50. 23 × 46 × 39.
51. 8 × 124 × 36.
52. 200 × 38 × 19.
53. 140 × 250 × 60.
54. 250 × 80 × 324.
55. 198 × 99 × 100.
56. 910 × 900 × 120.

EXERCISE 23.

1. What do 86 acres of land cost at $125 an acre?

SOLUTION. $125 = the cost of 1 acre.
∴ 86 × $125, or $10,750 = " " " 86 acres.

2. How many bushels of wheat can be raised on 28 acres, if each acre yields 32 bushels?

SOLUTION. 32 = the number of bushels raised on 1 acre.
∴ 28 × 32, or 896 = " " " " " " 28 acres.

3. A bushel of wheat weighs 60 pounds; how many pounds do 320 bushels weigh?

4. If a steamboat runs 16 miles an hour, how far will it run in 24 hours, or one day?

5. An excursion train consisted of 18 cars, each carrying 85 passengers; how many passengers did it carry?

6. There are 5280 feet in a mile; how many feet are there in 28 miles?

7. If the distance around a bicycle wheel is 90 inches, over how many inches will the bicycle have gone when the wheel has turned 240 times?

8. There are 2240 pounds in a long ton; how many pounds are there in 48 long tons?

9. If sound travels 1090 feet a second, how far will it travel in 60 seconds?

10. A barrel of flour weighs 196 pounds; how much do 645 barrels weigh?

11. If 16 men can build a certain wall in 24 days, how long would it take one man to build it?

12. A man bought 25 strips of carpet each 16 yards long at 85 cents a yard; how many cents did the carpet cost?

13. If 36 men can do a piece of work in 45 days, how many men could do it in 1 day?

14. A drover bought 49 cows at $32 a head and sold them at $48 a head; how much did he gain?

SUG. $48 − $32, or $16 = the gain on each head.

15. One pipe discharges 60 gallons of water per hour, and another 34 gallons per hour; how much more will the first discharge than the second in 24 hours?

16. A man earned $24 and his son $20 in a week; how much did they together earn in 52 weeks, or 1 year?

17. Show by an arrangement of dots that 6 × 7 = 7 × 6.

18. If sound travels 1090 feet per second and you see the flash of a gun 6 seconds before you hear the sound, how far are you from the gun?

Division.

47. A number or an amount may be separated into equal parts and the value of one of these parts found.

Thus, 12 apples may be separated into *three* equal parts; one of these parts is found to be *four apples.*

48. **Division** is the process of finding one of the equal parts of a number or amount.

49. The **Dividend** is the number or amount to be divided into equal parts; the **Divisor** is the number which denotes into how many equal parts the dividend is to be divided; the **Quotient** is one of the equal parts into which the dividend is divided.

Thus, one of the 3 equal parts of 18 is 6. Here 18 is the *dividend*; 3, the *divisor*; and 6, the *quotient.*

50. If in division we have an undivided part of the dividend less than the divisor, this undivided part of the dividend is called **The Remainder.**

Thus, 14 pounds ÷ 3 = 12 pounds + 2 pounds ÷ 3 = 4 pounds, and 2 pounds remaining. 2 pounds is *the remainder.*

Note 1. When there is a remainder, one of the equal parts of the dividend so far as divided is still called the quotient, though it is not the complete quotient.

Thus, in the above illustration 4 pounds is the quotient.

Note 2. It is evident that the divisor is always a *number*, and that if the dividend is an amount, the quotient and the remainder, if there be any, are amounts *similar* to the dividend.

51. The symbol of division is ÷, read *divided by*.

Thus, 18 apples ÷ 6 is read *18 apples divided by 6*.

52. One of the *two* equal parts of a number or amount is *one half* of it; one of the *three* equal parts, *one third* of it; . . . one of the *twenty-one* equal parts, one *twenty-first* of it; one of the *twenty-two* equal parts, one *twenty-second* of it; and so on.

One half is expressed $\frac{1}{2}$; one third, $\frac{1}{3}$; one eleventh, $\frac{1}{11}$; one twenty-second, $\frac{1}{22}$; and so on.

53. It follows from § 48 and § 52 that:

$$10 \div 2 = \tfrac{1}{2} \text{ of } 10.$$
$$12 \div 3 = \tfrac{1}{3} \text{ of } 12.$$
$$22 \div 11 = \tfrac{1}{11} \text{ of } 22.$$
$$66 \text{ feet} \div 22 = \tfrac{1}{22} \text{ of } 66 \text{ feet}.$$

EXERCISE 24. ORAL.

1. Find one half of 10.

Solution. One half of 10 is 5, since 2 times 5 is 10.

2. Find $\frac{1}{3}$ of 17.

Solution. One third of 17 is 5 and 2 over, since 3 times 5, plus 2, equals 17.

3. Divide 25 dollars by 4; that is, find $\frac{1}{4}$ of 25 dollars.

Solution. One fourth of 25 dollars equals 6 dollars and one dollar over, since 4 times 6 dollars, plus 1 dollar, equals 25 dollars.

4. In the table on the following page, divide at sight each number in large type by the numbers in small type in the same square.

	2 · · · 2 · · · ·	2 3 · · 3 · · · ·	2 3 4 · 4 · · · ·	2 3 4 · 5 5 · · ·	2 3 4 · 6 5 · · 6	2 3 4 · 7 5 · 7 6	2 3 4 · 8 5 8 7 6
2 3 4 9 9 5 8 7 6	2 3 4 9 10 5 8 7 6	2 3 4 9 11 5 8 7 6	2 3 4 9 12 5 8 7 6	2 3 4 9 13 5 8 7 6	2 3 4 9 14 5 8 7 6	2 3 4 9 15 5 8 7 6	2 3 4 9 16 5 8 7 6
2 3 4 9 17 5 8 7 6	2 3 4 9 18 5 8 7 6	2 3 4 9 19 5 8 7 6	3 4 5 · 20 6 9 8 7	3 4 5 · 21 6 9 8 7	3 4 5 · 22 6 9 8 7	3 4 5 · 23 6 9 8 7	3 4 5 · 24 6 9 8 7
3 4 5 · 25 6 9 8 7	3 4 5 · 26 6 9 8 7	3 4 5 · 27 6 9 8 7	3 4 5 · 28 6 9 8 7	3 4 5 · 29 6 9 8 7	4 5 6 · 30 7 · 9 8	4 5 6 · 31 7 · 9 8	4 5 6 · 32 7 · 9 8
4 5 6 · 33 7 · 9 8	4 5 6 · 34 7 · 9 8	4 5 6 · 35 7 · 9 8	4 5 6 · 36 7 · 9 8	4 5 6 · 37 7 · 9 8	4 5 6 · 38 7 · 9 8	4 5 6 · 39 7 · 9 8	5 6 7 · 40 8 · · 9
5 6 7 · 41 8 · · 9	5 6 7 · 42 8 · · 9	5 6 7 · 43 8 · · 9	5 6 7 · 44 8 · · 9	5 6 7 · 45 8 · · 9	5 6 7 · 46 8 · · 9	5 6 7 · 47 8 · · 9	5 6 7 · 48 8 · · 9
5 6 7 · 49 8 · · 9	6 7 8 · 50 9 · · ·	6 7 8 · 51 9 · · ·	6 7 8 · 52 9 · · ·	6 7 8 · 53 9 · · ·	6 7 8 · 54 9 · · ·	6 7 8 · 55 9 · · ·	6 7 8 · 56 9 · · ·
6 7 8 · 57 9 · · ·	6 7 8 · 58 9 · · ·	6 7 8 · 59 9 · · ·	7 8 9 · 60 · · · ·	7 8 9 · 61 · · · ·	7 8 9 · 62 · · · ·	7 8 9 · 63 · · · ·	7 8 9 · 64 · · · ·
7 8 9 · 65 · · · ·	7 8 9 · 66 · · · ·	7 8 9 · 67 · · · ·	7 8 9 · 68 · · · ·	7 8 9 · 69 · · · ·	8 9 · · 70 · · · ·	8 9 · · 71 · · · ·	8 9 · · 72 · · · ·
8 9 · · 73 · · · ·	8 9 · · 74 · · · ·	8 9 · · 75 · · · ·	8 9 · · 76 · · · ·	8 9 · · 77 · · · ·	8 9 · · 78 · · · ·	8 9 · · 79 · · · ·	9 · · · 80 · · · ·
9 · · · 81 · · · ·	9 · · · 82 · · · ·	9 · · · 83 · · · ·	9 · · · 84 · · · ·	9 · · · 85 · · · ·	9 · · · 86 · · · ·	9 · · · 87 · · · ·	9 · · · 88 · · · ·
9 · · · 89 · · · ·							

EXERCISE 25. ORAL.

1. If 5 oranges cost 25 cents, how much is that apiece?

SOLUTION. If 25 cents is the cost of 5 oranges, $\frac{1}{5}$ of 25 cents, or 5 cents, is the cost of each orange.

2. If 42 apples are placed in 6 rows, each containing the same number of apples, how many are there in each row?

SOLUTION. If 42 is the number of apples in 6 rows, $\frac{1}{6}$ of 42, or 7, is the number in each row.

3. If one man can do a piece of work in 28 days, in what time can 7 men do it?

SOLUTION. If 28 days is the time in which 1 man can do the work, $\frac{1}{7}$ of 28 days, or 4 days, is the time in which 7 men can do it.

4. If 6 pounds of sugar cost 36 cents, how much does 1 pound cost?

5. A father gave a Christmas present of $40 to his 8 children, giving each the same sum; how much did each child receive?

6. How much is paid per day to a workman who receives $24 for six days' work?

7. A dealer sold 4 pounds of butter for 80 cents; how much was that a pound?

8. 60 chairs are to be arranged in 12 rows of equal length; how many must be put in a row?

9. How many pounds of sugar at 6 cents a pound should a lady receive for 36 cents?

10. 4 thousand feet of lumber cost $96; how much was that a thousand feet?

11. A dealer buys milk at 3 cents a quart and retails it at 5 cents a quart; if his profit on a can is 36 cents, how many quarts does the can contain?

12. A lady exchanged 6 dozen of eggs at 20 cents a dozen for gingham at 12 cents a yard; how many yards did she receive?

13. A stationer buys pencils at 36 cents for 12, and retails them for 5 cents each; how much does he gain on each pencil?

14. If 8 yards of cloth cost 56 cents, how much will 7 yards cost at the same price per yard?

SOLUTION. If 56 cents is the cost of 8 yards, $\frac{1}{8}$ of 56 cents, or 7 cents, is the cost of one yard; and 7 times 7 cents, or 49 cents, will be the cost of 7 yards.

15. If 8 pair of shoes cost $16, what will 12 pair cost at the same price per pair?

16. If 6 oranges are worth 30 cents, how much more are 9 oranges worth?

17. If 6 pounds of butter are worth as much as 12 dozen of eggs, how many dozen of eggs are worth as much as 9 pounds of butter?

18. If 8 sheep cost $48, how much less will 5 sheep cost at the same price per head?

19. A bought 8 sheep for $40 and sold them at a profit of $1 a head; find the selling price of each sheep.

20. A dealer bought a box containing 3 dozen of oranges at 20 cents a dozen; if he sells one dozen for 40 cents, how much must he receive for each dozen remaining in order that he may gain 30 cents?

EXERCISE 26.

1. Divide 567 by 6; that is, find $\frac{1}{6}$ of 567.

FORM.

$$\begin{array}{r|l} 6\,) & \underline{567} \\ & 94^{3} \end{array}$$

SOLUTION. $\frac{1}{6}$ of 567 is $\frac{1}{6}$ of 56 tens plus $\frac{1}{6}$ of 7 units. $\frac{1}{6}$ of 56 tens is 9 tens and 2 tens, or 20 units, over. We write 9 tens in tens' place, and add the 20 units to the 7 units, making 27 units. $\frac{1}{6}$ of 27 units is 4 units, and 3 units over. We write the 4 units in units' place, and the 3 units remaining, as indicated in the form. The quotient is 94^{3}, read *94 and 3 over.*

Divide:

2. 498 by 3.	**5.** 372 by 6.	**8.** 747 by 9.
3. 596 by 4.	**6.** 875 by 7.	**9.** 1436 by 2.
4. 625 by 5.	**7.** 864 by 8.	**10.** 3471 by 3.

11. 6568 by 4.
12. 4985 by 5.
13. 5364 by 6.
14. 8128 by 8.
15. 6543 by 9.
16. 12475 by 7.
17. 25863 by 5.
18. 42781 by 6.
19. 15398 by 8.
20. 82749 by 4.
21. 52750 by 3.
22. 39563 by 9.

Find:

23. ½ of 6784.
24. ⅓ of 89765.
25. ¼ of 74896.
26. ⅕ of 8750.
27. ⅙ of 9879.
28. ⅐ of 6874.
29. ⅛ of 10010.
30. ⅑ of 10101.
31. ⅑ of 22222.

Find the value of:

32. 675 ÷ 2.
33. 8973 ÷ 3.
34. 7421 ÷ 4.
35. 10105 ÷ 5.
36. 601010 ÷ 6.
37. 170010 ÷ 7.
38. 11800 ÷ 8.
39. 100001 ÷ 9.
40. 202020 ÷ 9.

EXERCISE 27.

1. If 6 barrels of flour weigh 1176 pounds, what is the weight of 1 barrel?

Solution. 1176 pounds = the weight of 6 barrels.
∴ ⅙ of 1176 pounds, or 196 pounds = the weight of 1 barrel.

2. If a train ran a distance of 448 miles in 8 hours, how many miles did it run per hour?

Solution. 448 = the number of miles the train ran in 8 hours.
∴ ⅛ of 448, or 56 = " " " " " " " " 1 hour.

3. If in 9 weeks a man earns $315, how much does he earn per week?

4. If 6 bushels of wheat weigh 360 pounds, how many pounds does a bushel weigh?

5. $2764 was divided equally among 4 children; how much money did each receive?

6. The report of a gun 3270 feet distant was heard 3 seconds after it was fired; how far did sound travel per second at that time?

7. From the first to the tenth of a certain month a laborer working every day earned \$27; how much was that per day?

Note. The **Average** of two or more amounts is the quotient obtained by dividing their sum by the number of amounts.

Thus, if one of 4 sheep cost \$4, another \$6, and the others \$5 each, the average cost of them is the sum of \$4, \$6, \$5, and \$5, divided by 4, or \$5.

8. The sum of the lengths of 4 logs is 96 feet; what is the average length of the logs?

9. The sum of the weights of 5 persons is 655 pounds; what is their average weight?

10. Seven cars carried 6307 bushels of wheat; what was the average amount carried on a car?

11. A farmer hauled 4 loads of hay weighing respectively 1450 pounds, 1530 pounds, 1348 pounds, and 1456 pounds; what was the average weight per load?

12. A drover bought eight horses, paying for them respectively \$132, \$148, \$208, \$196, \$88, \$116, \$192, and \$96; what was the average price paid per head?

13. A bought 2 cows at \$55 a head and 3 at \$45 a head and sold them all at \$56 a head; what was the average gain per head?

14. A farmer had 3 acres of wheat that yielded 32 bushels an acre, 4 acres that yielded 30 bushels an acre, and 5 acres that yielded 24 bushels an acre; what was the average yield per acre?

EXERCISE 28. ORAL.

1. Divide 52 by 13; that is, find $\frac{1}{13}$ of 52.

SOLUTION. $\frac{1}{13}$ of 52 is 4, since 13 times 4 is 52.

Divide at sight:

2. 66 by 11; by 22; by 33.
3. 72 by 12; by 18; by 24; by 36.
4. 78 by 13; by 26; by 39.
5. 80 by 16; by 20; by 40.
6. 84 by 12; by 21; by 42.

State at sight the quotient in each of the following:

7. 100 divided by 20; by 25; by 50.
8. 104 divided by 13; by 26; by 52.
9. 105 divided by 15; by 21; by 35.
10. 114 divided by 19; by 38; by 57.
11. 174 divided by 29; by 58; by 87.
12. 225 divided by 25; by 45; by 75.
13. 210 divided by 30; by 35; by 42; by 70.
14. 504 divided by 56; by 63; by 72; by 84.
15. 252 divided by 28; by 36; by 42; by 63; by 84.
16. 288 divided by 32; by 36; by 48; by 72; by 96.
17. 336 divided by 42; by 48; by 56; by 84.
18. 168 divided by 21; by 24; by 28; by 42; by 56; by 84.
19. 792 divided by 99; by 88. 882 divided by 98.
20. 616 divided by 88; by 77. 855 divided by 95.
21. 981 ÷ 109; 666 ÷ 74; 639 ÷ 71; 776 ÷ 97.

EXERCISE 29.

1. Divide 78 by 23.

FORM.

23) 78 ($3^{\underline{9}}$
69
9

SOLUTION. $\frac{1}{23}$ of 78 is more than 3 and less than 4. 23 × 3 = 69, which taken from 78 leaves 9. Therefore, 78 ÷ 23 is 3, with 9 remaining.

Divide:

2. 167 by 33.
3. 176 by 55.
4. 195 by 38.
5. 546 by 94.
6. 528 by 88.
7. 547 by 60.
8. 499 by 95.
9. 759 by 84.
10. 801 by 84.
11. 987 by 98.
12. 800 by 87.
13. 709 by 89.
14. 900 by 97.
15. 795 by 88.
16. 601 by 79.

17. Divide 1508 by 26.

FORM.

26) 1508 (58
130
208
208

SOLUTION. 1508 = 150 tens + 8 units. $\frac{1}{26}$ of 150 tens is more than 5 tens and less than 6 tens. 26 × 5 tens = 130 tens, which taken from 150 tens leaves 20 tens, or 200 units, which added to the 8 units makes 208 units. $\frac{1}{26}$ of 208 units is 8 units. Taking 26 × 8 units from 208 units, nothing remains. Therefore, 1508 ÷ 26 is 5 tens and 8 units, or 58.

Remark. This method of dividing by expressing the successive partial dividends is called **Long Division.** The method of dividing without expressing the partial dividends is called **Short Division.** [See solution to problem 1, p. 42].

Divide:

18. 375 by 15.
19. 384 by 16.
20. 483 by 21.
21. 575 by 25.
22. 308 by 22.

23. 600 by 25.
24. 2176 by 32.
25. 4462 by 46.
26. 4984 by 56.
27. 5073 by 89.

28. 1005 by 67.
29. 1071 by 51.
30. 4012 by 59.
31. 9801 by 99.
32. 7569 by 87.

33. Divide 9792 by 32.

FORM.

```
32 ) 9792 ( 306
     96
     ---
      192
      192
      ---
```

SOLUTION. 9792 = 97 hun. + 9 tens + 2 units. $\frac{1}{32}$ of 97 hun. is more than 3 hun. and less than 4 hun. 32 × 3 hun. is 96 hun., which taken from 97 hun. leaves 1 hun., or 10 tens. 9 tens added to 10 tens makes 19 tens. $\frac{1}{32}$ of 19 tens is *no* tens. 19 tens equals 190 units, which added to 2 units makes 192 units. $\frac{1}{32}$ of 192 units is 6 units. Therefore, 9792 ÷ 32 = 3 hun. 0 tens 6 units, or 306.

Divide:

34. 2968 by 28.
35. 3456 by 32.
36. 8944 by 43.
37. 16686 by 54.
38. 27202 by 67.
39. 35956 by 89.

40. 66552 by 94.
41. 79282 by 98.
42. 70928 by 88.
43. 18240 by 24.
44. 18750 by 25.
45. 24750 by 25.

46. 57429 by 81.
47. 15436 by 17.
48. 47795 by 79.
49. 59073 by 97.
50. 27646 by 46.
51. 89991 by 99.

52. Divide 34968 by 215.

FORM.

```
215)34968(162 138
    215
    ----
    1346
    1290
    ----
      568
      430
      ---
      138
```

SOLUTION. 34968 = 349 h. + 6 t. + 8 u. $\frac{1}{215}$ of 349 h. = 1 h., and 134 h., or 1340 t., over. 1340 t. + 6 t. = 1346 t. $\frac{1}{215}$ of 1346 t. = 6 t., and 56 t., or 560 u., over. 560 u. + 8 u. = 568 u. $\frac{1}{215}$ of 568 u. = 2 u., and 138 u. over. Therefore, 34968 ÷ 215 = 162^{138}.

Note. To test the correctness of the result in division:

See if the divisor times the quotient, plus the remainder (if there be any), equals the dividend.

Divide:

53. 2160 by 135.
54. 6560 by 205.
55. 8190 by 182.
56. 13446 by 249.
~~**57.** 12144 by 276.~~
~~**58.** 19432 by 347.~~
59. 12144 by 276.
60. 19432 by 347.
61. 12208 by 436.
62. 24576 by 512.
63. 33228 by 639.
64. 13804 by 128.
65. 50504 by 214.
66. 44103 by 723.
67. 64640 by 808.
68. 60905 by 937.
69. 236418 by 327.
70. 496381 by 298.
71. 748000 by 845.
72. 100101 by 891.

EXERCISE 30.

1. Divide 3245 by 10; by 100; by 1000.

FORMS.

$$10\,)\,\underline{324|5}$$
$$324^{\underline{5}}$$

SOLUTION 1. $3245 = 324$ t. $+ 5$. $\frac{1}{10}$ of 324 t. $= 324$. Therefore, $\frac{1}{10}$ of $3245 = 324^{\underline{5}}$.

$$100\,)\,\underline{32|45}$$
$$32^{\underline{45}}$$

SOLUTION 2. $3245 = 32$ h. $+ 45$. $\frac{1}{100}$ of 32 h. $= 32$. Therefore, $\frac{1}{100}$ of $3245 = 32^{\underline{45}}$.

$$1000\,)\,\underline{3|245}$$
$$3^{\underline{245}}$$

SOLUTION 3. $3245 = 3$ thou. $+ 245$. $\frac{1}{1000}$ of 3 thou. $= 3$. Therefore, $\frac{1}{1000}$ of $3245 = 3^{\underline{245}}$.

From these solutions we have the following:

54. PRINCIPLES. *Cutting off the last figure of a number divides the number by 10, and the figure cut off expresses the remainder.*

Cutting off the last two figures of a number divides it by 100, and the figures cut off express the remainder; and so on.

2. Divide 14375 by 1600.

Form.

$$\begin{array}{l} 16|00\,)\,143|75\,(\,8^{\underline{1575}} \\ \qquad\quad\ \underline{128} \\ \qquad\quad\ \ 1575 \end{array}$$

Solution. 14375 ÷ 1600 = 14375 ÷ (100 × 16). 14375 ÷ 100 = 143, with 75 over. 143 ÷ 16 = 8 and 15 of the 143 over, or with 1500 of the 14300 over. Therefore, the quotient is 8 and the whole remainder 1500 + 75 or 1575.

Divide:

3. 320 by 10.
4. 460 by 20.
5. 540 by 30.
6. 640 by 40.
7. 650 by 50.
8. 840 by 70.
9. 960 by 80.
10. 990 by 90.
11. 875 by 30.
12. 938 by 60.
13. 1063 by 70.
14. 2466 by 80.
15. 3178 by 90.
16. 4284 by 50.
17. 3400 by 100.
18. 5600 by 200.
19. 8400 by 300.
20. 8400 by 400.
21. 9500 by 500.
22. 7850 by 600.
23. 8460 by 700.
24. 6942 by 800.
25. 9846 by 900.
26. 12480 by 120.
27. 14400 by 240.
28. 24600 by 360.
29. 42562 by 420.
30. 56843 by 560.
31. 18000 by 1000.
32. 46000 by 2000.
33. 18000 by 3000.
34. 56000 by 2000.
35. 72000 by 3000.
36. 56000 by 4000.
37. 75000 by 5000.
38. 108000 by 6000.
39. 21500 by 7000.
40. 36800 by 8000.
41. 52420 by 9000.
42. 37500 by 1200.
43. 54640 by 3250.
44. 98400 by 13000.
45. 67543 by 24000.
46. 34876 by 16500.
47. 40101 by 20400.

EXERCISE 31.

1. If 32 acres of land are worth $4096, what is the average value per acre?

Solution. $4096 = the value of 32 acres.
∴ $\frac{1}{32}$ of $4096, or $128 = the value per acre.

2. If 45 iron beams of equal weight weigh 16200 pounds, how many pounds does each beam weigh?

3. If 48 cows are worth $2688, what is the average value per cow?

4. A miller put 3400 pounds of flour into 136 bags, putting the same quantity into every bag; how many pounds did he put into each bag?

5. If the sum of the ages of 244 men is 7320 years, what is the average age of the men?

6. It was estimated that it would cost $38400 to pave 320 rods; what was the average sum allowed per rod?

7. If 84 town lots are valued at $44184, what is the average value per lot?

8. A salesman agreed to work for 12 months for $660, but he left at the end of 5 months; how much had he earned?

9. The population of a certain town was 8790 in 1890 and 9860 in 1900; what was the average gain per year for the 10 years?

10. If one man can do a piece of work in 270 days, how long would it take 45 men to do it?

11. A drover bought 38 cows for $2584 and sold them for $2736; what was the average gain per head?

12. A passenger train and a freight train started from the same place at the same time and ran in the same direction; at the end of 24 hours the former had gone 576 miles and the latter 360 miles; what was the average gain per hour of the passenger train over the freight train?

13. Find to the nearest acre the average size of the farms in the states named in the following table:

States.	Total Number of Farms.	Acres in Farms.	Average Acres in a Farm.
New York	226,223	21,961,562	
Pennsylvania	211,557	18,364,370	
New Jersey	30,828	2,662,009	
Delaware	9,381	1,055,692	
Maryland	40,798	4,952,390	

Mensuration.

55. One number or amount may contain another number or amount a number of times as a *unit of measure.*

Thus, 18 contains 6 *three* times as a *unit of measure,* since 3 × 6 = 18.

Also, 18 inches contains 6 inches *three* times as a *unit of measure,* since 3 × 6 inches = 18 inches.

56. The process of finding how many times one number or amount contains another number or amount as a unit of measure, is called **Mensuration.**

57. The number or amount measured is the **Antecedent**; the number or amount taken as the unit of measure is the **Consequent**; and the result of the measurement is the **Ratio.**

Thus, 24 cents measured by 6 cents is 4, since 24 cents contains 6 cents 4 times as a unit of measure. Here 24 cents is the *antecedent;* 6 cents, the *consequent;* and 4, the *ratio.*

Note. It is evident that the antecedent and the consequent, when amounts, must be similar; and that the ratio is always a number, never an amount.

58. PRIN. *One amount measured by another equals the number of units in the first measured by the number of units in the second.*

Thus, 15 pounds contains 3 pounds as a unit of measure, as many times as 15 contains 3.

Note. The number of units in an amount is called the **Numerical Value** of the amount.

59. The symbol of mensuration is : , read *measured by.*

Thus, 16 : 2 = 8 is read *16 measured by 2 equals 8.*

60. It is evident that 12 contains 1 as a unit of measure 12 times; hence, 12 contains 4 as a unit of measure $\frac{1}{4}$ of 12 times. Therefore, $12 : 4 = \frac{1}{4}$ of $12 = 12 \div 4$. Therefore,

PRIN. *One number measured by another equals the first divided by the second*, or,

The ratio of two numbers equals the antecedent divided by the consequent.

EXERCISE 32. ORAL.

1. Measure 18 by 3.

SOLUTION. 18 measured by 3 equals $\frac{1}{3}$ of 18, or 6 [§ 60, Prin.].

2. Measure 42 dollars by 7 dollars.

SOLUTION. 42 dollars measured by 7 dollars equals 42 measured by 7 [§ 58], which equals $\frac{1}{7}$ of 42, or 6.

Measure at sight:

3. 36 by 4.
4. 45 by 5.
5. 72 by 6.
6. 63 by 7.
7. 64 by 8.
8. 81 by 9.
9. 80 by 10.
10. 88 by 11.
11. 96 by 12.
12. 104 by 13.
13. 70 by 14.
14. 69 by 23.
15. 132 by 24.
16. 225 by 45.
17. 168 by 56.
18. 260 by 65.
19. 308 by 77.
20. 516 by 86.
21. 630 by 90.
22. 490 by 98.
23. 28 gallons by 7 gallons.
24. 36 men by 6 men.
25. 84 cents by 12 cents.
26. 88 miles by 22 miles.
27. 160 yards by 32 yards.
28. 258 bushels by 43 bushels.
29. 378 inches by 54 inches.
30. 248 hours by 62 hours.
31. 456 days by 76 days.
32. 258 dollars by 83 dollars.

Find the value of:

33. 126 : 42.
34. 399 : 57.
35. 384 : 64.
36. 568 : 71.
37. 255 : 85.
38. 542 : 92.
39. 408 : 102.
40. 750 : 125.
41. 288 men : 36 men.
42. 329 cows : 47 cows.
43. 498 acres : 83 acres.
44. 592 pounds : 97 pounds.

EXERCISE 33. ORAL.

1. When sugar costs 6 cents a pound, how many pounds can be bought for 42 cents?

SOLUTION. If one pound of sugar costs 6 cents, for 42 cents as many pounds can be bought as 6 cents is contained times in 42 cents, or 7.

2. There are 3 feet in 1 yard; how many yards are there in 12 feet?

3. Eight pints make one gallon; how many gallons in 56 pints?

4. A ball of twine containing 120 yards is cut into fishing lines each 12 yards long; how many lines are made?

5. How many crates does a market woman require to pack 42 dozen of eggs, if she puts 6 dozen in a crate?

6. How many melons at 24 cents apiece will 96 cents buy?

7. In how many hours will a boat sail 90 miles, if it sails 18 miles an hour?

8. There are 20 in a score; how many score are there in 120?

9. A bushel of wheat weighs 60 pounds; how many bushels weigh 240 pounds?

10. How long will it take a man walking 17 miles a day to go from Lancaster to Philadelphia, a distance of 68 miles?

11. How many trips must a farmer make to haul away 600 bushels of wheat, if he hauls 50 bushels at a load?

12. How many times can a 3-gallon pail be filled from a hogshead which contains 63 gallons?

13. In a certain drawing 12 inches represent a mile; how many miles are represented in this drawing by a line 36 inches long?

14. At an average price of $15 a set, how many sets of books can be bought for $150?

15. If 6 pounds of sugar cost 36 cents, how much sugar will 48 cents buy?

SOLUTION. If 6 pounds cost 36 cents, $\frac{1}{6}$ of 36 cents, or 6 cents, will buy 1 pound; and 48 cents, or 8 times 6 cents, will buy 8 times 1 pound, or 8 pounds.

16. If in 6 days a mechanic earns $24, in how many days will he earn $56?

17. If 3 oranges are worth 5 cents, how many oranges will 10 times 5 cents, or 50 cents, buy?

18. A customer exchanged eggs at 12 cents a dozen for store goods; the eggs exchanged amounted to 72 cents, and the goods purchased, to 96 cents; how many more dozen of eggs would have paid the bill?

19. If 2 boxes of matches can be bought for 5 cents, how many boxes can be bought for 25 cents?

20. If plums are sold at 2 quarts for 25 cents, how many quarts can be bought for 75 cents?

21. In 4 rods there are 22 yards; how many yards long is a line that measures 20 rods?

EXERCISE 34.

1. Measure 868 by 7.

SOLUTION. 868 : 7 = $\frac{1}{7}$ of 868 [§ 60, Prin.], or 124.

2. Measure 13104 bushels by 56 bushels.

SOLUTION. 13104 bu. : 56 bu. = 13104 : 56 [§ 58] = $\frac{1}{56}$ of 13104, or 234.

3. Measure 74915 yards by 240 yards.

SOLUTION. 74915 yd. : 240 yd. = 74915 : 240 =
$\frac{1}{240}$ of 74915 = $312^{\underline{35}}$.

74915 yd. : 240 yd. = $312^{\underline{35\text{ yd.}}}$, read *312, and 35 yd. remaining.*

Measure:

4. 1308 by 6.
5. 1635 by 5.
6. 2842 by 7.
7. 5104 by 8.
8. 2047 by 23.
9. 28946 by 41.
10. 36000 by 30.
11. 110484 by 54.
12. 42000 by 6000.

13. 3080 bu. : 8 bu.
14. 7200 lb. : 10 lb.
15. $1728 : $12.
16. $68530 : $22.
17. 38110 qt. : 185 qt.
18. $68650 : $170.
19. 66054 min. : 327 min.
20. 87264 gal. : 808 gal.
21. 73225 rd. : 320 rd.
22. 74836 sec. : 3600 sec.

EXERCISE 35.

1. A man invested $11220 in land at $85 an acre; how many acres did he buy?

SOLUTION. $11220 = the amount invested in land.
$85 = the cost of 1 acre.
∴ $11220 : $85, or 11220 : 85, or 132 = the number of acres bought.

2. A farmer sold 13328 pounds of corn; how many bushels did he sell, if one bushel weighs 56 pounds?

3. A mail carrier drives 25 miles a day; in how many days will he drive 5000 miles?

4. The distance around a wheel is 3 feet; how many times will it turn in going 5280 feet, or one mile?

5. A miller shipped 9800 pounds of flour in barrels; how many barrels did he ship, if one barrel contains 196 pounds?

6. If a steamer goes on an average 25 miles an hour, how many days will it take to go 4200 miles, running 24 hours a day?

7. A house that rents for $15 a month cost $3600; in how many years will the rent amount to the cost of the house?

8. A drover bought young mules at an average of $85 a head and sold them at $100 a head, thereby gaining $900; how many mules did he buy and sell?

9. A contractor built a row of houses on which he gained $6048; this was an average gain of $216 a house; how many houses were there in the row?

10. The average age of the citizens above 70 years old in a certain city is 76 years, and the sum of their ages is 3572 years; how many of these citizens are there?

11. Mr. Jones bought a drove of cattle for $8170 and sold them for $10750, which was an average gain of $12 a head; how many cattle were there in the drove?

12. A dealer bought a number of tons of coal, receiving 2240 pounds for a ton; he sold the same number of tons, giving 2000 pounds for a ton, and had 2400 pounds over; how many tons did he buy?

13. A farm that cost $50 an acre was sold for $35 an acre, which made the loss on the entire farm $900; how many acres were there in the farm?

14. A farmer mixed an equal number of bushels of corn and oats for horse feed, and found that the mixture weighed 1760 pounds; how many bushels of feed had he, if a bushel of corn weighs 56 pounds and a bushel of oats 32 pounds?

SUG. ½ of (56 + 32) lb., or 44 lb. = the weight of 1 bu. of feed.

15. A laborer who saves $20 a month must labor how many years to pay for 60 acres of land at $40 an acre?

Arithmetical Expressions.

61. An **Arithmetical Expression**, or simply an **Expression**, is any number or combination of numbers expressed in arithmetical symbols.

Thus, 12, $6 + 4 + 3$, $8 - 2 + 4$, $2 \times 3 \times 5$, and $8 \times 4 \div 2$ are *expressions*.

62. The parts of an expression connected by + or − are called the **Terms** of the expression.

Thus, in the expression $8 - 6 + 3 \times 4$, 8, 6, and 3×4 are the *terms*.

63. The operations of multiplication and division indicated in any term of an expression must be performed before the terms are combined; and if two or more of

such operations are indicated in the same term, they must be performed in the order of their occurrence from left to right.

Thus, the expression, 3 × 4 ÷ 2 + 36 ÷ 3 × 4 − 24 ÷ 6 ÷ 2, has three terms.

The first term 3 × 4 ÷ 2 = 12 ÷ 2, or 6.
The second term 36 ÷ 3 × 4 = 12 × 4, or 48.
The third term 24 ÷ 6 ÷ 2 = 4 ÷ 2, or 2.
The whole expression = 6 + 48 − 2, or 52.

64. The **Value** of an expression is the result obtained by performing the operations indicated.

Thus, the value of the expression 3 × 4 ÷ 2 + 36 ÷ 3 × 4 − 24 ÷ 6 ÷ 2 is 52.

65. Two expressions are equal when they have the same value.

Thus, 8 × 5 + 6 = 30 + 16, since the value of each expression is 46.

66. The **Parenthesis** ( ) indicates that the value of the expression enclosed by it is to be found and used instead of the expression as a whole.

Thus, 8 × (6 − 2) denotes that the difference of 6 and 2, or 4, is to be multiplied by 8.

Note. The brackets [ ], the braces { }, and the vinculum ——, may be used instead of the parenthesis.

EXERCISE 36.

1. Find the value of 6 × 8 ÷ 6 + 3 × (8 − 7 + 5) − (18 − 6) ÷ 3.

SOLUTION. 6 × 8 ÷ 6 = 48 ÷ 6 = 8;
3 × (8 − 7 + 5) = 3 × 6 = 18;
(18 − 6) ÷ 3 = 12 ÷ 3 = 4;
∴ 6 × 8 ÷ 6 + 3 × (8 − 7 + 5) − (18 − 6) ÷ 3 = 8 + 18 − 4 = 22.

Find the value of:

2. (24 + 36 − 15) − (26 − 13 + 8).
3. ($48 + $36 − $15) − 3 × ($18 − $5 − $8).
4. 116 × 4 ÷ 8 − 84 ÷ 12 × 3 + 63 ÷ 9 × 7.
5. 39 ÷ 13 × 5 + 56 × 7 ÷ 8 − 144 ÷ 36 × 5.
6. $4 \times \overline{125 + 250} - 6 \times (250 - 125) + \overline{500 - 250} \div 25$.
7. (320 − 160) ÷ 8 − (324 − 248) ÷ 4 + (5 × 720 − 640).
8. 375 × 40 ÷ 50 + 1200 ÷ 60 × 40 − (1000 − 600).
9. 1760 yd. ÷ 40 − 3 × 1200 yd. ÷ 60 + 4 × 1500 yd. ÷ 30.
10. $\overline{\$180 + \$120} \div 60 - (\$192 - \$168) \div 24 - \$75 \div 25$.
11. 12 × (350 − 280) ÷ 6 − (540 − 480) ÷ 15 + (412 − 216) × (24 − 18).
12. (36 + 48) × (48 − 36) − (120 − 84) ÷ (18 − 6) + (140 − 70) ÷ (70 − 35).

Factors, Exact Divisors, and Exact Measures.

67. The **Factors** of a number are such numbers as multiplied together will produce the number.

Thus, the factors of 30 are 2, 3, and 5, since 2 × 3 × 5 = 30. Other factors of 30 are 10 and 3, 6 and 5, 2 and 15.

68. A number composed of no other factors than itself and *one* is a **Prime Number**; as, 7 = 7 × 1.

69. A number composed of other factors than itself and *one* is a **Composite Number**; as, 12 = 3 × 4.

70. The same factor may be used more than once in forming a composite number. Each time it is so used it is considered a separate factor.

Thus, since 2 × 2 × 3 × 3 × 3 = 108, the factors of 108 are 2, 2, 3, 3, and 3.

71. When the factors of a number are prime numbers, they are called the **Prime Factors** of the number.

Thus, 2, 2, 3, 3, and 3 are the prime factors of 108.

72. A number that divides a given number without a remainder is an **Exact Divisor**, or simply a **Divisor**, of the given number.

Thus, 6 is a divisor of 24, since $\frac{1}{6}$ of 24 = 4, with no remainder.

73. A prime number that divides a given number without a remainder is a **Prime Divisor** of the number.

Thus, 7 is a prime divisor of 14.

74. An **Exact Measure**, or simply a **Measure**, of a given number or amount is a number or amount that is contained in it a whole number of times without a remainder.

Thus, 5 is an exact measure of 35, since 5 is contained in 35 7 times without a remainder.

75. Since one number measured by another equals the former divided by the latter [§ 60], it follows that:

PRIN. *A divisor of a number is also a measure of the number.*

76. An **Even Number** is a number that is divisible by 2, and an **Odd Number**, one that is not divisible by 2.

Thus, the numbers 1, 2, 3, 4, 5, 6, etc., are alternately odd and even.

PRINCIPLES OF DIVISIBILITY.

77. The following principles will be found useful in factoring numbers:

78. *A number is divisible by 2 or 5 if the number expressed by the units' figure is divisible by 2 or 5.*

Thus, 136 is divisible by 2, since 6 is divisible by 2; and 235 is divisible by 5, since 5 is divisible by 5.

Note. *Zero* is regarded divisible by any number, and the quotient is always *zero.*

79. *A number is divisible by 3 or 9 if the number expressed by the sum of its digits is divisible by 3 or 9.*

Thus, 2346 is divisible by 3, since 2 + 3 + 4 + 6, or 15, is divisible by 3; and 4671 is divisible by 9, since 4 + 6 + 7 + 1, or 18, is divisible by 9.

80. *A number is divisible by 4 or 25 if the number expressed by the two right-hand digits is divisible by 4 or 25.*

Thus, 1424 is divisible by 4, since 24 is divisible by 4; and 1575 is divisible by 25, since 75 is divisible by 25.

EXERCISE 37. ORAL.

Name at sight:

1. The two factors of 4, 6, 9, 10, 14, 15, 21, 22, 25, 33, 35, 49, 55, 77, 121.

2. The three factors of 8, 12, 18, 20, 28, 30, 42, 50, 66, 70, 75.

3. The exact divisors of 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 26, 27, 29.

4. The exact measures of 6 men, 9 horses, 10 inches, 15 pounds, 28 miles, 39 years, 45 bushels, 48 cows, 56 toys.

5. The prime factors of 12, 18, 21, 24, 25, 27, 28, 30, 33, 34, 35, 38, 39, 40.

Tell at sight which of the numbers, 2, 3, 4, 5, 9, and 25 will divide:

6. 78.

7. 75.

8. 146.

9. 230.

10. 324.

11. 425.

12. 423.

13. 552.

14. 627.

15. 1359.

16. 2495.

17. 6318.

18. 7423.

19. 5700.

20. 10008.

81. If a number is divided by one of its prime divisors, the quotient by one of its prime divisors, the next quotient by one of its prime divisors, and so on until the quotient becomes 1, these successive divisors are called the **Continued Prime Divisors** of the number.

$$\begin{array}{r|r} 3 & 105 \\ \cline{2-2} 5 & 35 \\ \cline{2-2} 7 & 7 \\ \cline{2-2} & 1 \end{array}$$

Thus, 3, 5, and 7 are the continued prime divisors of 105.

82. Prin. *The continued prime divisors of a number are the prime factors of the number.*

$$\begin{array}{r|r} 3 & 105 \\ \cline{2-2} 5 & 35 \\ \cline{2-2} 7 & 7 \\ \cline{2-2} & 1 \end{array}$$

Thus, 3, 5, and 7, the continued prime divisors of 105, are the prime factors of 105.

For, $105 = 3 \times 35$; and $35 = 5 \times 7$.

$\therefore 105 = 3 \times 5 \times 7$.

EXERCISE 38.

1. Find the prime factors of 350.

Form.

$$\begin{array}{r|r} 2 & 350 \\ \cline{2-2} 5 & 175 \\ \cline{2-2} 5 & 35 \\ \cline{2-2} 7 & 7 \\ \cline{2-2} & 1 \end{array}$$

Solution. The continued prime divisors of 350 are 2, 5, 5, and 7.

∴ 2, 5, 5, and 7 are the prime factors of 350 [§ 82].

Find the prime factors of:

2. 30.
3. 54.
4. 72.
5. 60.
6. 75.
7. 90.
8. 110.
9. 156.
10. 280.
11. 182.
12. 735.
13. 765.
14. 630.
15. 1001.
16. 3150.

Common Divisors and Common Measures.

83. A **Common Divisor** of two or more numbers is a number that is an exact divisor of each of them.

Thus, 4 is a common divisor of 16, 24, 32 and 48.

84. A **Common Measure** of two or more numbers or amounts is a number or amount that is an exact measure of each of them.

Thus, 4 hours is a common measure of 16 hours, 24 hours, 32 hours, and 48 hours.

85. The **Greatest Common Divisor** of two or more numbers is the number that is the greatest exact divisor of each of them.

86. The **Greatest Common Measure** of two or more numbers is the number that is the greatest exact measure of each of them.

87. Since the continued prime divisors of a number are the prime factors of the number, it follows that the continued common prime divisors of two or more numbers are their **Common Prime Factors.**

$$\begin{array}{r|rrr} 2 & 36 & 48 & 60 \\ \cline{2-4} 2 & 18 & 24 & 30 \\ \cline{2-4} 3 & 9 & 12 & 15 \\ \cline{2-4} & 3 & 4 & 5 \end{array}$$

Thus, 2, 2, and 3, the continued common prime divisors of 36, 48, and 60, are the common prime factors of 36, 48, and 60.

88. PRIN. *The product of any of the common prime factors of two or more numbers is a common divisor of them.*

Thus, 2 × 2, or 4; 2 × 3, or 6; and 2 × 2 × 3, or 12, are common divisors of 36, 48, and 60.

89. PRIN. *The product of all the common prime factors of two or more numbers is their greatest common divisor.*

Thus, 2 × 2 × 3, or 12, is the greatest common divisor of 36, 48, and 60.

90. PRIN. *The greatest common divisor of two or more numbers is also their greatest common measure.*

For any divisor of a number is also a measure of it [§ 75 Prin.].

91. PRIN. *The greatest common measure of two or more similar amounts equals their common unit of amount multiplied by the greatest common divisor of their numerical values.*

Thus, the greatest common measure of 36 yards, 48 yards, and 60 yards equals 1 yard multiplied by the greatest common divisor of 36, 48, and 60; or 12 × 1 yard, or 12 yards.

92. When two numbers have no common divisor, they are said to be **Prime to Each Other.**

Thus, 7 and 15 are prime to each other.

Note. The abbreviations *g. c. d.* and *g. c. m.* stand for *greatest common divisor* and *greatest common measure.*

EXERCISE 39. ORAL.

Name at sight the common divisors of:

1. 9 and 12.
2. 8 and 12.
3. 10 and 20.
4. 18 and 24.
5. 8, 12, and 16.
6. 12, 18, and 24.
7. 14, 21, and 28.
8. 22, 44, and 66.
9. 25, 50, and 75.
10. 15, 45, and 60.
11. 13, 26, and 39.
12. 17, 34, and 51.

Name at sight the greatest common measures of:

13. 18 men and 24 men.
14. 16 horses and 48 horses.
15. 15 cents and 25 cents.

16. 18 yards, 24 yards, and 36 yards.

17. \$14, \$21, and \$28.

18. 20 inches, 30 inches, and 40 inches.

19. Sara has 3 ribbons, 12 feet, 14 feet, and 16 feet long respectively, which she wishes to cut without waste into shorter ribbons of equal length and as long as possible; what is the length of the shorter ribbons?

SOLUTION. The length of the longest ribbons of equal length that can be cut without waste from ribbons 12 feet, 14 feet, and 16 feet long is the g. c. m. of 12 feet, 14 feet, and 16 feet, or 2 feet.

20. What are the longest strings of equal length that can be cut without waste from three fishing lines whose lengths are 20 feet, 25 feet, and 30 feet?

21. A boy has 18 apples, 24 peaches, and 30 plums, which he wishes to put in separate packages each containing the same number of pieces, without mixing the fruit; what is the largest number that he may put in each package?

EXERCISE 40.

1. Find the greatest common divisor of 294, 378, and 462.

FORM.

$$\begin{array}{r|rrr} 2 & 294 & 378 & 462 \\ \hline 3 & 147 & 189 & 231 \\ \hline 7 & 49 & 63 & 77 \\ \hline & 7 & 9 & 11 \end{array}$$

SOLUTION. 2, 3, and 7 are the common prime factors of 294, 378, and 462 [§ 87].

∴ g. c. d. of 294, 378, and 462 = 2 × 3 × 7, or 42 [§ 89].

2. What is the length of the longest boards of equal length that can be used to lay the floors in rooms 28 feet wide and 42 feet wide without cutting the boards?

FORM.

$$\begin{array}{r|rr} 2 & 28 & 42 \\ \hline 7 & 14 & 21 \\ \hline & 2 & 3 \end{array}$$

SOLUTION. Since the floors are to be laid without cutting the boards, the length of each board must be an exact measure of 28 feet and 42 feet; and since the boards are to be of equal length and as long as possible, the g. c. m. of 28 feet and 42 feet, or 2 × 7 × 1 foot, or 14 feet = the length of each board. [§ 91].

Find the greatest common divisor of:

3. 24 and 36.
4. 56 and 72.
5. 70 and 98.
6. 63 and 91.
7. 90 and 108.
8. 75 and 175.
9. 112 and 168.
10. 210 and 270.
11. 42, 49, and 56.
12. 40, 56, and 72.
13. 70, 80, and 90.
14. 52, 78, and 104.
15. 245, 280, and 315.
16. 288, 384, and 528.
17. 432, 540, and 648.
18. 384, 576, and 768.
19. 1875, 3125, and 4375.
20. 138, 276, 414, and 552.
21. 330, 495, 660, and 825.
22. 224, 336, 560, and 784.
23. 234, 312, 390, and 546.
24. 392, 504, 560, and 616.

25. Two pieces of metal weighing 18 pounds and 24 pounds were each cast without waste into balls; what was the weight of each of the balls if they were of equal weight and as heavy as possible?

26. What is the length of the longest panel that can be used in fencing a lot 160 feet long and 136 feet wide, making the panels of equal length? How many panels will it require to enclose the lot?

27. What is the highest average price that may be paid for sheep in order that $24, $30, or $36 may be invested in them?

28. What is the length of the longest sticks of timber of equal length that may be sawed without waste from three logs 28 feet, 42 feet, and 56 feet long?

29. What is the longest measure that is contained a whole number of times in the length, width, and height of a room 32 feet long, 28 feet wide, and 16 feet high?

30. A man has three balls of fishing twine containing 210 feet, 350 feet, and 525 feet, which he desires to cut into fishing lines of equal length, making them as long as possible; how long must the lines be cut?

31. A speculator bought three tracts of land containing 385 acres, 462 acres, and 539 acres, which he wishes to divide into small farms of equal size, making them as large as possible; how many acres must he put in each farm? How many farms can he make?

32. A farmer has 225 bushels of corn, 315 bushels of wheat, and 360 bushels of barley, which he wishes to haul away separately in loads of equal size and as large as possible; how many bushels can he haul at a load?

33. What is the highest average price that may be paid for horses in order that a whole number may be purchased for $852, $973, or $1094, and have $5 over in each case to pay for their care?

Common Multiples.

93. A **Multiple** of a given number is such a number as contains the given number as a measure.

Thus, 15 is a multiple of 5, since 15 contains 5 as a measure.

94. A **Common Multiple** of two or more numbers is a number that contains each of them as a measure.

Thus, 36 is a common multiple of 6 and 9, since it contains each of them as a measure.

95. The **Least Common Multiple** of two or more numbers is the least number that contains each of them as a measure.

Thus, 18 is the least common multiple of 6 and 9, since it is the least number that contains each of them as a measure.

Note. The abbreviation *l. c. m.* stands for *least common multiple.*

96. PRIN. 1. *The least common multiple of two or more numbers equals the product of all their different prime factors, each used the greatest number of times it occurs in any one of the numbers.*

Let it be required to find the least common multiple of 18, 24, and 36.

$18 = 2 \times 3 \times 3.$
$24 = 2 \times 3 \times 2 \times 2.$
$36 = 2 \times 3 \times 3 \times 2.$

The greatest number of times 2 occurs as a factor in any of the numbers is *three;* the greatest number of times that 3 occurs as a factor in any of the numbers is *two;* and 2 and 3 are all the different prime factors of the numbers: it is to be proved that the l. c. m. of the numbers $= 2 \times 2 \times 2 \times 3 \times 3$.

To contain 18 the l. c. m. must contain the prime factors of 18, or 2, 3, and 3; to contain 24 it must contain the additional prime factors 2 and 2; and as these are the only factors required to contain 36 also, the l. c. m. of 18, 24, and $36 = 2 \times 3 \times 3 \times 2 \times 2$, or $2 \times 2 \times 2 \times 3 \times 3$, or 72.

Remark. The least common multiple of two or more similar amounts equals their common unit of amount multiplied by the least common multiple of their numerical values.

EXERCISE 41. ORAL.

Name at sight the least common multiple of:

1. 2 and 4.
2. 3 and 5.
3. 4 and 6.
4. 8 and 10.
5. 10 and 15.
6. 3, 4, and 6.
7. 2, 5, and 10.
8. 5, 10, and 15.
9. 6, 8, and 12.
10. 7, 14, and 21.
11. 6, 9, and 12.
12. 11, 22, and 44.
13. 9, 18, and 27.
14. 10, 15, and 20.
15. 8, 12, and 16.

16. What is the length of the shortest rope that can be cut without a remnant into pieces of equal length either 4 feet, 6 feet, or 8 feet long?

SOLUTION. The length required is the least length that contains 4 feet, 6 feet, and 8 feet each as a measure, and is therefore the l. c. m. of 4 feet, 6 feet, and 8 feet, or 24 feet [§ 96 *Remark*].

17. What is the least sum of money that will buy a whole number of apples at 3 cents apiece, a whole number of oranges at 5 cents apiece, or a whole number of pineapples at 10 cents apiece?

18. What is the least weight that can be weighed with 2-pound weights, 3-pound weights, 4-pound weights, or 6-pound weights, and how many of each kind are required?

19. What is the least amount of vinegar that could be measured by a 2-quart, a 4-quart, or a 6-quart measure, and have in each case 1 quart over?

EXERCISE 42.

1. Find the l. c. m. of 24, 36, and 48.

FORM.

2	24	36	48
2	12	18	24
3	6	9	12
2	2	3	4
	1	3	2

SOLUTION. We see in the form that 2, 2, and 3 are prime factors of each of the numbers, 24, 36, and 48; 2 again a prime factor of 24 and 48; and 3 again a prime factor of 36 and 2 of 48.

The different prime factors are 2 and 3; 2 occurs *four* times in 48, and 3 *twice* in 36.

∴ the l. c. m. of 24, 36, and 48 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3$, or 144 [§ 96].

2. What is the least sum of money that will buy cows at $45 a head, colts at $60 a head, or horses at $110 a head, and leave no remainder?

SOLUTION. The required sum must be the least sum that will contain $45, $60, and $110 each as a measure, or it must be the l. c. m. of these amounts, which is $1980 [§ 96, *Remark*].

Find the least common multiple of:

3. 28 and 42.
4. 84 and 96.
5. 63 and 84.
6. 66 and 99.
7. 56 and 120.
8. 75 and 125.
9. 52 and 130.
10. 105 and 135.
11. 42, 49, and 56.
12. 72, 96, and 120.
13. 45, 75, and 90.
14. 52, 78, and 104.
15. 36, 54, and 81.
16. 30, 90, and 150.
17. 56, 70, and 84.
18. 110, 275, and 605.
19. 154, 242, and 539.
20. 65, 91, and 143.
21. 91, 117, and 130.
22. 108, 180, and 243.

23. What is the smallest amount of hay that can be put up in bales of 100 pounds or 125 pounds each?

24. What is the least sum of money that a boy must take to a store in order that he may be able to buy a whole number of pounds of coffee at either 24 cents or 28 cents a pound? How many pounds of each could he buy?

25. Find the width of the narrowest walk that can be paved with stones cut uniformly 20 inches long and 12 inches wide, allowing them to be laid without waste either lengthwise or across the walk.

26. Two men start at the same time and place to ride around a race track. The first can go around it in 6 minutes and the second in 8 minutes; if they ride in the same direction, in how many minutes after starting will the first pass the second?

27. Find the length of the shortest iron rod that may be cut without waste into bolts 6 inches, 8 inches, or 10 inches long.

28. Three steamers leave a certain port, the first every 5 days, the second every 12 days, and the third every 15 days. They all left on January 1; how many days will elapse before they leave together again? How many times will the first and the third have left together in the meantime?

29. What is the least number of eggs that a man can have in a basket so that they may be counted 8, 12, or 20 at a time and have none remaining?

30. What is the smallest tract of land that may be divided into farms of 60, 120, and 180 acres each?

31. What is the shortest log that may be sawed into 12, 10, or 24 feet boards and leave in each case a remainder of 2 feet of log?

32. What is the least sum of money that will buy town lots at $100, $120, or $150 apiece and lack $50 of paying for them?

33. What is the weight of the smallest block of granite that may be sawed into blocks weighing 160 pounds, 240 pounds, or 360 pounds each, deducting in each case 20 pounds for the weight of the sawdust?

34. A farmer observed that he could haul all his grain, except 40 bushels which he wished to reserve for seed, to a warehouse, by hauling either 48, 56, or 60 bushels a load; how many bushels of grain had he?

Cancellation.

97. Cancellation is the process of crossing out a number or an amount and writing in its stead the result obtained

by multiplying, dividing, or measuring the number or the amount.

Thus, $8 \times 25 = 2 \times \cancel{4} \times \overset{100}{\cancel{25}} = 200$; $6 \div 2 = \overset{3}{\cancel{6}}$;
$\$12 : \$4 = \$\underset{3}{\cancel{12}}$.

98. PRIN. *Multiplying one number and dividing another by the same number does not alter their product.*

Thus, $8 \times 12 = 96$ and $(4 \times 8) \times (12 \div 4)$, or $32 \times 3 = 96$.

99. PRIN. *Multiplying both dividend and divisor by the same number does not alter the quotient.*

Thus, $12 \div 3 = 4$, and $(5 \times 12) \div (5 \times 3)$, or $60 \div 15 = 4$.

100. PRIN. *Multiplying both antecedent and consequent by the same number does not alter the ratio.*

Thus, 12 men : 3 men = 4, and (5 × 12 men) : (5 × 3 men), or 60 men : 15 men = 4.

101. PRIN. *Dividing both dividend and divisor by the same number does not alter the quotient.*

Thus, $18 \div 6 = 3$, and $(18 \div 3) \div (6 \div 3)$, or $6 \div 2 = 3$.

102. PRIN. *Dividing both antecedent and consequent by the same number does not alter the ratio.*

Thus, \$36 : \$9 = 4, and (\$36 ÷ 3) : (\$9 ÷ 3), or \$12 : \$3 = 4.

103. PRIN. *Measuring both antecedent and consequent by the same number or amount does not alter the ratio.*

Thus, $36 : 9 = 4$, and $(36 : 3) : (9 : 3)$, or $12 : 3 = 4$.

EXERCISE 43.

1. Find the product of 25 and 36.

FORM.

$\overset{100}{\cancel{25}} \times \overset{9}{\cancel{36}} = 900.$

SOLUTION. Multiplying 25 by 4 and dividing 36 by 4, we have $25 \times 36 = 100 \times 9$ [§ 98] = 900.

2. Divide 375 by 125.

FORM.

$$\overset{15}{\cancel{375}} \div \overset{5}{\cancel{125}} = 3.$$

SOLUTION. Dividing 375 and 125 each by 25, we have $375 \div 125 = 15 \div 5$ [§ 101] $= 3$.

3. Measure 72 yards by 18 yards.

FORM.

$$\overset{12}{\cancel{72}}\ \cancel{\text{yards}} : \overset{3}{\cancel{18}}\ \cancel{\text{yards}} = 4.$$

SOLUTION. Measuring both 72 yards and 18 yards by 6 yards, we have 72 yards : 18 yards = 12 : 3 [§ 103] = 4.

Note. For the purpose of cancellation it is often preferable to write the divisor or consequent under the dividend or antecedent with a line between them.

Thus, $24 \div 6 = \frac{24}{6}$, and 18 bushels : 3 bushels $= \frac{18 \text{ bushels}}{3 \text{ bushels}}$.

4. Find the value of $\dfrac{24 \times 36 \times 45}{9 \times 16 \times 5}$, read *24 times 36 times 45, divided by* (or *measured by*) *9 times 16 times 5.*

FORM.

$$\frac{\overset{3}{\cancel{24}} \times \overset{18}{\cancel{36}} \times \overset{9}{\cancel{45}}}{\cancel{9} \times \underset{2}{\cancel{16}} \times \cancel{5}} = 54.$$

SOLUTION. Since dividing both dividend and divisor by the same number does not alter the quotient, we divide 24 and 16 each by 8, and write the quotients 3 and 2; we next divide 45 by 5 and write the quotient 9; then 9 by 9 and 36 by 2, and write the quotient 18. There are now left the factors of the quotient, 3 and 18, whose product is 54.

5. How many loads of 25 bags, each holding 3 bushels, will be required to haul the wheat that grows on 30 acres producing 20 bushels an acre?

SOLUTION. 20 = the number of bushels 1 acre produces;
∴ 30 × 20 = " " " " 30 acres produce;
3 = " " " " 1 bag holds;
∴ 25 × 3 = " " " " 25 bags hold;
whence, 25 × 3 = " " " " in 1 load.
Now, (30 × 20) : (25 × 3) = the number of loads required.

or, $\dfrac{\overset{\cancel{10}\,2}{\cancel{30}} \times \overset{4}{\cancel{20}}}{\underset{\cancel{5}}{\cancel{25}} \times \cancel{3}}$, or 2 × 4, or 8 = the number of loads required.

Find the value of:

6. 12 × 13.
7. 13 × 14.
8. 14 × 15.
9. 15 × 17.
10. 15 × 18.
11. 16 × 17.
12. 17 × 18.
13. 18 × 19.
14. 18 × 20.
15. 15 × 25.
16. 16 × 50.
17. 19 × 14.
18. 20 × 15 × 8.
19. 25 × 16 × 20.
20. 32 × 15 × 4.
21. 25 × 12 × 3.

Find the value of:

22. 36 ÷ 18.
23. 96 ÷ 24.
24. 150 ÷ 75.
25. 204 ÷ 51.
26. 315 ÷ 63.
27. 616 ÷ 88.
28. 405 ÷ 45.
29. 2376 ÷ 216.
30. 1008 ÷ 168.
31. 3528 : 252.
32. 264 yd. : 66 yd.
33. 576 hr. : 192 hr.
34. 2430 hr. : 810 hr.
35. 3680 min. : 1840 min.
36. 2400 da. : 400 da.
37. $\frac{24 \times 56 \times 84}{36 \times 28 \times 14}$.
38. $\frac{24 \times 35 \times 72}{32 \times 54}$.
39. $\frac{96 \times 90 \times 150 \text{ bu.}}{72 \times 180 \text{ bu.}}$.
40. $\frac{12 \times 39 \times 42 \times 36 \text{ yd.}}{4 \times 13 \times 14 \times 9}$.

41. A farmer exchanged 18 pounds of butter at 20 cents a pound, for cloth at 12 cents a yard; how many yards did he obtain?

42. A quantity of corn consisting of 273 bushels, each bushel weighing 56 pounds, was hauled in 6 loads; what was the average weight of the loads?

43. How many years would it require a laborer who saves 15 dollars a month for 12 months in a year, to pay for 54 acres of land at 50 dollars an acre?

44. If 6 men are allowed 24 days to do a piece of work, how long should 18 men be given to do it?

45. The yield of wheat from a 10-acre field is estimated at 30 bushels per acre; if a bushel of wheat weighs 60 pounds, how many tons of 2000 pounds each are there in the yield?

46. If 24 men can do a piece of work in 12 days, working 10 hours a day, how many men can do it in 10 days, working 8 hours a day?

47. How many horses, each eating 20 quarts of oats daily, will eat as much oats in 12 days as 8 horses, each eating 16 quarts daily, will eat in 60 days?

Supplement.

HIGHEST COMMON DIVISOR OF NUMBERS NOT READILY FACTORED.

104. PRIN. *A divisor of a number is a divisor of any multiple of that number.*

Thus, 3, a divisor of 15, is a divisor of 4 × 15, or 60.

105. PRIN. *A common divisor of two numbers is a divisor of their sum.*

Thus, 3 a common divisor of 18 and 12, is a divisor of 18 + 12, or 30.

106. PRIN. *A common divisor of two numbers is a divisor of their difference.*

Thus, 3, a common divisor of 18 and 12, is a divisor of 18 − 12, or 6.

107. If one number is divided by another, then the divisor by the remainder, then the next divisor by the next remainder, and so on until there is no remainder, the process is called **Successive Division**.

108. PRIN. *The last divisor obtained by the successive division of two numbers is their greatest common divisor.*

The following explanation proves that 47, the last divisor obtained by the successive division of 329 and 423, is the greatest common divisor of these two numbers:

329)423(1
329
94)329(3
282
47)94(2
94

(1) 47 is a divisor of 94, since the division has terminated; hence it is a divisor of 3 × 94, or 282 [§ 104], and of 282 + 47, or 329 [§ 105], and also of 329 + 94, or 423. Therefore, 47 is a common divisor of 329 and 423.

(2) There can be no greater common divisor of 329 and 423 than 47; for if there could be, it would be a divisor of 423 – 329, or 94 [§ 106]; also of 3 × 94, or 282 [§ 104] and of 329 – 282, or 47; that is, a number greater than 47 would be a divisor of 47, which is impossible. Therefore, 47 is the g. c. d. of 329 and 423.

109. Prin. *The greatest common divisor of three or more numbers may be obtained by finding the greatest common divisor of two of them, then the greatest common divisor of it and the third number, and so on.*

Thus, by successive division it is found that the g. c. d. of 312 and 585 is 39, and that the g. c. d. of 39 and 338 is 13. Then, 13 is the g. c. d. of 312, 585, and 338.

EXERCISE 44.

1. Find the g. c. d. of 312 and 585.

Form.

312)585(1
312
273)312(1
273
39)273(7
273

Solution. The last divisor obtained by the successive division of 312 and 585 is 39.

∴ g. c. d. of 312 and 585 = 39 [§ 108].

Find the g. c. d. of:

2. 102 and 119.
3. 114 and 209.
4. 253 and 299.
5. 290 and 609.
6. 403 and 527.
7. 703 and 925.
8. 287 and 451.
9. 301, 387, and 430.
10. 376, 423, and 517.
11. 147, 294, and 637.
12. 459, 612, and 816.
13. 852, 1278, and 1349.
14. 949, 1898, and 2190.
15. 1185, 1580, and 1896.

16. What is the highest average price at which mules may be purchased for $381, $889, or $1143? How many would each sum buy?

17. A invested $1608 in land, B $2345, and C $2814, at the same price per acre, and they found that the price paid per acre was the highest that would procure for each a whole number of acres; what was the price paid per acre, and how many acres did each receive?

18. What is the largest sum of money that is contained in $1489 and in $1663, with a remainder of $10 in each case?

LEAST COMMON MULTIPLE OF NUMBERS NOT READILY FACTORED.

110. Prin. *The least common multiple of two numbers may be found by dividing one of them by their greatest common divisor and multiplying the other by the quotient thus obtained.*

The following explanation proves that the l. c. m. of 68 and 85 equals $\frac{68}{17} \times 85$, the g. c. d. of 68 and 85 being 17:

$68 = 4 \times 17$ and $85 = 5 \times 17$.

$\therefore$ l. c. m. of 68 and 85 $= 4 \times 17 \times 5$ [§ 96].

But, $4 = \frac{68}{17}$, and $17 \times 5 = 85$.

$\therefore 4 \times 17 \times 5 = \frac{68}{17} \times 85$.

Hence, l. c. m. of 68 and 85 $= \frac{68}{17} \times 85$.

111. Prin. *The least common multiple of three or more numbers may be found by finding the least common multiple of two of them, then the least common multiple of that and the third number, and so on.*

Thus, let it be required to find the l. c. m. of 68, 85, and 112. The l. c. m. of 68 and 85 $= \frac{68}{17} \times 85$, or 340 [§ 110]; the l. c. m. of 340 and 112 $= \frac{340}{34} \times 112$, or 1120.

Then, 1120 is the l. c. m. of 68, 85, and 112.

EXERCISE 45.

1. Find the l. c. m. of 104 and 143.

SOLUTION. The g. c. d. of 104 and 143 = 13.
The l. c. m. of 104 and 143 = $\frac{104}{13} \times 148$ [§ 110]
$= 8 \times 143 = 1144$.

2. Find the l. c. m. of 102, 136, and 221.

SOLUTION. The g. c. d. of 102 and 136 = 34.
Whence, the l. c. m. of 102 and 136 = $\frac{102}{34} \times 136 = 408$.
The g. c. d. of 408 and 221 = 17.
Whence, the l. c. m. of 408 and 221 = $\frac{408}{17} \times 221 = 5304$.
∴ the l. c. m. of 102, 136, and 221 = 5304 [§ 111].

Find l. c. m. of:

3. 117 and 143.
4. 102 and 119.
5. 95 and 133.
6. 115 and 184.
7. 290 and 377.
8. 372 and 527.
9. 231, 297, and 330.
10. 442, 663, and 1105.
11. 111, 148, and 185.
12. 117, 208, and 273.
13. 169, 338, and 507.
14. 841, 1682, and 2523.

15. What is the least sum of money which may be wholly expended for automobiles at either $680 or $850 apiece?

16. What is the least amount of iron that may be cast into wheels weighing 273 pounds, 420 pounds, or 504 pounds each, allowing a waste of 40 pounds for casting?

17. I have the least quantity of flour with which I can fill an exact number of sacks holding 25 pounds, 50 pounds, or 100 pounds each, or barrels holding 196 pounds each; how much flour have I?

18. Show that the product of 42 and 70 divided by their g. c. d. equals their l. c. m.

19. Show that the product of two numbers equals the product of their g. c. d. and l. c. m.

CHAPTER II.

FRACTIONS.

General Definitions.

112. One or more units may each be divided into a number of equal parts called **Fractional Units.**

If one or more units are each divided into *two* equal parts, each fractional unit is called *a half;* if into *three* equal parts, *a third;* if into *four* equal parts, *a fourth;* if into *twenty-one* equal parts, *a twenty-first;* if into *twenty-two* equal parts, *a twenty-second;* and so on.

113. Any number of equal fractional units is a **Fraction.**

Thus, *one third, four fifths,* and *sixteen tenths* are fractions.

114. If similar units of quantity are each divided into a number of equal parts, each part is a **Fractional Unit of Quantity.**

Thus, a *third of a yard* is a fractional unit of quantity.

115. Any number of equal fractional units of quantity is a **Fractional Amount.**

Thus, *twelve fifths of a bushel* is a fractional amount.

116. The number of equal parts into which a unit is divided to form fractional units is the **Denominator,** and the number of equal fractional units taken to make a fraction is the **Numerator** of the fraction.

Thus, if each of a number of units is divided into fifths, and twelve of these fifths are taken to make a fraction, *five* is the *denominator*, and *twelve* the *numerator* of the fraction *twelve fifths.*

117. The numerator and the denominator are together called the **Terms** of the fraction.

118. A fraction is expressed by writing the numerator above the denominator with a short line between them.

Thus, $\frac{4}{7}$ expresses *four sevenths,* and $\frac{29}{21}$, *twenty-nine twenty-firsts.*

119. A **Proper Fraction** is one whose numerator is less than its denominator.

Thus, $\frac{7}{9}$ is a proper fraction.

120. An **Improper Fraction** is one whose numerator is equal to or greater than its denominator.

Thus, $\frac{9}{9}$ and $\frac{16}{9}$ are improper fractions.

121. A **Mixed Number** is a whole number and a fraction combined.

Thus, $12\frac{2}{5}$, which is equal to $12 + \frac{2}{5}$, is a mixed number.

Note. A whole number is also called an **Integral Number**, or an **Integer.**

122. Since $3 \div 2 = \frac{1}{2}$ of $3 = \frac{1}{2}$ of $(1+1+1) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$, $\frac{3}{2} = 3 \div 2$. Therefore,

A fraction may be considered to denote that the numerator is to be divided by the denominator.

123. The **Inverse** of a given fraction is the fraction obtained by interchanging the terms of the given fraction.

Thus, $\frac{5}{4}$ is the inverse of $\frac{4}{5}$.

124. The **Reciprocal** of a number is *one* divided by that number.

Thus, $1 \div 2\frac{3}{4}$, or $\frac{1}{2\frac{3}{4}}$, is the reciprocal of $2\frac{3}{4}$.

EXERCISE 46. ORAL.

1. Read $\frac{7}{8}$, and tell what it expresses.

Solution. *Seven eighths.* It expresses seven fractional units, each of which is one eighth.

2. Read $\frac{12}{5}$ of a bu., and tell what it expresses.

Solution. *Twelve fifths of a bushel.* It expresses twelve fractional units of quantity, each of which is one fifth of a bushel.

Read the following expressions and give their meaning:

3. $\frac{4}{5}$. **6.** $\frac{8}{3}$. **9.** $\frac{9}{10}$ of a yd. **12**: $\frac{18}{7}$ of an orange.
4. $\frac{7}{9}$. **7.** $\frac{10}{7}$. **10.** $\frac{12}{17}$ of a bu. **13.** $\frac{16}{5}$ of an apple.
5. $\frac{12}{13}$. **8.** $\frac{23}{15}$. **11.** $\frac{14}{23}$ of a lb. **14.** $\frac{13}{8}$ of a melon.

15. $2\frac{4}{5}$ bu.

Solution. *Two and four fifths bushels.* It denotes two bushels and four fifths of a bushel.

16. $3\frac{7}{9}$. **18.** $22\frac{1}{7}$ yd. **20.** $1\frac{3}{5}$ lb.
17. $14\frac{5}{8}$. **19.** $16\frac{1}{2}$ ft. **21.** $66\frac{2}{3}$ ct.

Name the inverse of each of the following fractions:

22. $\frac{7}{8}$. **23.** $\frac{19}{25}$. **24.** $\frac{41}{7}$. **25.** $\frac{101}{99}$.

General Principles.

125. Prin. *Multiplying the numerator of a fraction multiplies the value of the fraction.*

Thus, $3 \times \frac{4}{5} = \frac{3 \times 4}{5}$, or $\frac{12}{5}$, because if the numerator is multiplied by 3 and the denominator remains unchanged, there will be 3 times as many fractional units each of the same value as before.

126. Prin. *Multiplying the denominator of a fraction divides the value of the fraction.*

Thus, $\frac{4}{5} \div 3 = \frac{4}{3 \times 5}$, or $\frac{4}{15}$, because if the denominator is multiplied by 3 and the numerator remains unchanged, there will be the same number of fractional units as before, but each will be only one third as great.

127. PRIN. *Dividing the numerator of a fraction divides the value of the fraction.*

Thus, $\frac{6}{7} \div 3 = \frac{6 \div 3}{7}$, or $\frac{2}{7}$, because if the numerator is divided by 3 and the denominator remains unchanged, there will be only one third as many fractional units, each of the same value as before.

128. PRIN. *Dividing the denominator of a fraction multiplies the value of the fraction.*

Thus, $3 \times \frac{7}{12} = \frac{7}{12 \div 3}$, or $\frac{7}{4}$, because if the denominator is divided by 3 and the numerator remains unchanged, there will be the same number of fractional units as before, but each will be three times as great.

129. PRIN. *Multiplying both terms of a fraction by the same number does not alter the value of the fraction.*

Thus, $\frac{4}{5} = \frac{3 \times 4}{3 \times 5}$, or $\frac{12}{15}$, because multiplying the numerator by 3 multiplies the value of the fraction by 3, and multiplying the denominator by 3 divides the value of the fraction by 3, and doing both, multiplies and divides the value of the fraction by 3, which does not alter its value.

130. PRIN. *Dividing both terms of a fraction by the same number does not alter the value of the fraction.*

Thus, $\frac{12}{15} = \frac{12 \div 3}{15 \div 3}$, or $\frac{4}{5}$, because dividing the numerator by 3 divides the value of the fraction by 3, and dividing the denominator by 3 multiplies the value of the fraction by 3, and doing both, divides and multiplies the value of the fraction by 3, which does not alter its value.

Reduction of Fractions.

131. **Reduction** is the process of changing the form an expression without altering its value.

EXERCISE 47. ORAL.

1. Reduce $2\frac{3}{4}$ to fourths.

Solution. *One* equals $\frac{4}{4}$; hence, 2 equals 2 times $\frac{4}{4}$, or $\frac{8}{4}$; and $2\frac{3}{4}$ equals $\frac{8}{4}$ plus $\frac{3}{4}$, or $\frac{11}{4}$.

Reduce:

2. 3 to halves.
3. $4\frac{2}{3}$ to thirds.
4. $5\frac{3}{4}$ to fourths.
5. $6\frac{3}{5}$ to fifths.
6. $7\frac{5}{6}$ to sixths.
7. $8\frac{4}{7}$ to sevenths.
8. $9\frac{3}{8}$ to eighths.
9. $10\frac{2}{9}$ to ninths.
10. $11\frac{3}{10}$ to tenths.

11. Reduce $\frac{19}{4}$ to a mixed number.

Solution. $\frac{19}{4}$ equals $\frac{1}{4}$ of 19, which equals 4, and 3 over; $\frac{1}{4}$ of 3 equals $\frac{3}{4}$ [§ 122]; hence, $\frac{19}{4}$ equals $4\frac{3}{4}$.

Reduce to whole or mixed numbers or amounts:

12. $\frac{14}{2}$.
13. $\frac{15}{3}$.
14. $\frac{17}{3}$.
15. $\frac{21}{4}$.
16. $\frac{33}{10}$.
17. $\frac{28}{9}$.
18. $\frac{45}{9}$ of a yard.
19. $\frac{36}{7}$ of a pound.

20. Reduce $\frac{3}{4}$ to 12ths.

Solution. $\frac{3}{4}$ equals $\frac{9}{12}$, since multiplying both terms of $\frac{3}{4}$ by 3 does not alter its value.

Reduce:

21. $\frac{1}{2}$ to 6ths.
22. $\frac{2}{3}$ to 9ths.
23. $\frac{3}{7}$ to 28ths.
24. $\frac{6}{5}$ to 20ths.
25. $\frac{7}{9}$ to 27ths.
26. $\frac{3}{10}$ to 50ths.

27. Reduce $\frac{8}{12}$ to 3ds.

Solution. $\frac{8}{12}$ equals $\frac{2}{3}$, since dividing both terms of $\frac{8}{12}$ by 4 does not alter its value.

Reduce:

28. $\frac{6}{8}$ to 4ths.
29. $\frac{10}{12}$ to 6ths.
30. $\frac{15}{10}$ to halves.
31. $\frac{24}{16}$ to halves.
32. $\frac{12}{20}$ to 5ths.
33. $\frac{18}{42}$ to 7ths.

132. A fraction is in its **Lowest Terms** when its numerator and denominator are prime to each other.

Thus, $\frac{12}{15}$ when reduced to its lowest terms equals $\frac{4}{5}$.

133. Prin. *Dividing both terms of a fraction by their greatest common divisor reduces the fraction to its lowest terms.*

Thus, $\frac{160}{224} = \frac{160 \div 32}{224 \div 32} = \frac{5}{7}$, which is in its lowest terms, for 32, the g. c. d. of 160 and 224, contains all the common factors of 160 and 224, and hence the quotients 5 and 7 are prime to each other. The value of the fraction is not changed according to § 130.

134. Prin. *A mixed number is equal to the whole number multiplied by the denominator of the fraction, plus the numerator of the fraction, all divided by the denominator of the fraction.*

Thus, $24\frac{4}{5} = \frac{5 \times 24 + 4}{5}$, because 24 × 5, or 5 × 24 = the number of 5ths in 24; whence, $5 \times 24 + 4 =$ the number of 5ths in $24\frac{4}{5}$.

135. Fractions are **Similar** when they are composed of like fractional units; that is, when they have a common denominator.

Thus, $\frac{3}{9}$ and $\frac{5}{9}$ are similar fractions.

136. Prin. *Fractions having different denominators may be reduced to fractions having a common denominator equal to the least common multiple of their denominators.*

Thus, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ are fractions having different denominators.

The l. c. m. of 3, 4, and 5 is 60. $60 \div 3 = 20$; $60 \div 4 = 15$; and $60 \div 5 = 12$.

Now, $\frac{2}{3} = \frac{20 \times 2}{20 \times 3} = \frac{40}{60}$; $\frac{3}{4} = \frac{15 \times 3}{15 \times 4} = \frac{45}{60}$; and $\frac{4}{5} = \frac{12 \times 4}{12 \times 5} = \frac{48}{60}$.

The values of the fractions are not changed according to § 129.

Note 1. Any common multiple of the denominators may be taken as a common denominator. Thus, in the above illustration, 120, 180, 240, etc., may each be taken as a common denominator.

Note 2. If fractions having different denominators are in their lowest terms, the least common multiple of their denominators is their Least Common Denominator.

Note 3. l. c. d. stands for Least Common Denominator.

EXERCISE 48.

1. Reduce $\frac{168}{192}$ to its lowest terms.

SOLUTION. The g. c. d. of 168 and 192 is 24.
Now, $\frac{168}{192} = \frac{168 \div 24}{192 \div 24}$ [§ 130] $= \frac{7}{8}$.

2. Reduce $56\frac{5}{8}$ to an improper fraction.

SOLUTION. $56\frac{5}{8} = \frac{8 \times 56 + 5}{8}$ [§ 134] $= \frac{453}{8}$.

3. Reduce $\frac{425}{13}$ to a mixed number.

SOLUTION. $\frac{425}{13} = \frac{1}{13}$ of $425 = 32\frac{9}{13}$.

4. Reduce $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{9}{16}$ to fractions having the least common denominator.

SOLUTION. The l. c. m. of the denominators, 8, 12, and 16, is 48.
∴ 48 = l. c. d. of the fractions.
$48 \div 8 = 6$; $48 \div 12 = 4$; and $48 \div 16 = 3$.
Now, $\frac{3}{8} = \frac{6 \times 3}{6 \times 8} = \frac{18}{48}$; $\frac{7}{12} = \frac{4 \times 7}{4 \times 12} = \frac{28}{48}$; and $\frac{9}{16} = \frac{3 \times 9}{3 \times 16} = \frac{27}{48}$.

Reduce to lowest terms:

5. $\frac{45}{60}$. **8.** $\frac{98}{168}$. **11.** $\frac{270}{288}$. **14.** $\frac{125}{175}$. **17.** $\frac{369}{423}$.
6. $\frac{100}{120}$. **9.** $\frac{180}{195}$. **12.** $\frac{168}{175}$. **15.** $\frac{279}{288}$. **18.** $\frac{405}{480}$.
7. $\frac{128}{144}$. **10.** $\frac{144}{224}$. **13.** $\frac{162}{168}$. **16.** $\frac{232}{248}$. **19.** $\frac{108}{324}$.

Reduce to improper fractions:

20. $15\frac{7}{8}$. **23.** $123\frac{3}{8}$. **26.** $36\frac{12}{13}$. **29.** $145\frac{7}{25}$. **32.** $506\frac{3}{20}$.
21. $39\frac{5}{6}$. **24.** $156\frac{8}{9}$. **27.** $48\frac{12}{15}$. **30.** $256\frac{1}{19}$. **33.** $300\frac{2}{25}$.
22. $87\frac{3}{4}$. **25.** $312\frac{2}{15}$. **28.** $98\frac{8}{16}$. **31.** $220\frac{5}{17}$. **34.** $1001\frac{1}{100}$

Reduce to whole or mixed numbers:

35. $\frac{89}{7}$. **38.** $\frac{242}{5}$. **41.** $\frac{348}{10}$. **44.** $\frac{432}{27}$. **47.** $\frac{1001}{11}$.
36. $\frac{92}{8}$. **39.** $\frac{312}{4}$. **42.** $\frac{555}{11}$. **45.** $\frac{908}{31}$. **48.** $\frac{1475}{35}$.
37. $\frac{96}{6}$. **40.** $\frac{581}{3}$. **43.** $\frac{729}{18}$. **46.** $\frac{673}{2}$. **49.** $\frac{2020}{70}$.

Reduce to fractions having the lowest common denominator:

50. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$. **54.** $\frac{5}{6}$, $\frac{7}{9}$, and $\frac{5}{18}$. **58.** $\frac{17}{36}$ and $\frac{19}{60}$.
51. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$. **55.** $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{3}{28}$. **59.** $\frac{8}{63}$ and $\frac{2}{105}$.
52. $\frac{4}{9}$, $\frac{7}{12}$, $\frac{5}{18}$, and $\frac{7}{36}$. **56.** $\frac{12}{13}$, $\frac{5}{26}$, and $\frac{4}{39}$. **60.** $\frac{24}{125}$ and $\frac{48}{175}$.
53. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{10}$, and $\frac{2}{15}$. **57.** $\frac{9}{15}$, $\frac{8}{20}$, and $\frac{15}{25}$. **61.** $\frac{37}{105}$ and $\frac{52}{385}$.

Addition of Fractions.

137. PRIN. *The sum of two or more similar fractions equals the sum of their numerators divided by their common denominator.*

Thus, $\frac{2}{5} + \frac{3}{5} + \frac{4}{5} = (2 + 3 + 4)$ fractional units each of which is $\frac{1}{5} = \frac{2+3+4}{5}$, or $\frac{9}{5}$.

138. The sum of two or more dissimilar fractions may be found by first reducing them to fractions having a common denominator and then adding them according to § 137.

Thus, $\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$.

139. The sum of two fractional units whose denominators are prime to each other is most readily obtained by taking for the numerator the sum of the denominators and for the denominator the product of the denominators.

Thus, $\frac{1}{3} + \frac{1}{4} = \frac{3+4}{3 \times 4}$, or $\frac{7}{12}$.

EXERCISE 49. ORAL.

At sight find the value of:

1. $\frac{1}{4}+\frac{3}{4}+\frac{5}{4}$.

Solution. $\frac{1}{4}+\frac{3}{4}+\frac{5}{4}=\frac{9}{4}$, or $2\frac{1}{4}$.

2. $\frac{1}{2}+\frac{1}{3}$.

Solution. $\frac{1}{2}+\frac{1}{3}=\frac{3+2}{3\times 2}=\frac{5}{6}$.

3. $\frac{5}{6}+\frac{3}{4}$.

Solution. $\frac{5}{6}+\frac{3}{4}=\frac{10}{12}+\frac{9}{12}=\frac{19}{12}$, or $1\frac{7}{12}$.

4. $\frac{1}{7}+\frac{2}{7}+\frac{3}{7}$.
5. $\frac{3}{8}+\frac{5}{8}+\frac{9}{8}$.
6. $\frac{5}{9}+\frac{2}{9}+\frac{7}{9}$.
7. $\frac{1}{3}+\frac{1}{4}$.
8. $\frac{1}{4}+\frac{1}{5}$.
9. $\frac{1}{3}+\frac{1}{5}$.
10. $\frac{3}{4}+\frac{4}{5}$.
11. $\frac{4}{3}+\frac{5}{6}$.
12. $\frac{2}{5}+\frac{2}{3}$.

13. $2\frac{1}{2}+3\frac{2}{3}$.

Solution. $2\frac{1}{2}+3\frac{2}{3}=2\frac{3}{6}+3\frac{4}{6}=5\frac{7}{6}=6\frac{1}{6}$.

14. $2\frac{3}{4}+1\frac{1}{4}$. **15.** $7\frac{1}{4}+3\frac{3}{8}$. **16.** $3\frac{2}{3}+2\frac{1}{6}$. **17.** $5\frac{2}{3}+2\frac{3}{4}$.

18. If a man does $\frac{1}{5}$ of a piece of work one day and $\frac{1}{4}$ of it the next day, what part of it has he done?

19. A storekeeper sold at auction $\frac{1}{5}$ of his stock and at private sale $\frac{3}{10}$ of it; what part did he sell in all?

20. What part of a bushel of oats is required to feed one horse $\frac{3}{4}$ of a bushel and another $\frac{3}{8}$ of a bushel?

21. A boy carried from a store $12\frac{1}{2}$ pounds of meat and $6\frac{1}{4}$ pounds of sugar; what weight did he carry?

22. A can weighing $2\frac{5}{8}$ pounds contains $12\frac{1}{4}$ pounds of lard; find the weight of the can and lard.

23. A fishing line $12\frac{2}{3}$ yards long was lengthened $6\frac{1}{4}$ yards; what was then its length?

24. Into a can containing $4\frac{3}{4}$ gallons of milk $2\frac{3}{8}$ gallons more were poured; how many gallons did the can then contain?

25. A farmer hauled a load of wheat weighing $2\frac{3}{10}$ tons in a wagon weighing $1\frac{4}{5}$ tons; what weight did he haul?

26. A farmer planted $\frac{1}{2}$ of an acre in potatoes, $\frac{1}{3}$ of an acre in tobacco, and $\frac{1}{4}$ of an acre in sweet corn; how much did he plant in all?

EXERCISE 50.

1. Find the sum of $\frac{2}{15}$, $\frac{9}{35}$, and $\frac{11}{75}$.

Solution. $\frac{2}{15} = \frac{2}{3 \times 5}$; $\frac{9}{35} = \frac{9}{5 \times 7}$; $\frac{11}{75} = \frac{11}{3 \times 5 \times 5}$.

$\therefore$ l. c. m. of the denominators $= 3 \times 5 \times 5 \times 7 =$ the l. c. d. of the fractions.

Now, $\frac{2}{15} = \frac{5 \times 7 \times 2}{5 \times 7 \times 15} = \frac{70}{525}$, multiplying both terms by 5×7.

$\frac{9}{35} = \frac{3 \times 5 \times 9}{3 \times 5 \times 35} = \frac{135}{525}$, " " " " 3×5.

$\frac{11}{75} = \frac{7 \times 11}{7 \times 75} = \frac{77}{525}$, " " " " 7.

$\therefore \frac{2}{15} + \frac{9}{35} + \frac{11}{75} = \frac{70}{525} + \frac{135}{525} + \frac{77}{525} = \frac{282}{525} = \frac{94}{175}$.

Note. By keeping the l. c. m. of the denominators in a factored form, it will be readily seen by what factors both terms of each fraction must be multiplied to reduce it to a fraction having the common denominator.

2. Find the sum of $124\frac{7}{8}$, $346\frac{5}{16}$, and $452\frac{7}{32}$.

Form.

$124\frac{7}{8} = 124\frac{28}{32}$.
$346\frac{5}{16} = 346\frac{10}{32}$.
$452\frac{7}{32} = 452\frac{7}{32}$.
$\therefore$ Sum $= 923\frac{13}{32}$.

Sug. The sum of the fractions equals $1\frac{13}{32}$. Write the $\frac{13}{32}$, add the 1 to the column of units, and proceed as in addition of whole numbers.

3. If A can do $\frac{2}{7}$ of a piece of work in a day, B $\frac{2}{5}$ of it in a day, and C $\frac{2}{9}$ of it in a day, what part of it can they together do in one day?

Solution. $\frac{2}{7} =$ the part A can do in a day.
$\frac{2}{5} =$ " " B " " " "
$\frac{2}{9} =$ " " C " " " "

$\therefore \frac{2}{7} + \frac{2}{5} + \frac{2}{9}$, or $\frac{286}{315}$ = the part A, B, and C can do in a day.

Find the value of:

4. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$.
5. $\frac{3}{4} + \frac{7}{8} + \frac{5}{12}$.
6. $\frac{5}{6} + \frac{7}{12} + \frac{11}{18}$.
7. $\frac{3}{8} + \frac{5}{16} + \frac{3}{32}$.
8. $\frac{1}{15} + \frac{1}{10}$.
9. $\frac{1}{13} + \frac{1}{12}$.
10. $\frac{1}{25} + \frac{1}{26}$.
11. $\frac{1}{17} + \frac{1}{19}$.
12. $2\frac{3}{8} + 3\frac{5}{16}$.
13. $7\frac{1}{12} + 6\frac{5}{6}$.
14. $8\frac{3}{8} + 10\frac{2}{3}$.
15. $12\frac{3}{4} + 5\frac{1}{32}$.

16. $\frac{2}{9} + \frac{2}{7}$.
17. $\frac{3}{8} + \frac{3}{9}$.
18. $\frac{4}{7} + \frac{4}{10}$.
19. $\frac{5}{13} + \frac{5}{12}$.
20. $12\frac{1}{2} + 13\frac{2}{3} + 15\frac{1}{6}$.
21. $16\frac{2}{3} + 12\frac{1}{2} + 18\frac{3}{4}$.
22. $42\frac{2}{7} + 35\frac{5}{14} + 40\frac{2}{21}$.
23. $15\frac{2}{3} + 18\frac{1}{6} + 20\frac{3}{4}$.

24. There are three slates in a blackboard; the first is $4\frac{2}{3}$ feet long, the second $3\frac{3}{4}$ feet long, and the third $5\frac{1}{8}$ feet long; how long is the blackboard?

25. A owns $\frac{2}{15}$ of a certain capital, B $\frac{7}{20}$ of it, and C $\frac{3}{25}$ of it; what part of the capital do they together own?

26. If a train runs $16\frac{2}{3}$ miles in one hour, $18\frac{3}{4}$ miles the next hour, and $17\frac{5}{6}$ miles the next hour, how far does it go in 3 hours?

27. Mr. Smith bought $\frac{1}{24}$ of a certain tract of land, and Mr. Jones $\frac{1}{36}$ of it; what part of it did they together buy?

28. Mr. Hill gave $\frac{1}{16}$ of his money to a church, $\frac{1}{24}$ of it to a hospital, and $\frac{1}{12}$ of it to a college; what part of it did he give away?

Subtraction of Fractions.

140. PRIN. *The difference of two similar fractions equals the difference of their numerators divided by their common denominator.*

Thus, $\frac{7}{8} - \frac{3}{8} = (7 - 3)$ fractional units each of which is $\frac{1}{8}$ $= \frac{7-3}{8} = \frac{4}{8}$.

141. The difference of two dissimilar fractions may be found by first reducing them to fractions having a common denominator and then subtracting according to § 140.

142. The difference of two fractional units whose denominators are prime to each other is most readily found by taking for the numerator the difference of their denominators, and for the denominator the product of their denominators.

Thus, $\frac{1}{3} - \frac{1}{4} = \frac{4-3}{3 \times 4}$, or $\frac{1}{12}$.

EXERCISE 51. ORAL.

At sight find the value of:

1. $1-\frac{1}{3}$.

SOLUTION. $1-\frac{1}{3}=\frac{3}{3}-\frac{1}{3}=\frac{2}{3}$.

2. $1-\frac{4}{5}$.
3. $1-\frac{7}{9}$.
4. $1-\frac{4}{7}$.
5. $\frac{7}{3}-\frac{2}{3}$.
6. $\frac{5}{9}-\frac{2}{9}$.
7. $\frac{13}{8}-\frac{5}{8}$.
8. $\frac{1}{4}-\frac{1}{5}$.
9. $\frac{1}{5}-\frac{1}{6}$.
10. $\frac{1}{5}-\frac{1}{7}$.
11. $\frac{3}{4}-\frac{1}{2}$.
12. $\frac{5}{6}-\frac{2}{3}$.
13. $\frac{6}{7}-\frac{5}{14}$.
14. $2-\frac{2}{3}$.
15. $5-\frac{3}{4}$.
16. $2\frac{3}{4}-\frac{1}{4}$.
17. $3\frac{5}{6}-2$.

18. $3\frac{1}{4}-1\frac{3}{4}$.

SOLUTION. $3\frac{1}{4}-1\frac{3}{4}=2\frac{5}{4}-1\frac{3}{4}=1\frac{2}{4}=1\frac{1}{2}$.

19. $5\frac{5}{6}-3\frac{1}{2}$.

SUG. $5\frac{5}{6}-3\frac{1}{2}=5\frac{5}{6}-3\frac{3}{6}$.

20. $5\frac{3}{5}-2\frac{2}{5}$.
21. $6\frac{1}{2}-2\frac{1}{3}$.
22. $7\frac{3}{5}-4\frac{4}{5}$.
23. $2-1\frac{1}{4}$.
24. $4-2\frac{2}{3}$.
25. $1-\frac{4}{5}$.
26. $20\frac{4}{5}-17\frac{3}{5}$.
27. $22\frac{2}{7}-20\frac{4}{7}$.
28. $16\frac{1}{10}-14\frac{3}{5}$.

29. If James does $\frac{3}{5}$ of a piece of work and John does the remainder, what part of the work does John do?

30. Out of a bag containing $\frac{3}{4}$ of a bushel of oats, $\frac{1}{8}$ of a bushel was taken; how much remained?

31. A boy who has earned $\$2\frac{3}{4}$ must earn how much more to have $5?

32. A lady bought a pair of shoes for $\$2\frac{3}{8}$ and gave in payment $\$2\frac{1}{2}$; how much change should she receive?

33. A man bought a hat and gave in payment a ten-dollar bill; he received $\$6\frac{1}{4}$ in change. Find the cost of the hat.

34. B's lot contains $\frac{3}{4}$ of an acre, which is $\frac{2}{3}$ of an acre more than A's; how much does A's contain?

35. A jug that holds $2\frac{1}{2}$ gallons of vinegar lacking $\frac{3}{8}$ of a gallon, holds how much?

EXERCISE 52.

1. From $\frac{9}{14}$ take $\frac{7}{21}$.

SOLUTION. $\frac{9}{14} = \frac{9}{2 \times 7}$ and $\frac{7}{21} = \frac{7}{3 \times 7}$.

The l. c. m. of the denominators $= 2 \times 3 \times 7 =$ the lowest common denominator.

Now, $\frac{9}{14} = \frac{3 \times 9}{3 \times 2 \times 7} = \frac{27}{42}$, and $\frac{7}{21} = \frac{2 \times 7}{2 \times 3 \times 7} = \frac{14}{42}$.

$\therefore \frac{9}{14} - \frac{7}{21} = \frac{27}{42} - \frac{14}{42} = \frac{13}{42}$.

2. Find the difference of $124\frac{3}{8}$ and $98\frac{3}{4}$.

FORM.

$124\frac{3}{8} = 123\frac{11}{8}$.
$98\frac{3}{4} = 98\frac{6}{8}$.
$\therefore$ Diff. $= 25\frac{5}{8}$.

SUG. Since $\frac{3}{4}$ is more than $\frac{3}{8}$, take *one* from 124 and add it to $\frac{3}{8}$ before subtracting.

3. From $\frac{3}{7}$ take $\frac{3}{8}$.

SOLUTION. $\frac{3}{7} - \frac{3}{8} = 3 \times (\frac{1}{7} - \frac{1}{8}) = 3 \times \frac{1}{56}$ [§ 142] $= \frac{3}{56}$.

4. A farmer has a field containing $26\frac{3}{8}$ acres, from which he takes $5\frac{9}{16}$ acres for an orchard; how many acres remain in the field?

SOLUTION. $26\frac{3}{8} =$ the number of acres in the original field.
$5\frac{9}{16} =$ the number of acres taken for the orchard.
$\therefore 26\frac{3}{8} - 5\frac{9}{16}$, or $20\frac{13}{16} =$ the number of acres remaining in the field.

Find the value of:

5. $\frac{7}{8} - \frac{5}{12}$.
6. $\frac{8}{9} - \frac{7}{12}$.
7. $\frac{13}{15} - \frac{6}{25}$.
8. $\frac{17}{18} - \frac{5}{24}$.
9. $\frac{6}{7} - \frac{9}{35}$.
10. $\frac{1}{8} - \frac{1}{12}$.
11. $\frac{1}{9} - \frac{1}{10}$.
12. $\frac{1}{12} - \frac{1}{15}$.
13. $\frac{1}{20} - \frac{1}{30}$.
14. $\frac{1}{30} - \frac{1}{35}$.
15. $\frac{3}{5} - \frac{3}{7}$.
16. $\frac{3}{10} - \frac{3}{11}$.
17. $\frac{5}{12} - \frac{5}{16}$.
18. $\frac{7}{15} - \frac{7}{20}$.
19. $\frac{9}{20} - \frac{9}{25}$.
20. $18\frac{3}{4} - 5\frac{3}{5}$.
21. $25\frac{7}{12} - 13\frac{5}{8}$.
22. $128\frac{3}{10} - 39\frac{4}{5}$
23. $47\frac{3}{14} - 28\frac{2}{21}$
24. $100\frac{1}{9} - 37\frac{3}{8}$.

Find the value of:

25. $\frac{3}{4} + \frac{5}{6} - \frac{7}{8}$.
26. $\frac{5}{3} - \frac{5}{9} + \frac{5}{6}$.
27. $1 - (\frac{1}{5} + \frac{1}{6})$.
28. $2 + (\frac{3}{5} - \frac{1}{3})$.
29. $18\frac{3}{4} + 16\frac{2}{3} - 12\frac{1}{2}$.
30. $2\frac{15}{16} - (\frac{2}{5} + \frac{3}{10})$.
31. $1 - (\frac{3}{26} + \frac{5}{13})$.
32. $10 - (3\frac{3}{4} + 2\frac{5}{6})$.
33. $\frac{15}{16} - (\frac{3}{8} + \frac{1}{24})$.
34. $\frac{23}{21} - (\frac{6}{7} + \frac{3}{21})$.
35. $12 - (5\frac{5}{16} + 2\frac{3}{8})$.
36. $25 + (17\frac{1}{2} - 8\frac{2}{15})$

37. A man sold $\frac{3}{14}$ of his farm; what part of his farm had he remaining?

38. Mr. Jones owed \$124$\frac{3}{8}$ and paid \$25$\frac{3}{4}$; how much of his debt remained unpaid?

39. A farmer has $\frac{1}{6}$ of his farm in wheat, $\frac{1}{4}$ of it in corn, $\frac{1}{3}$ of it in grass, and the rest in barley; what part of it has he in barley?

40. A man has gone 14$\frac{3}{7}$ miles on his journey, which is 5$\frac{5}{14}$ miles farther than he has yet to go; what is the length of his journey?

41. A had 100 bushels of wheat. He used 24$\frac{3}{4}$ bushels for seed and had 16$\frac{7}{8}$ bushels made into flour; how many bushels had he remaining?

Multiplication of Fractions.

143. *To multiply by a whole number* is to take the multiplicand as many times as is denoted by the multiplier.

Thus, to multiply $\frac{2}{3}$ by 4 is to take $\frac{2}{3}$ *four* times.

EXERCISE 53. ORAL.

Find the value of:

1. $3 \times \frac{4}{5}$.

SOLUTION. Since multiplying the numerator of a fraction multiplies the value of the fraction [§ 125], 3 times $\frac{4}{5} = \frac{12}{5}$, or 2$\frac{2}{5}$.

2. $3 \times \frac{6}{7}$.
3. $4 \times \frac{5}{3}$.
4. $5 \times \frac{3}{7}$.
5. $6 \times \frac{5}{9}$.
6. $7 \times \frac{3}{4}$.
7. $8 \times \frac{5}{11}$.
8. $9 \times \frac{5}{8}$.
9. $10 \times \frac{3}{7}$.
10. $11 \times \frac{5}{6}$.
11. $12 \times \frac{4}{5}$.
12. $13 \times \frac{7}{4}$.
13. $14 \times \frac{5}{9}$.
14. $3 \times \frac{5}{6}$.

SOLUTION. Since dividing the denominator multiplies the value of a fraction [§ 128], 3 times $\frac{5}{6} = \frac{5}{2}$, or 2$\frac{1}{2}$.

15. $2 \times \frac{3}{4}$.
16. $3 \times \frac{5}{6}$.
17. $4 \times \frac{7}{8}$.
18. $5 \times \frac{3}{10}$.
19. $6 \times \frac{7}{12}$.
20. $7 \times \frac{4}{21}$.
21. $8 \times \frac{7}{16}$.
22. $9 \times \frac{4}{27}$.
23. $10 \times \frac{17}{20}$.
24. $5 \times \frac{7}{25}$.
25. $7 \times \frac{5}{42}$.
26. $8 \times \frac{5}{56}$.

27. $3 \times 4\frac{2}{3}$.

Solution. 3 times $4\frac{2}{3} = 3 \times 4 + 3 \times \frac{2}{3}$, or $12 + 2$, or 14.

28. $2 \times 5\frac{1}{2}$.
29. $3 \times 7\frac{5}{6}$.
30. $4 \times 5\frac{5}{8}$.
31. $4 \times 2\frac{1}{3}$.
32. $6 \times 7\frac{2}{5}$.
33. $7 \times 5\frac{5}{7}$.
34. $8 \times 4\frac{3}{4}$.
35. $9 \times 3\frac{4}{5}$.
36. $10 \times 8\frac{7}{8}$.
37. $6 \times 12\frac{1}{2}$.
38. $5 \times 14\frac{2}{3}$.
39. $7 \times 11\frac{1}{7}$.

144. *To multiply by a fraction* is to take such a part of the multiplicand as is denoted by the denominator as many times as is indicated by the numerator.

Thus, $\frac{3}{4}$ of $12 = 3 \times \frac{1}{4}$ of 12.

EXERCISE 54. ORAL.

Find the value of:

1. $\frac{2}{3} \times 15$.

Solution. $\frac{2}{3}$ of 15 equals 2 times $\frac{1}{3}$ of 15, which equals 2 times 5, or 10.

2. $\frac{3}{4} \times 9$.

Solution. $\frac{3}{4}$ of 9 equals $\frac{1}{4}$ of 3 times 9, which equals $\frac{1}{4}$ of 27, or $6\frac{3}{4}$.

3. $\frac{2}{5}$ of 10.
4. $\frac{5}{6}$ of 12.
5. $\frac{7}{8}$ of 24.
6. $\frac{2}{3}$ of 4.
7. $\frac{4}{7}$ of 5.
8. $\frac{5}{9}$ of 8.
9. $\frac{3}{8}$ of 40.
10. $\frac{4}{5}$ of 35.
11. $\frac{7}{3}$ of 5.
12. $\frac{5}{4}$ of 10.
13. $\frac{7}{15}$ of 30.
14. $\frac{12}{7}$ of 2.

15. $\frac{2}{3} \times \frac{4}{5}$.

Solution. $\frac{2}{3}$ of $\frac{4}{5}$ equals 2 times $\frac{1}{3}$ of $\frac{4}{5}$, which equals 2 times $\frac{4}{15}$ [§ 126], or $\frac{8}{15}$.

16. $\frac{2}{5}$ of $\frac{5}{7}$.
17. $\frac{3}{4}$ of $\frac{8}{9}$.
18. $\frac{3}{5}$ of $\frac{1}{2}$.
19. $\frac{2}{7}$ of $\frac{2}{3}$.
20. $\frac{3}{8}$ of $\frac{6}{5}$.
21. $\frac{4}{7}$ of $\frac{5}{8}$.
22. $\frac{7}{8}$ of $\frac{6}{5}$.
23. $\frac{4}{9}$ of $\frac{9}{10}$.
24. $\frac{3}{7}$ of $\frac{6}{5}$.
25. $\frac{3}{8}$ of $\frac{16}{3}$.
26. $\frac{4}{9}$ of $\frac{9}{2}$.
27. $\frac{3}{8}$ of $\frac{16}{5}$.

28. $2\frac{3}{4} \times 12$.

Solution. $2\frac{3}{4}$ times 12 equals 2 times $12 + \frac{3}{4}$ of 12, or $24 + 9$, or 33

29. $3\frac{1}{2} \times 6$.
30. $4\frac{2}{3} \times 9$.
31. $5\frac{1}{6} \times 12$
32. $3\frac{4}{5} \times 8$.
33. $6\frac{1}{4} \times 4$.
34. $5\frac{5}{7} \times 14$.
35. $7\frac{3}{8} \times 5$.
36. $8\frac{3}{5} \times 6$.
37. $9\frac{3}{4} \times 10$.
38. $10\frac{1}{2} \times 8$.
39. $7\frac{1}{2} \times 7$.
40. $6\frac{3}{4} \times 6$.

41. $2\frac{5}{6} \times 1\frac{1}{2}$.

Solution. $2\frac{5}{6}$ times $1\frac{1}{2}$ equals $\frac{17}{6}$ of $\frac{3}{2}$, or 17 times $\frac{1}{6}$ of $\frac{3}{2}$, or 17 times $\frac{1}{4}$, or $4\frac{1}{4}$.

42. $3\frac{1}{2} \times 1\frac{1}{4}$. **44.** $3\frac{1}{3} \times \frac{5}{6}$. **46.** $\frac{5}{6}$ of $2\frac{2}{3}$. **48.** $5\frac{1}{2} \times \frac{4}{5}$.
43. $2\frac{1}{4} \times 2\frac{1}{4}$. **45.** $1\frac{1}{2} \times \frac{4}{5}$. **47.** $\frac{7}{8}$ of $1\frac{1}{4}$. **49.** $\frac{3}{4} \times 1\frac{3}{5}$.

EXERCISE 55. ORAL.

1. At $\frac{3}{4}$ of a cent a pound, what is the value of 12 pounds of old iron?

Solution. If $\frac{3}{4}$ of a cent is the value of one pound, 12 times $\frac{3}{4}$ of a cent, or 9 cents, is the value of 12 pounds.

2. If it requires $1\frac{3}{4}$ yards of cloth to make a vest, how many yards are required for 6 vests of the same pattern?

3. A man walks a mile in $\frac{2}{5}$ of an hour; what time would he require to walk 8 miles at the same rate?

4. A rod is $5\frac{1}{2}$ yards; how many yards are there in 3 rods?

5. The distance around an iron hoop is $4\frac{2}{3}$ ft.; how far will it go in turning 4 times?

6. A snail was observed to go $1\frac{3}{4}$ feet in a minute, and it crossed a pavement in 8 minutes; how wide was the pavement?

7. What does $\frac{2}{3}$ of a yard of gingham cost at 12 cents a yard?

Solution. If 12 cents is the cost of one yard of gingham, $\frac{2}{3}$ of 12 cents, or 8 cents, is the cost of $\frac{2}{3}$ of a yard.

8. If one boy sold 24 newspapers and another $\frac{2}{3}$ as many, how many did the second boy sell?

9. At 12 cents a dozen, what will $2\frac{3}{4}$ dozen of eggs cost?

10. One race-horse trotted $\frac{2}{5}$ of a mile in a minute and another only $\frac{3}{4}$ as far in the same time; how far did the second trot?

11. There are $5\frac{1}{2}$ yards in a rod; how many yards in a line $\frac{2}{3}$ of a rod long?

12. A yard is $\frac{2}{11}$ as long as a rod, and a foot is $\frac{1}{3}$ as long as a yard; what part of a rod is a foot?

13. A man owned $\frac{4}{5}$ of a property and sold $\frac{2}{3}$ of his share; what part of the property did he retain?

14. Two men start at the same point to walk in opposite directions around a field; if the first walks on an average 50 yards a minute, and the second 40 yards a minute, and they meet in $6\frac{5}{8}$ minutes, how far is it around the field?

145. PRIN. *The product of two or more fractions equals the product of their numerators divided by the product of their denominators.*

Thus, $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$.

EXPLANATION. $\frac{2}{3} \times \frac{4}{5} = 2 \times \frac{1}{3}$ of $\frac{4}{5} = 2 \times \frac{4}{3 \times 5}$ [§126] $= \frac{2 \times 4}{3 \times 5}$ [§ 125].

146. PRIN. *Multiplying the numerator of one fraction and the denominator of another by the same number does not change the value of their product.*

Thus, $\frac{6}{5} \times \frac{4}{9} = \frac{3 \times 6}{5} \times \frac{4}{3 \times 9}$.

EXPLANATION. Multiplying the numerator of $\frac{6}{5}$ by 3 and the denominator of $\frac{4}{9}$ by 3, multiplies $\frac{6}{5}$ by 3 [§ 125] and divides $\frac{4}{9}$ by 3 [§ 126] and, therefore, does not alter the value of their product [§ 98].

147. PRIN. *Dividing the numerator of one fraction and the denominator of another by the same number does not change the value of their product.*

Thus, $\frac{6}{5} \times \frac{4}{9} = \frac{6 \div 3}{5} \times \frac{4}{9 \div 3}$.

EXPLANATION. Dividing the numerator of $\frac{6}{5}$ by 3 and the denominator of $\frac{4}{9}$ by 3, divides $\frac{6}{5}$ by 3 [§ 127] and multiplies $\frac{4}{9}$ by 3 [§ 128] and, therefore, does not change the value of their product [§ 98].

COR. *Cancelling a factor common to the numerator of one fraction and the denominator of another does not alter the value of their product.*

Thus, $\frac{5}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{6}}}{7} = \frac{10}{21}$.

For, 9, the denominator of the first fraction, is divided by 3, and 6, the numerator of the second fraction, is divided by 3, which does not change the value of their product [§147].

EXERCISE 56.

1. Multiply $\frac{5}{13}$ by 7.

SUG. $7 \times \frac{5}{13} = \frac{7 \times 5}{13}$ [§ 125].

2. Multiply $\frac{7}{18}$ by 9.

SUG. $9 \times \frac{7}{18} = \frac{7}{18 \div 9}$ [§ 128].

3. Multiply $\frac{7}{15}$ by 6.

SUG. $\overset{2}{\cancel{6}} \times \frac{7}{\underset{5}{\cancel{15}}} = 2 \times \frac{7}{5}$ [§ 98].

Find the value of:

4. $\frac{3}{17}$ of 51.
5. $\frac{5}{19}$ of 95.
6. $\frac{31}{32}$ of 224.
7. $\frac{7}{15}$ of 17.
8. $\frac{19}{21}$ of 100.
9. $\frac{13}{47}$ of 120.
10. $\frac{4}{25}$ of 45.
11. $\frac{17}{48}$ of 80.
12. $\frac{23}{36}$ of 126.
13. $\frac{52}{17}$ of 68.
14. $\frac{31}{9}$ of 42
15. $\frac{67}{15}$ of 35.

16. Multiply $\frac{5}{7}$ by $\frac{3}{8}$.

SUG. $\frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7}$ [§ 145].

17. Multiply $\frac{19}{25}$ by $\frac{15}{38}$.

SUG. $\frac{\cancel{19}}{\underset{5}{\cancel{25}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{38}}} = \frac{1}{5} \times \frac{3}{2}$ [§ 147, Cor.].

Find the value of:

18. $\frac{1}{12} \times \frac{1}{11}$.
19. $\frac{12}{13} \times \frac{6}{13}$.
20. $\frac{3}{17} \times \frac{15}{4}$.
✓ **21.** $\frac{14}{15} \times \frac{35}{48}$.
22. $\frac{39}{40} \times \frac{28}{65}$.
23. $\frac{14}{65} \times \frac{25}{21}$.
24. $\frac{3}{4} \times \frac{5}{9} \times \frac{12}{25}$.
25. $\frac{7}{12} \times \frac{5}{21} \times \frac{3}{25}$.
26. $\frac{17}{11} \times \frac{22}{85} \times \frac{5}{4}$.

27. Multiply $18\frac{3}{4}$ by 16.

Sug. $16 \times 18\frac{3}{4} = 16 \times 18 + 16 \times \frac{3}{4}$.

28. Multiply 24 by $4\frac{3}{4}$.

Sug. $4\frac{3}{4} \times 24 = 4 \times 24 + \frac{3}{4}$ of 24.

29. Multiply $14\frac{2}{3}$ by $9\frac{5}{6}$.

Sug. $9\frac{5}{6} \times 14\frac{2}{3} = 9 \times 14 + 9 \times \frac{2}{3} + \frac{5}{6}$ of $14 + \frac{5}{6}$ of $\frac{2}{3}$.

30. Multiply $1\frac{11}{12}$ by $3\frac{9}{13}$.

Sug. $3\frac{9}{13} \times 1\frac{11}{12} = \frac{48}{13} \times \frac{23}{12}$.

Find the value of:

31. $12 \times 18\frac{3}{4}$.

32. $36 \times 20\frac{5}{6}$.

33. $21\frac{3}{7} \times 42$.

34. $42\frac{5}{8} \times 96$.

35. $36\frac{3}{4} \times 200\frac{5}{6}$.

36. $2\frac{102}{103} \times 6\frac{7}{16}$.

37. Find the cost of $18\frac{3}{4}$ cords of wood at \$$4\frac{1}{2}$ a cord.

Solution. \$$4\frac{1}{2}$ = the cost of one cord.
∴ $18\frac{3}{4}$ × \$$4\frac{1}{2}$, or \$$84\frac{3}{8}$ = the cost of $18\frac{3}{4}$ cords.

38. On a map, $\frac{7}{12}$ of an inch is made to represent a mile; what length on the map would represent 25 miles?

39. Find the selling price of a flock of 20 sheep that sold at an average of \$$5\frac{3}{4}$ each.

40. A cubic foot of water weighs $62\frac{1}{2}$ pounds, and platinum is about 22 times as heavy as water; find the weight of a cubic foot of platinum.

41. There are $16\frac{1}{2}$ feet in a rod; how many feet long is a field whose length is $35\frac{1}{2}$ rods?

42. A barrel holding $31\frac{1}{2}$ gallons is $\frac{3}{4}$ full of vinegar; how many gallons of vinegar are there in the barrel?

43. A mechanic was employed for \$65 a month, and paid \$$37\frac{1}{2}$ a month for boarding and other expenses; how much did he save in 12 months?

44. A foot is 12 times an inch, and a rod is $16\frac{1}{2}$ times a foot; how many inches are there in $20\frac{1}{11}$ rods?

45. On $10\frac{1}{2}$ acres of land a farmer sows $1\frac{1}{2}$ bushels of clover seed valued at \$$5\frac{1}{2}$ per bushel, and on each acre of the field he sows $2\frac{3}{4}$ bushels of wheat valued at $\frac{4}{5}$ of a dollar a bushel; find the value of the seed sown on this field.

46. A dealer bought $12\frac{1}{2}$ yards of cloth at $\$\frac{7}{8}$ a yard; but the cloth having become damaged by water, shrunk to $\frac{11}{12}$ of its original length, and was then sold at $\frac{4}{5}$ as much per yard as it cost; find the amount received.

47. If a cubic foot of water weighs $62\frac{1}{2}$ pounds, and a gallon is $\frac{231}{1728}$ of a cubic foot, find the weight of a gallon of water.

Division of Fractions.

148. *To divide by a whole number* is to take such a part of the dividend as is denoted by the divisor.

Thus, to divide $\frac{4}{5}$ by 3 is to find $\frac{1}{3}$ of $\frac{4}{5}$.

EXERCISE 57. ORAL.

Find the value of:

1. $\frac{6}{7} \div 3$, or $\frac{1}{3}$ of $\frac{6}{7}$.

Solution. Since dividing the numerator of a fraction divides the value of the fraction, $\frac{1}{3}$ of $\frac{6}{7}$ is $\frac{2}{7}$.

2. $\frac{4}{5} \div 2$.
3. $\frac{9}{4} \div 3$.
4. $\frac{12}{5} \div 4$.
5. $\frac{15}{8} \div 5$.
6. $\frac{18}{5} \div 6$.
7. $\frac{35}{9} \div 7$.
8. $\frac{64}{11} \div 8$.
9. $\frac{72}{5} \div 9$.
10. $\frac{90}{13} \div 10$.
11. $\frac{88}{3} \div 11$.
12. $\frac{72}{7} \div 12$.
13. $\frac{52}{3} \div 13$.

14. $\frac{2}{5} \div 4$, or $\frac{1}{4}$ of $\frac{2}{5}$.

Solution. Since multiplying the denominator of a fraction divides the value of the fraction, $\frac{1}{4}$ of $\frac{2}{5}$ is $\frac{2}{20}$, or $\frac{1}{10}$.

15. $\frac{5}{3} \div 2$.
16. $\frac{5}{4} \div 3$.
17. $\frac{7}{8} \div 4$.
18. $\frac{17}{3} \div 5$.
19. $\frac{13}{8} \div 6$.
20. $\frac{8}{11} \div 7$.
21. $\frac{6}{5} \div 8$.
22. $\frac{6}{7} \div 9$.
23. $\frac{15}{4} \div 10$.
24. $\frac{7}{10} \div 11$.
25. $\frac{8}{9} \div 12$.
26. $\frac{10}{7} \div 20$.

27. $15\frac{3}{4} \div 4$.

Solution. $\frac{1}{4}$ of $15\frac{3}{4}$ is 3, and $3\frac{3}{4}$, or $\frac{15}{4}$ over; $\frac{1}{4}$ of $\frac{15}{4}$ is $\frac{15}{16}$; hence, $\frac{1}{4}$ of $15\frac{3}{4}$ is $3\frac{15}{16}$.

28. $12\frac{4}{5} \div 2$.
29. $15\frac{3}{5} \div 3$.
30. $24\frac{8}{9} \div 4$.
31. $16\frac{1}{4} \div 5$.
32. $18\frac{3}{5} \div 6$.
33. $21\frac{7}{8} \div 7$.
34. $1\frac{1}{5} \div 8$.
35. $2\frac{3}{4} \div 9$.
36. $6\frac{1}{2} \div 10$.
37. $22\frac{1}{2} \div 11$.
38. $16 \div 12$.
39. $15 \div 13$.

149. *To divide by a fraction* is to find a result which multiplied by the fraction will produce the dividend.

Thus, 12 divided by $\frac{2}{3}$ gives a result, $\frac{2}{3}$ of which is 12.

EXERCISE 58. ORAL.

Find the value of:

1. $12 \div \frac{2}{3}$.

Solution. $\frac{2}{3}$ of the quotient is 12; hence, $\frac{1}{3}$ of the quotient is $\frac{1}{2}$ of 12, or 6, and the quotient is 3 times 6, or 18.

2. $9 \div \frac{3}{4}$.
3. $12 \div \frac{2}{3}$.
4. $15 \div \frac{5}{6}$.
5. $16 \div \frac{4}{5}$.
6. $18 \div \frac{6}{7}$.
7. $21 \div \frac{3}{4}$.
8. $24 \div \frac{4}{3}$.
9. $25 \div \frac{5}{4}$.
10. $28 \div \frac{4}{7}$.
11. $26 \div \frac{2}{3}$.
12. $30 \div \frac{3}{5}$.
13. $32 \div \frac{8}{7}$.
14. $35 \div \frac{7}{8}$.
15. $36 \div \frac{9}{10}$.
16. $40 \div \frac{10}{3}$.
17. $45 \div \frac{5}{2}$.

18. $8 \div \frac{3}{4}$.

Solution. $\frac{3}{4}$ of the quotient is 8; hence, $\frac{1}{4}$ of the quotient is $\frac{1}{3}$ of 8, or $\frac{8}{3}$, and the quotient is 4 times $\frac{8}{3}$, or $\frac{32}{3}$, or $10\frac{2}{3}$.

19. $6 \div \frac{5}{2}$.
20. $5 \div \frac{4}{5}$.
21. $8 \div \frac{7}{3}$.
22. $9 \div \frac{4}{3}$.
23. $7 \div \frac{5}{3}$.
24. $10 \div \frac{4}{7}$.
25. $12 \div \frac{5}{3}$.
26. $15 \div \frac{1}{2}$.

27. $\frac{2}{3} \div \frac{3}{4}$.

Solution. $\frac{3}{4}$ of the quotient is $\frac{2}{3}$; hence, $\frac{1}{4}$ of the quotient is $\frac{1}{3}$ of $\frac{2}{3}$, or $\frac{2}{9}$, and the quotient is 4 times $\frac{2}{9}$, or $\frac{8}{9}$.

28. $\frac{2}{3} \div \frac{2}{5}$.
29. $\frac{3}{4} \div \frac{3}{2}$.
30. $\frac{5}{6} \div \frac{2}{3}$.
31. $\frac{1}{4} \div \frac{3}{4}$.
32. $\frac{1}{7} \div \frac{5}{6}$.
33. $\frac{2}{9} \div \frac{2}{3}$.
34. $\frac{7}{8} \div \frac{1}{3}$.
35. $\frac{4}{5} \div \frac{2}{5}$.
36. $\frac{3}{8} \div \frac{1}{4}$.
37. $\frac{3}{10} \div \frac{6}{5}$.
38. $\frac{7}{5} \div \frac{3}{2}$.
39. $\frac{5}{12} \div \frac{5}{6}$.

40. $12 \div 2\frac{2}{3}$.

Solution. Multiplying both dividend and divisor by 3, we have 36 divided by 8, which is $4\frac{1}{2}$.

41. $5 \div 2\frac{1}{2}$.
42. $7 \div 3\frac{1}{2}$.
43. $9 \div 2\frac{1}{4}$.
44. $6 \div 1\frac{1}{2}$.
45. $8 \div 2\frac{2}{3}$.
46. $10 \div 3\frac{1}{3}$.
47. $11 \div 2\frac{1}{2}$.
48. $12 \div 1\frac{2}{3}$.

49. $2\frac{1}{2} \div 3\frac{1}{3}$.

Solution. Multiplying both dividend and divisor by 6, we have 15 divided by 20, which is $\frac{15}{20}$, or $\frac{3}{4}$.

50. $\frac{3}{5} \div 2\frac{1}{5}$.
51. $\frac{2}{3} \div 3\frac{1}{3}$.
52. $2\frac{1}{2} \div 1\frac{1}{4}$.
53. $1\frac{1}{2} \div 1\frac{3}{4}$.
54. $3\frac{2}{3} \div \frac{5}{6}$.
55. $\frac{5}{6} \div 3\frac{2}{3}$.
56. $7\frac{1}{2} \div 3\frac{3}{4}$.
57. $3\frac{3}{4} \div 7\frac{1}{2}$.

EXERCISE 59. ORAL.

1. Four bottles of equal size together hold 1 gill; what part of a gill does each bottle hold?

2. When eggs are selling at 16 cents per dozen, what is the price of one egg?

3. If a man takes 10 steps to go 8 yards, what is the average length of his steps?

4. A lot containing $\frac{4}{11}$ of an acre is divided into 2 lots of equal size; how much does each lot contain?

5. A string $\frac{7}{8}$ of a yard long will just reach three times around a tree; what is the distance around the tree?

6. A measuring wheel makes 6 turns in measuring $18\frac{1}{2}$ inches; what is the distance around the wheel?

7. A man wishes to plant a row of 6 shade trees at equal distances apart along one side of a lawn $12\frac{1}{2}$ rods long; how far apart must he plant the trees, if there is one at each end of the row?

8. A milliner wishes to make 20 bows out of $2\frac{1}{2}$ yards of ribbon; what part of a yard must she use for each bow?

9. A lady paid 6 cents for $\frac{3}{4}$ of a yard of muslin; what was the price per yard?

Solution. If 6 cents was the cost of $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of 6 cents, or 2 cents, was the cost of $\frac{1}{4}$ of a yard, and 4 × 2 cents, or 8 cents, was the cost of a yard.

10. If 24 bushels of potatoes were raised on $\frac{3}{4}$ of an acre, how many bushels at the same rate could be raised on an acre?

11. If $\frac{3}{4}$ of a bushel of oats weighs 24 pounds, what does a bushel weigh?

12. After spending 12 cents, a boy finds he has spent $\frac{4}{5}$ of his money; how much money had he?

13. A man after walking 8 miles finds he has yet $\frac{5}{9}$ of his journey to go; what is the length of his journey?

14. What is the price of nails per pound when 2½ pounds cost 10 cents?

15. A lady was given 6¼ pounds of sugar for 25 cents; how much was charged for it per pound?

16. If a barrel, or 2½ bushels, of corn sells for $1, what part of a dollar does a bushel cost?

17. There are 5½ yards in a rod; what part of a rod is there in a yard?

18. A farmer uses 7 bushels of seed in sowing a field; how many acres are there in the field, if he sows 2½ bushels of seed to the acre?

19. If ⅔ of the distance from A to B is ¾ of a mile, what is the distance?

20. The length of a pole is 7½ feet, which is ¾ of the length of its shadow; how long is the shadow?

21. When eggs sell at 25 cents for 20, how much is that a dozen?

22. When matches sell at 3 boxes for 5 cents, how much is that per dozen boxes?

23. If ⅔ of an inch on a map represents a mile, what length does ¾ of an inch on the same map represent?

24. A planted 1¼ acres in potatoes. This was ⅓ of his lot If he planted ⅖ of the lot in corn, how many acres did he plant in corn?

25. A grocer charges 45 cents for ¾ of a pound of tea; at this rate what should he charge for ½ of a pound?

150. PRIN. *The quotient obtained by dividing one fraction by another equals the dividend multiplied by the inverse of the divisor.*

Thus, $\frac{5}{6} \div \frac{3}{7} = \frac{7}{3} \times \frac{5}{6}$.
For, $\frac{3}{7}$ of the quotient $= \frac{5}{6}$ [§ 149].
$\therefore$ $\frac{1}{7}$ of the quotient $= \frac{1}{3}$ of $\frac{5}{6}$.
And, $\frac{7}{7}$ of the quotient $= 7 \times \frac{1}{3}$ of $\frac{5}{6}$, or $\frac{7}{3}$ of $\frac{5}{6}$.
That is, the quotient $= \frac{7}{3} \times \frac{5}{6}$.

Note. An integer may be regarded as a fraction whose denominator is 1.

Thus, $8 = \frac{8}{1}$.

EXERCISE 60.

1. Divide $\frac{8}{43}$ by 4.

Sug. $\frac{8}{43} \div 4 = \frac{8 \div 4}{43}$ [§ 127].

2. Divide $\frac{15}{4}$ by 7.

Sug. $\frac{15}{4} \div 7 = \frac{15}{7 \times 4}$ [§ 126].

3. Divide $167\frac{5}{6}$ by 3.

Form.

$$\begin{array}{r} 3\,\big)\,\underline{167\frac{5}{6}} \\ 55\frac{17}{18} \end{array}$$

Solution. $\frac{1}{3}$ of $167\frac{5}{6} = 55$, and $2\frac{5}{6}$, or $\frac{17}{6}$ over. $\frac{1}{3}$ of $\frac{17}{6} = \frac{17}{18}$.
$\therefore \frac{1}{3}$ of $167\frac{5}{6} = 55\frac{17}{18}$.

Find the value of:

4. $\frac{16}{17} \div 8$. **8.** $\frac{13}{7} \div 9$. **12.** $\frac{18}{5} \div 24$. **16.** $\frac{56}{121} \div 14$.
5. $\frac{28}{45} \div 14$. **9.** $\frac{9}{17} \div 14$. **13.** $\frac{32}{33} \div 20$. **17.** $\frac{33}{4} \div 37$.
6. $\frac{45}{37} \div 15$. **10.** $\frac{12}{8} \div 24$. **14.** $\frac{144}{7} \div 80$. **18.** $\frac{35}{17} \div 25$.
7. $\frac{150}{11} \div 25$. **11.** $\frac{145}{13} \div 13$. **15.** $\frac{24}{37} \div 96$. **19.** $\frac{324}{41} \div 36$.

20. Divide 12 by $\frac{2}{3}$.

Sug. $12 \div \frac{2}{3} = \frac{3}{2} \times 12$ [§ 150].

21. Divide $\frac{2}{3}$ by $\frac{5}{9}$.

Sug. $\frac{2}{3} \div \frac{5}{9} = \frac{9}{5} \times \frac{2}{3}$ [§ 150].

Find the value of:

22. $45 \div \frac{5}{9}$. **25.** $\frac{8}{9} \div \frac{7}{15}$. **28.** $1 \div \frac{43}{45}$. **31.** $\frac{12}{17} \div \frac{3}{85}$.
23. $90 \div \frac{3}{10}$. **26.** $\frac{31}{32} \div \frac{15}{16}$. **29.** $123 \div \frac{7}{13}$. **32.** $5 \div \frac{15}{28}$.
24. $240 \div \frac{13}{9}$. **27.** $\frac{45}{31} \div \frac{9}{11}$. **30.** $\frac{120}{23} \div \frac{65}{46}$. **33.** $\frac{1}{3} \div \frac{1}{21}$.

34. Divide $18\frac{3}{4}$ by $12\frac{1}{2}$.

Sug. $18\frac{3}{4} \div 12\frac{1}{2} = (4 \times 18\frac{3}{4}) \div (4 \times 12\frac{1}{2})$ [§ 99].

35. Divide $1\frac{5}{6}$ by $3\frac{2}{7}$.

Sug. $1\frac{5}{6} \div 3\frac{2}{7} = \frac{11}{6} \div \frac{23}{7} = \frac{7}{23} \times \frac{11}{6}$.

Find the value of:

36. $12\frac{1}{2} \div 4$. **39.** $6 \div 12\frac{1}{2}$. **42.** $\frac{5}{8} \div 2\frac{3}{8}$. **45.** $120\frac{3}{4} \div 12\frac{1}{2}$.
37. $16\frac{2}{3} \div 6$. **40.** $20 \div 3\frac{1}{3}$. **43.** $\frac{3}{4} \div 10\frac{1}{2}$. **46.** $64\frac{7}{8} \div 4\frac{3}{8}$.
38. $36\frac{5}{8} \div 9$. **41.** $48 \div 15\frac{1}{5}$. **44.** $6\frac{1}{4} \div \frac{5}{8}$. **47.** $2\frac{1}{3} \div 39\frac{2}{3}$.
48. $\frac{3}{4} \times \frac{5}{6} \div \frac{7}{12}$. **50.** $(\frac{7}{8} - \frac{3}{8}) \div \frac{3}{40}$. **52.** $(12\frac{1}{2} + 6\frac{1}{4}) \div 3\frac{1}{8}$.
49. $(\frac{3}{4} + \frac{5}{6}) \div \frac{7}{12}$. **51.** $(\frac{3}{10} \div \frac{2}{5}) \times \frac{5}{6}$. **53.** $(12\frac{1}{2} \times 6\frac{1}{4}) \div 3\frac{1}{8}$.

54. A man who owned a two-thirds interest in a mine, divided his interest equally among his four sons; what part of the entire interest was given to each son?

SOLUTION. $\frac{2}{3}$ = the part of the interest owned by the father.
4 = the number of sons receiving this interest.
∴ $\frac{1}{4}$ of $\frac{2}{3}$, or $\frac{1}{6}$ = the part of the entire interest received by each son.

55. The product of two fractions is $\frac{9}{10}$ and one of them is $\frac{4}{5}$; what is the other?

56. If $4\frac{1}{2}$ bushels of good wheat make 196 pounds, or 1 barrel, of flour, how many pounds of flour does each bushel of wheat make?

57. If 1 cubic foot of water will make $1\frac{1}{10}$ of a cubic foot of ice, what part of a cubic foot of water will make 1 cubic foot of ice?

58. A young man's salary was increased by $\frac{1}{3}$ of itself every year; the third year it was $640; what was it each of the other years?

59. A cannon ball traveling at the rate of 2500 feet in $2\frac{1}{2}$ seconds travels how far per second?

60. What is the distance around a race track, if a cyclist goes around it $2\frac{2}{3}$ times to ride a mile?

61. A horse cost $5\frac{5}{8}$ times as much as a set of harness and they together cost $118; find the cost of each.

62. A bushel of oats is $\frac{8}{15}$ as heavy as a bushel of wheat and the difference of their weights is 28 pounds; what is the weight of a bushel of each?

63. The product of three fractions is $1\frac{24}{25}$ and two of them are $\frac{8}{9}$ and $\frac{7}{12}$; what is the third?

Mensuration of Fractions.

151. *To measure one number or amount by another* is to find how many times, or what part of, the second is the first.

Thus, to measure 5 by 4 is to find how many times 4 is 5; to measure 2 by 3 is to find what part of 3 is 2; to measure 4 by $\frac{2}{3}$ is to find how many times $\frac{2}{3}$ is 4; to measure $\frac{2}{3}$ by 4 is to find what part of 4 is $\frac{2}{3}$.

EXERCISE 61. ORAL.

1. Measure 3 by 2; that is, find how many times 2 is 3.

SOLUTION 1. 3 measured by 1 is 3; therefore, 3 measured by 2 is $\frac{1}{2}$ of 3, or $\frac{3}{2}$, or $1\frac{1}{2}$.

SOLUTION 2. *One* is $\frac{1}{2}$ of 2, and 3 is 3 times $\frac{1}{2}$ of 2, or $\frac{3}{2}$ of 2.

2. Measure 4 by 3; 3 by 4.

3. What part of 4 is 3? 3 is 4?

4. Measure $\frac{2}{3}$ by 4; that is, find what part of 4 is $\frac{2}{3}$.

SOLUTION 1. $\frac{2}{3}$ measured by 1 is $\frac{2}{3}$; hence, $\frac{2}{3}$ measured by 4 is $\frac{1}{4}$ of $\frac{2}{3}$, or $\frac{1}{6}$.

SOLUTION 2. *One* is $\frac{1}{4}$ of 4; hence, $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{1}{4}$ of 4, or $\frac{1}{12}$ of 4; and $\frac{2}{3}$ is 2 times $\frac{1}{12}$ of 4, or $\frac{1}{6}$ of 4.

Measure:

5. $\frac{5}{8}$ by 5. **6.** $\frac{13}{2}$ by 3. **7.** $\frac{3}{4}$ by 12. **8.** $\frac{7}{8}$ by 4.

What part of:

9. 3 is $\frac{2}{5}$? **10.** 9 is $\frac{4}{5}$? **11.** 6 is $\frac{4}{9}$? **12.** 3 is $\frac{3}{4}$?

13. Measure 6 by $\frac{2}{3}$; that is, find how many times $\frac{2}{3}$ is 6.

SOLUTION 1. 6 measured by 1 is 6; hence, 6 measured by $\frac{1}{3}$ is 3 times 6, or 18; and 6 measured by $\frac{2}{3}$ is $\frac{1}{2}$ of 18, or 9.

SOLUTION 2. $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$; hence, 1 is $\frac{3}{2}$ of $\frac{2}{3}$; and 6 is 6 times $\frac{3}{2}$ of $\frac{2}{3}$, or 9 times $\frac{2}{3}$.

Measure:

14. 8 by $\frac{3}{4}$. **15.** 15 by $\frac{2}{5}$. **16.** 21 by $\frac{7}{8}$. **17.** 18 by $\frac{9}{2}$.

What part of:

18. 9 is $\frac{5}{6}$? **19.** 14 is $\frac{3}{4}$? **20.** 10 is $\frac{5}{2}$? **21.** 25 is $\frac{5}{9}$?

22. Measure $\frac{2}{3}$ by $\frac{3}{4}$; that is, find what part of $\frac{3}{4}$ is $\frac{2}{3}$.

Solution 1. $\frac{2}{3}$ measured by 1 is $\frac{2}{3}$; hence, $\frac{2}{3}$ measured by $\frac{1}{4}$ is 4 times $\frac{2}{3}$, or $\frac{8}{3}$; and $\frac{2}{3}$ measured by $\frac{3}{4}$ is $\frac{1}{3}$ of $\frac{8}{3}$, or $\frac{8}{9}$.

Solution 2. $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$; hence, 1 is $\frac{4}{3}$ of $\frac{3}{4}$; and $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{4}{3}$ of $\frac{3}{4}$, or $\frac{4}{9}$ of $\frac{3}{4}$; whence, $\frac{2}{3}$ is 2 times $\frac{4}{9}$ of $\frac{3}{4}$, or $\frac{8}{9}$ of $\frac{3}{4}$.

Measure:

23. $\frac{1}{3}$ by $\frac{1}{4}$. **24.** $\frac{1}{8}$ by $\frac{2}{5}$. **25.** $\frac{3}{4}$ by $\frac{2}{3}$. **26.** $\frac{4}{3}$ by $\frac{2}{9}$.

What part of:

27. $\frac{1}{6}$ is $\frac{1}{5}$? **28.** $\frac{4}{5}$ is $\frac{2}{3}$? **29.** $\frac{4}{5}$ is $\frac{3}{10}$? **30.** $\frac{2}{3}$ is $\frac{3}{4}$?

31. Measure $2\frac{1}{2}$ by 6; that is, find what part of 6 is $2\frac{1}{2}$.

Solution 1. Since multiplying both antecedent and consequent by the same number does not alter the ratio, $2\frac{1}{2}$ measured by 6 equals 2 times $2\frac{1}{2}$, or 5, measured by 2 times 6, or 12, which is $\frac{5}{12}$.

Solution 2. $2\frac{1}{2}$ is the same part of 6 that $\frac{5}{2}$ is of $\frac{12}{2}$, or that 5 is of 12, which is $\frac{5}{12}$.

Measure:

32. $3\frac{1}{3}$ by 9. **33.** 4 by $2\frac{2}{3}$. **34.** $4\frac{1}{6}$ by $\frac{1}{12}$. **35.** $6\frac{1}{4}$ by $12\frac{1}{2}$.

What part of:

36. 11 is $5\frac{1}{2}$? **37.** $3\frac{1}{2}$ is $2\frac{1}{2}$? **38.** $1\frac{1}{4}$ is $\frac{7}{8}$? **39.** $18\frac{3}{4}$ is $12\frac{1}{2}$?

40. A earns \$3 a day and B \$5; what part of A's wages equals B's?

Solution. Since \$3 measured by \$5 equals $\frac{3}{5}$, A's wages are $\frac{3}{5}$ of B's.

41. B owned a 3-acre lot and sold 2 acres of it; what part of the lot did he sell?

42. When oranges are bought at 20 cents a dozen and sold at a gain of 5 cents a dozen, what part of the cost equals the gain?

43. A lady spends $\frac{2}{3}$ of a dollar for cloth at \$2 a yard; what part of a yard does she buy?

44. A yard is $\frac{2}{11}$ of a rod; how many yards in a rod?

45. In 10 bushels of mixed grain $3\frac{1}{3}$ bushels are corn; what part of the grain is corn?

46. How many strings $\frac{3}{4}$ of a foot long can be cut from a line 9 feet long?

Solution. Since 9 measured by $\frac{3}{4}$ equals $\frac{4}{3}$ of 9, or 12, the number of strings $\frac{3}{4}$ of a foot long that can be made from a line 9 feet long is 12.

47. If $\frac{3}{4}$ of a yard of lining is needed for a yard of dress goods, how much dress goods can be lined with a yard of lining?

48. How long will 4 pounds of coffee last a family that uses half a pound a day?

49. A rod is $16\frac{1}{2}$ feet and a yard is 3 feet; how many yards are there in a rod?

50. At $\frac{3}{4}$ of a dollar a yard, what part of a yard of dress goods should $\frac{1}{2}$ of a dollar buy?

51. How many bags each holding $2\frac{1}{2}$ bushels are required to hold $12\frac{1}{2}$ bushels of grain?

EXERCISE 62.

152. Prin. *The ratio of two numbers equals the antecedent divided by the consequent.*

Note 1. This principle, before stated in § 60, is here shown to apply to both integral and fractional numbers.

Note 2. Since mixed numbers may be reduced to improper fractions, the principle also applies to mixed numbers.

Thus, **1.** $2 : 5 = 2 \div 5$.

For, $2 : 1 = 2$; hence, $2 : 5 = \frac{1}{5}$ of $2 = 2 \div 5$.

2. $\frac{3}{4} : 4 = \frac{3}{4} \div 4$.

For, $\frac{3}{4} : 1 = \frac{3}{4}$; hence, $\frac{3}{4} : 4 = \frac{1}{4}$ of $\frac{3}{4} = \frac{3}{4} \div 4$.

3. $5 : \frac{4}{7} = 5 \div \frac{4}{7}$.

For, $5 : 1 = 5$; hence, $5 : \frac{1}{7} = 7 \times 5$, and $5 : \frac{4}{7} = \frac{1}{4}$ of $7 \times 5 = \frac{7}{4}$ of $5 = 5 \div \frac{4}{7}$ [§ 150].

4. $\frac{3}{5} : \frac{2}{3} = \frac{3}{5} \div \frac{2}{3}$.

For, $\frac{3}{5} : 1 = \frac{3}{5}$; hence, $\frac{3}{5} : \frac{1}{3} = 3 \times \frac{3}{5}$, and $\frac{3}{5} : \frac{2}{3} = \frac{1}{2}$ of $3 \times \frac{3}{5} = \frac{3}{2}$ of $\frac{3}{5} = \frac{3}{5} \div \frac{2}{3}$ [§ 150].

Measure:

5. $\frac{3}{4}$ by 7.

Solution. $\frac{3}{4} : 7 = \frac{3}{4} \div 7$ [§ 152] $= \frac{3}{28}$.

6. $\frac{5}{6}$ by 9.
7. $\frac{12}{25}$ by 6.
8. $\frac{8}{9}$ by 12.
9. $\frac{15}{16}$ by 25.
10. $\frac{14}{33}$ by 28.
11. $\frac{8}{125}$ by 16.
12. $\frac{12}{11}$ by 36.
13. $\frac{28}{31}$ by 21.

14. 6 by $\frac{4}{9}$.

Solution. $6 : \frac{4}{9} = 6 \div \frac{4}{9} = \frac{9}{\underset{2}{\cancel{4}}} \times \overset{3}{\cancel{6}} = \frac{27}{2} = 13\,\frac{1}{2}$.

15. 18 by $\frac{6}{7}$.
16. 24 by $\frac{12}{11}$.
17. 1 by $\frac{15}{16}$.
18. 5 by $\frac{65}{63}$.
19. 23 by $\frac{46}{7}$.
20. 40 by $\frac{25}{26}$.
21. 100 by $\frac{1}{7}$.
22. 250 by $\frac{1}{25}$.

23. $\frac{3}{4}$ by $\frac{9}{20}$.

Solution. $\frac{3}{4} : \frac{9}{20} = \frac{3}{4} \div \frac{9}{20} = \frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{\cancel{4}} = \frac{5}{3} = 1\,\frac{2}{3}$.

24. $\frac{3}{8}$ by $\frac{21}{24}$.
25. $\frac{21}{25}$ by $\frac{3}{5}$.
26. $\frac{4}{9}$ by $\frac{12}{27}$.
27. $\frac{16}{21}$ by $\frac{18}{35}$.
28. $\frac{14}{27}$ by $\frac{35}{36}$.
29. $\frac{102}{121}$ by $\frac{39}{77}$.

30. $2\frac{1}{2}$ by $3\frac{1}{3}$.

Solution. $2\frac{1}{2} : 3\frac{1}{3} = (6 \times 2\frac{1}{2}) : (6 \times 3\frac{1}{3})$ [§ 100] $= 15 : 20 = 15 \div 20 = \frac{3}{4}$.

31. $12\frac{1}{2}$ by $18\frac{3}{4}$.
32. $87\frac{1}{2}$ by $62\frac{1}{2}$.
33. $37\frac{1}{2}$ by 25.
34. 75 by $62\frac{1}{2}$.
35. $\frac{3}{8}$ by $12\frac{1}{2}$.
36. $18\frac{3}{4}$ by $\frac{3}{16}$.

Find the value of:

37. $\frac{5}{16} : \frac{3}{8}$.
38. $14\frac{2}{7} : 100$.
39. $28\frac{4}{7} : 42\frac{6}{7}$.
40. $(37\frac{1}{2} \times 18\frac{3}{4}) : (37\frac{1}{2} : 18\frac{3}{4})$.
41. $(\frac{2}{3} + \frac{3}{4}) : (17 \div 12)$.
42. $(\frac{5}{4}$ of $57\frac{1}{7}) : (1\frac{1}{16} - \frac{2}{3})$.
43. $(37\frac{1}{2} + 25) : (37\frac{1}{2} - 25)$.
44. $(\frac{5}{16} + 37\frac{1}{2} - 12\frac{1}{2}) : (62\frac{1}{2} - 50 + 6\frac{1}{4})$.

45. How many sticks, each $2\frac{1}{2}$ feet long, can be sawed from a log 15 feet long?

Solution. 15 feet = the length of the log.
$2\frac{1}{2}$ feet = the length of 1 stick.
∴ **15 feet : $2\frac{1}{2}$ feet, or 6 = the required number.**

46. At $6\frac{1}{4}$ cents a pound, how many pounds of sugar can be bought for 50 cents?

47. If the distance around a carriage wheel is $7\frac{1}{3}$ feet, how many times has it turned when the carriage has gone 1 mile, or 5280 feet?

48. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how many cubic feet of water are there in a ton of 2000 lb.?

49. The cloth for a suit cost $\$15\frac{3}{4}$ at $\$2\frac{1}{4}$ a yard; how many yards are there in the suit?

50. How many pounds of lard at $11\frac{1}{4}$ cents a pound can be bought for 6 dozen of eggs at $12\frac{1}{2}$ cents a dozen?

51. How many times can a bucket holding $2\frac{3}{4}$ gallons be filled from a barrel holding $31\frac{1}{2}$ gallons?

52. There are 365 days in a year and 31 days in July; what part of a year is July?

53. A dealer bought a cow for $\$42\frac{1}{2}$ and sold her at a loss of $\$2\frac{1}{8}$; what part of the cost was the loss?

54. An agent who sold a machine for $\$75\frac{3}{4}$ was paid $\$6\frac{3}{50}$; what part of the selling price was paid him?

55. It requires about $18\frac{3}{16}$ pounds of barbed wire to build 100 feet of fence having 3 wires; how many rods of fence, each rod containing $16\frac{1}{2}$ feet, can be built with 1000 pounds of wire? (Give answer to the nearest fourth of a rod.)

Complex Fractions.

153. A **Complex Fraction** is an expression in the form of a fraction, one or both of whose terms are either fractions or mixed numbers.

Thus, $\frac{\frac{3}{4}}{2}$, $\frac{2}{\frac{3}{4}}$, $\frac{\frac{2}{3}}{\frac{3}{4}}$, and $\frac{2\frac{1}{2}}{3\frac{1}{5}}$ are complex fractions.

Note. A complex fraction may be regarded as denoting that the numerator is to be either divided by or measured by the denominator.

EXERCISE 63.

1. **Reduce** $\dfrac{\frac{3}{4}}{2}$ **to a simple fraction.**

Solution. $\dfrac{\frac{3}{4}}{2} = \dfrac{3}{4 \times 2}$ [§ 126] $= \dfrac{3}{8}$.

2. **Reduce** $\dfrac{2}{\frac{3}{4}}$ **to a simple fraction.**

Solution. $\dfrac{2}{\frac{3}{4}} = \frac{4}{3}$ of 2 [§ 150] $= \frac{8}{3}$, or $2\frac{2}{3}$.

3. **Reduce** $\dfrac{\frac{2}{3}}{\frac{3}{4}}$ **to a simple fraction.**

Solution. $\dfrac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4} = \frac{4}{3} \times \frac{2}{3}$ [§ 150] $= \frac{8}{9}$.

4. **Reduce** $\dfrac{3\frac{1}{5} - 2\frac{1}{2}}{3\frac{1}{4} + 1\frac{1}{2}}$ **to a simple fraction.**

Solution. $\dfrac{3\frac{1}{5} - 2\frac{1}{2}}{3\frac{1}{4} + 1\frac{1}{2}} = \dfrac{20\,(3\frac{1}{5} - 2\frac{1}{2})}{20\,(3\frac{1}{4} + 1\frac{1}{2})} = \dfrac{64 - 50}{65 + 30} = \dfrac{14}{95}$.

Reduce to simple fractions:

5. $\dfrac{\frac{4}{5}}{2}$.
6. $\dfrac{\frac{10}{3}}{8}$.
7. $\dfrac{4}{\frac{8}{3}}$.
8. $\dfrac{5}{\frac{9}{20}}$.
9. $\dfrac{10\frac{5}{12}}{5}$.
10. $\dfrac{3\frac{1}{3}}{10}$.
11. $\dfrac{100}{16\frac{2}{3}}$.
12. $\dfrac{50}{33\frac{1}{3}}$.
13. $\dfrac{\frac{4}{5}}{\frac{7}{8}}$.
14. $\dfrac{12\frac{1}{2}}{\frac{3}{8}}$.
15. $\dfrac{\frac{7}{8}}{62\frac{1}{2}}$.
16. $\dfrac{12\frac{1}{2}}{6\frac{1}{4}}$.
17. $\dfrac{16\frac{2}{3}}{83\frac{1}{3}}$.
18. $\dfrac{37\frac{1}{2}}{87\frac{1}{2}}$.
19. $\dfrac{14\frac{2}{7}}{71\frac{3}{7}}$.
20. $\dfrac{66\frac{2}{3}}{4\frac{1}{6}}$.
21. $\dfrac{2 + \frac{2}{3}}{2 - \frac{2}{3}}$.
22. $\dfrac{4 + 2\frac{1}{12}}{5 - 1\frac{5}{6}}$.
23. $\dfrac{\frac{8}{9} - \frac{2}{3}}{\frac{5}{9} - \frac{1}{3}}$.
24. $\dfrac{5\frac{3}{4} - \frac{2}{3}}{3\frac{1}{4} + \frac{1}{12}}$.
25. $\dfrac{12\frac{1}{2} + 6\frac{1}{4}}{18\frac{3}{4} - 3\frac{1}{8}}$.
26. $\dfrac{\frac{3}{100} + \frac{9}{50} + \frac{4}{25} + \frac{3}{5}}{\frac{92}{100} - \frac{23}{50} + \frac{12}{25} - \frac{16}{25}}$.
27. $\dfrac{\dfrac{3 - \frac{1}{5}}{4} - \dfrac{3 + \frac{1}{5}}{8}}{16}$.
28. $\dfrac{(\frac{2}{3} + \frac{3}{4}) \times (\frac{3}{4} - \frac{2}{3})}{(\frac{9}{16} - \frac{4}{9}) \div (\frac{2}{3} + \frac{3}{4})}$.

EXERCISE 64. ORAL.

Miscellaneous Problems.

1. Reduce $\frac{27}{3}$, $\frac{18}{4}$, $\frac{21}{15}$, $\frac{25}{8}$ to whole or mixed numbers.

2. Reduce $2\frac{2}{3}$, $1\frac{9}{12}$, $3\frac{4}{15}$, $12\frac{1}{2}$ to improper fractions.

3. Reduce $\frac{2}{3}$, $\frac{5}{6}$, 2, $3\frac{1}{4}$, $\frac{2}{6}$ to 12ths.

4. Reduce $\frac{9}{15}$, $\frac{20}{25}$, 2, $1\frac{5}{15}$ to 5ths.

5. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

6. Find the sum of $2\frac{1}{2}$ and $3\frac{2}{5}$.

7. Find the difference of $\frac{4}{5}$ and $\frac{2}{3}$; also of $1\frac{2}{3}$ and $\frac{3}{4}$.

8. Find the value of $4 \times \frac{3}{5}$; $4 \times \frac{3}{8}$; $\frac{2}{3}$ of 9; $\frac{2}{3}$ of 4.

9. Find the value of $\frac{3}{4}$ of $\frac{8}{9}$; $\frac{3}{4}$ of $\frac{5}{6}$; $\frac{3}{4}$ of $4\frac{4}{5}$; $\frac{3}{4}$ of $5\frac{1}{4}$.

10. Divide 1 by 6; 5 by 6; $\frac{4}{5}$ by 4; $\frac{8}{9}$ by 4; $4\frac{8}{9}$ by 4; $3\frac{3}{4}$ by 4.

11. Divide 1 by $\frac{2}{3}$; 4 by $\frac{4}{5}$; 4 by $\frac{7}{8}$; $\frac{2}{3}$ by $\frac{3}{4}$; $2\frac{1}{2}$ by $\frac{2}{5}$; $2\frac{1}{2}$ by $1\frac{3}{4}$.

12. Measure 1 by 5; 4 by 5; $\frac{5}{6}$ by 5; $\frac{3}{4}$ by 5; $5\frac{5}{6}$ by 5; $1\frac{3}{4}$ by 5.

13. Measure 1 by $\frac{7}{8}$; 6 by $\frac{7}{8}$; $\frac{2}{3}$ by $\frac{7}{8}$; $7\frac{14}{15}$ by $\frac{7}{8}$; $1\frac{1}{3}$ by $\frac{7}{8}$.

14. What part of 4 is 1? 9 is 8? 2 is $\frac{2}{3}$? 3 is $1\frac{1}{2}$? $2\frac{1}{2}$ is $1\frac{1}{2}$?

15. How many times 2 is 5? $\frac{2}{3}$ is 4? $1\frac{1}{2}$ is $4\frac{1}{2}$? $\frac{2}{3}$ is $\frac{3}{4}$?

16. What part of 10 is $\frac{2}{3}$ of $\frac{3}{4}$?

17. $\frac{3}{5}$ of $2\frac{1}{2}$ is $\frac{2}{3}$ of what number?

18. $\frac{3}{4}$ of a number is $3\frac{3}{4}$; what is $\frac{2}{3}$ of the number?

19. $\frac{4}{5}$ of a number is $\frac{2}{3}$; what is $\frac{2}{3}$ of the number?

20. A lady who charges \$$1\frac{3}{4}$ a day for board should charge how much for 6 days' board?

21. When coal costs \$5 a ton, what part of a ton can be bought for \$1? \$$2\frac{1}{2}$? \$$1\frac{1}{4}$?

22. How many cords of chestnut wood can be bought for \$$2\frac{1}{2}$ at \$$\frac{3}{4}$ a cord?

23. A laborer who receives \$$2\frac{1}{2}$ a day must work how many days to pay a rent of \$15?

24. If $1\frac{3}{4}$ bushels of corn will last a horse 7 days, how long will 20 bushels last him?

25. A bottle that holds $\frac{1}{8}$ of a pint can be filled how often from $\frac{1}{2}$ of a pint?

26. Bought $2\frac{1}{2}$ dozen of eggs at 20 cents a dozen and $\frac{3}{4}$ of a pound of dried beef at 24 cents a pound. What was my bill?

27. $3\frac{1}{2}$ yards of ribbon are cut into pieces $\frac{1}{4}$ of a yard long; how many pieces are made?

28. If two pounds of cheese can be bought for $\frac{1}{4}$ of a dollar, for how many pounds should $\frac{7}{8}$ of a dollar be charged?

29. A quart of milk weighs about $2\frac{1}{2}$ pounds; how many quarts weigh 100 pounds?

30. How long will $4\frac{3}{4}$ bushels of oats last a horse which is fed $\frac{3}{4}$ of a bushel a day?

31. A owned $\frac{2}{3}$ of an acre of land and sold $\frac{3}{4}$ of it; what part of an acre had he then?

32. Black tea forms $\frac{2}{5}$ of a mixture of black and green tea. There are $2\frac{1}{2}$ pounds of green tea in the mixture; what is the weight of the mixture?

33. From an acre on which $2\frac{1}{2}$ bushels of seed was sown, the yield is 20 bushels. What is the yield from each bushel of seed?

34. A merchant paid 4 times as much for a pound of coffee as for a pound of sugar, and for both he paid $31\frac{1}{4}$ cents; find the cost per pound of each.

35. $\frac{2}{7}$ of the weight of a bushel of corn equals $\frac{1}{2}$ of the weight of a bushel of oats, and the difference of their weights is 24 pounds; find the weight of a bushel of each.

36. A dealer bought an article for $20 and sold it at a gain of $\frac{2}{5}$ of the cost; find the selling price.

37. A stove was sold for $$13\frac{1}{2}$ which was at a gain of $\frac{2}{7}$ of the cost; find the cost.

38. The gain made in selling a pair of shoes was $$1\frac{2}{5}$; if this was $\frac{7}{20}$ of the cost, what was the selling price?

39. On an article costing \$$5\frac{1}{3}$ there was a loss in selling of $\frac{3}{8}$ of the cost; find the selling price.

40. Sugar was sold for $5\frac{3}{4}$ cents per pound, which was at a loss of $\frac{2}{25}$ of the cost; find the cost.

41. A dealer lost $2\frac{2}{3}$ cents a yard on damaged goods by selling them for $\frac{5}{6}$ of the cost; find the selling price.

42. If a machine makes a keg of nails in 4 hours, what part of a keg will it make in 1 hour?

43. One pipe can carry off the water in a tank in 6 hours, and another pipe in 4 hours; what part of it can both together carry off in an hour?

44. A train that can run a certain distance in $\frac{3}{4}$ of an hour can run what part of the distance in $\frac{1}{4}$ of an hour? What part of the distance can it run in an hour?

45. One engine burns a ton of coal in $\frac{3}{4}$ of an hour, and another engine a ton in $\frac{2}{3}$ of an hour; what part of a ton does each burn in an hour? What part of a ton do they together burn in an hour?

46. At a certain creamery a gallon of milk passes through the separator in $1\frac{3}{4}$ minutes; what part of a gallon is separated in $\frac{1}{4}$ of a minute? What part of a gallon in one minute?

47. One hopper in a mill grinds a bushel of corn in $6\frac{1}{4}$ minutes and another grinds a bushel in $7\frac{1}{2}$ minutes; what part of a bushel can both together grind in a minute?

48. A boat that can run 20 miles an hour can run 1 mile at the same rate in what part of an hour?

49. One train running at the rate of 25 miles an hour and another running toward it at the rate of 20 miles an hour, together run a mile in what part of an hour?

50. A can do $\frac{3}{4}$ of a piece of work in a day; what time would he require to do $\frac{1}{4}$ of the work? To do all of the work?

51. If one lamp burns $\frac{3}{4}$ of a pint of oil in a night and another burns $\frac{5}{8}$ of a pint, what part of a night would a pint last each? Both?

52. A man can cut $\frac{7}{8}$ of an acre of corn in a day and his son can cut $\frac{5}{8}$ of an acre; how long would it take both to cut $\frac{1}{8}$ of an acre? How long to cut 1 acre?

53. One pipe can fill a tank in 2 hours and another can fill it in 3 hours; how long would it require both running together to fill it?

54. A man and his wife use a pound of coffee in 4 days, and it will last the wife alone 6 days; how long would it last the man alone?

55. One boy takes $\frac{1}{4}$ of an hour to pick a quart of berries and another takes $\frac{1}{3}$ of an hour; how long would it require both to pick a quart?

Sug. The first boy picks 4 quarts in an hour, and the second, 3 quarts.

56. One stove consumes $\frac{3}{4}$ of a gallon of oil in a day and another consumes $\frac{2}{3}$ of a gallon; what part of a day would a gallon last both stoves?

57. A and B together can do a piece of work in $2\frac{1}{2}$ days; how long will it take each, if A can do only $\frac{2}{3}$ as much in a day as B?

EXERCISE 65.

Miscellaneous Problems.

1. $\frac{1}{3}$ of a barrel of vinegar leaked away, and there remained in the barrel $20\frac{5}{6}$ gallons; how many gallons leaked away?

2. There are 2000 pounds in a short ton, which is $\frac{25}{28}$ as heavy as a long ton; how many pounds are there in a long ton?

3. At \$$15\frac{3}{5}$ a ton, find the cost of $12\frac{1}{2}$ tons of hay.

4. Divide two times the sum of the fractions $\frac{9}{15}$ and $\frac{17}{40}$ by $\frac{1}{3}$ of their difference.

5. If $3\frac{1}{2}$ barrels of lime are used in 100 square yards of plastering, how many barrels are required for 1256 square yards?

6. A bankrupt's property is \$3000, and with it he can pay $\frac{56\frac{2}{3}}{100}$ of his debts; what are his debts?

7. A farmer raised $286\frac{2}{3}$ bushels of wheat on $14\frac{1}{3}$ acres; what was the yield per acre?

8. Find the value of $\frac{8}{9} \times \frac{3}{4} + \frac{1}{3} \div \frac{1}{4} \times 2\frac{1}{2} - 1\frac{1}{4}$.

9. Allowing each of 15 sailors $1\frac{3}{4}$ pounds of meat daily, how much meat must be supplied for a cruise of 20 days?

10. Bought a house and 60 acres of land for $6800. If the house was valued at $3000, what was the value of the land per acre?

11. Bought a horse and paid cash $100, giving my note for the balance; the cash paid was $\frac{4}{5}$ of the cost. For what sum was the note given?

12. A gave to his son and daughter $125, giving 3 times as much to the son as to the daughter. How much did he give each?

13. If green coffee loses $\frac{1}{10}$ of its weight in roasting, how many pounds of green coffee were used to make 144 pounds of roasted coffee?

14. $\frac{3}{4}$ of a number was diminished by 10, and then there remained $\frac{7}{12}$ of the number; find the number.

15. If $\frac{2}{3}$ of the length of a field is $\frac{3}{4}$ of its width, what part of the length is $\frac{2}{3}$ of the width?

16. If $2\frac{1}{2}$ inches on a certain map represent 10 miles, what is the distance between two places on this map, if their real distance apart is $37\frac{1}{2}$ miles?

17. Express in words $\frac{304}{622}$.

18. A farmer makes a mixture of oats and corn, using 3 bushels of oats for every 5 bushels of corn; how many bushels of each in a mixture of 20 bushels?

19. 60 pounds of good wheat produce about $43\frac{5}{9}$ pounds of flour; how many pounds of wheat will produce a barrel of 196 pounds of flour?

20. A and B invest $5000 in a store, A investing $\frac{2}{3}$ as much as B. Find the investment of each.

21. A milkman pours 1 quart of water into every 8 quarts of pure milk; on a certain day he sells 108 quarts of the mixture; how much of this was pure milk?

22. A merchant sells $\frac{3}{4}$ of an article for $\frac{7}{8}$ of its cost; at the same rate, what part of the cost would be received for the whole article?

23. If on a certain scale $\frac{3}{8}$ is taken to represent 10, what represents $3\frac{1}{3}$?

24. A boy bought a number of bananas at the rate of 2 for 3 cents, and as many apples at the rate of 3 for 2 cents. If he paid 52 cents for all, how many of each did he buy?

25. 66 feet make 4 rods; how many feet make $2\frac{1}{2}$ rods?

26. If a ten-foot pole casts a shadow of $12\frac{1}{2}$ feet, what is the length of the flag pole that casts a shadow of $60\frac{2}{3}$ feet at the same time?

27. Find by cancellation the quotient, if the factors of the dividend are $14\frac{2}{7}$, $18\frac{3}{4}$, $87\frac{1}{2}$, and $37\frac{1}{2}$, and those of the divisor $12\frac{1}{2}$, 100, $1\frac{9}{16}$, and $3\frac{1}{8}$.

28. How many vests, each containing $\frac{3}{4}$ of a yard, can be cut from $10\frac{3}{4}$ yards of cloth, and how much cloth will remain?

29. A gravity car climbs a mountain and descends again in 75 minutes. If it requires 3 times as long to go up as to descend, how long does it take to go each way?

30. A can do a piece of work in 9 days, and A and B together can do it in 6 days; in what time could B alone do it?

31. A gained $50 by selling his horse for $300; what part of the cost was the gain?

32. From a barrel containing $31\frac{1}{2}$ gallons of vinegar $4\frac{1}{2}$ gallons were drawn; what part of the barrel remained?

33. After paying $\frac{4}{9}$ of his salary for board, a clerk has $\$15\frac{3}{4}$ left for a certain week; find his wages for that week.

34. How many jars, each holding $\frac{3}{8}$ of a pint, can be filled with $7\frac{7}{8}$ pints of maple syrup?

35. Two wheels run by the same belting have circumferences of $3\frac{3}{4}$ feet and $2\frac{1}{2}$ feet; how many times does the smaller turn while the larger turns 100 times?

36. A man 54 years old should live $17\frac{1}{2}$ years; to what age should he live?

37. 14 pounds of seed are put up in $\frac{1}{2}$ pound, $\frac{1}{4}$ pound, and $\frac{1}{8}$ pound packages, of each an equal number; how many of each are there?

38. From a piece of cloth containing 24 yards there were sold $2\frac{1}{2}$ yards, $3\frac{3}{4}$ yards, and $12\frac{1}{8}$ yards. Find the value of the remainder at \$$1\frac{1}{3}$ a yard.

39. The product of three fractions is $1\frac{1}{3}$; if two of them are $\frac{5}{4}$ and $\frac{8}{9}$, what is the third?

40. How much money must a contractor draw from a bank to pay a force of 20 men for the 6 working days of a week, each man being paid \$$1\frac{1}{4}$ a day?

41. A can saw 4 cords of wood in a day and split 2 cords in a day; what time would he require to cut and split 10 cords?

42. A bushel of oats weighs 32 pounds, which is $\frac{4}{7}$ of the weight of a bushel of corn; what is the weight of a bushel of corn?

43. A laborer who earns \$$1\frac{2}{3}$ a day spends $\frac{3}{4}$ of a dollar a day for support and saves the remainder. If he has already saved \$$24\frac{1}{4}$, how many more days must he work to save his rent, which is \$60?

44. If one machine can skim 75 gallons of milk in $1\frac{3}{4}$ hours, and another 60 gallons per hour, how long must both run to skim 150 gallons of milk?

45. A dealer bought a live turkey that weighed $7\frac{1}{2}$ pounds at 10 cents a pound. If it lost $\frac{1}{4}$ of its weight in dressing, at what price per pound must the dressed turkey be sold that the gain on the transaction may be 15 cents?

46. At what price per pound should a mixed tea be sold that is made from black tea that sells for 60 cents a pound and green tea that sells for 45 cents a pound, if 3 pounds of the green are used as often as 1 pound of the black?

47. A cubic foot of water weighs $62\frac{1}{2}$ pounds, and ice is $\frac{9}{10}$ times as heavy as water; how many cubic feet in a ton, or 2000 pounds, of ice? (Cancel.)

48. How many cubic feet of water weigh a ton?

49. Simplify $\dfrac{1-\frac{1}{4}\times 1+\frac{1}{4}\div 1+\frac{1}{4}\times 1-\frac{1}{4}}{1\times\frac{1}{4}+1\div\frac{1}{4}}$.

50. A newsboy bought "The Press" every week day for 1 cent a copy, and sold it for 2 cents. He bought "The Sunday Press" for $2\frac{1}{2}$ cents a copy, and sold it for 5 cents. His gain for a certain week was 75 cents; how many copies of "The Sunday Press" did he sell, if he sold 1 copy of it for every 10 of the daily?

51. Divide 120 into two such parts that $\frac{2}{3}$ of the smaller part shall equal $\frac{2}{5}$ of the larger.

52. If the rent charged for a house valued at \$5000 is $\$33\frac{1}{3}$ per month, what rent should be charged per month, at the same rate, for a house valued at \$6000?

53. Show that cancelling a factor common to the numerators of two fractions does not change their quotient.

54. Show that cancelling a factor common to the denominators of two fractions does not change their quotient.

55. A and B together purchase $160\frac{2}{3}$ acres of land, of which A takes $29\frac{3}{4}$ acres more than B; how many acres does each take?

56. Simplify $\dfrac{\frac{3}{4}\times\frac{2}{3}\div 2+\frac{4}{5}-\frac{3}{10}\div\frac{1}{20}\times\frac{1}{6}}{\frac{\frac{3}{4}}{\frac{2}{3}}\times\frac{\frac{2}{3}}{\frac{3}{4}}\div 20}$.

57. An article was sold for $\$8\frac{3}{4}$, which was $\dfrac{83\frac{1}{3}}{100}$ of the cost; find the loss.

CHAPTER III.

DECIMALS.

General Definitions.

154. One tenth of a unit is *a tenth;* one tenth of a tenth is *a hundredth;* one tenth of a hundredth is *a thousandth;* one tenth of a thousandth is *a ten-thousandth;* and so on.

155. A tenth, a hundredth, a thousandth, a ten-thousandth, and so on, are **Decimal Units.** A tenth is a decimal unit of the *first order;* a hundredth, one of the *second order*, and so on.

156. Any number of decimal units of the same order is a **Decimal.**

Thus, 9 tenths, 25 hundredths, 375 thousandths are decimals.

157. If the line of notation in the decimal system is extended rightward from units' place, and a point (.), called the **Decimal Point**, is placed after the units' order, a figure standing in the *first* place to the right of this point represents a number of *tenths;* in the *second* place, a number of *hundredths;* in the *third* place, a number of *thousandths;* in the *fourth* place, a number of *ten-thousandths;* and so on.

Thus, 3.245 equals 3 units 2 tenths 4 hundredths 5 thousandths; .0425 equals 4 hundredths 2 thousandths 5 ten-thousandths.

Numeration of Decimals.

EXERCISE 66.

Read .325.

Solution. $.325 = \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} = \frac{300}{1000} + \frac{20}{1000} + \frac{5}{1000} = \frac{325}{1000}$, which is read *three hundred twenty-five thousandths.* Therefore, to read a decimal expressed by the decimal system of notation, use the following:

158. Rule. *Read the number expressed by the figures of the decimal and pronounce the name of the place occupied by the right-hand figure.*

Read, and write in words:

1. .3.
2. .05.
3. .75.
4. .008.
5. .045.
6. .607.
7. .675.
8. .650.
9. .700.
10. .0004.
11. .0017.
12. .0756.
13. .8452.
14. .9804.
15. .2005.
16. .00001.
17. .00120.
18. .45005.
19. .37568.
20. .04040.

159. An expression that contains no other than decimal figures is called a **Pure Decimal.**

Thus, all the above expressions are pure decimals.

160. An expression composed of a whole number and a decimal is called a **Mixed Decimal.**

Thus, 3.25, read *three and twenty-five hundredths*, or *three hundred twenty-five hundredths*, is a mixed decimal.

Read in two ways:

21. 12.5.
22. 6.25.
23. 30.125.
24. 35.004.
25. 100.01.
26. 75.875.

161. A decimal that does not end in a common fraction is a **Simple Decimal.**

Thus, all the preceding decimals in this exercise are simple decimals.

162. A decimal that ends in a fraction is a **Complex Decimal.**

Thus, $.2\frac{1}{4}$, read *two and one fourth tenths ;* $3.16\frac{2}{3}$, read *three and sixteen and two thirds hundredths* or *three hundred sixteen and two thirds hundredths*, are complex decimals.

Note. The fractional part of a complex decimal does not occupy a place by itself, but belongs to the preceding figure.

Thus, $.0\frac{1}{3}$ is read *one third of a tenth*, not *one third of a hundredth.*

Read :

27. $.12\frac{1}{2}$.	**29.** $.00\frac{2}{3}$.	**31.** $5.01\frac{2}{9}$.
28. $.02\frac{7}{8}$.	**30.** $4.78\frac{1}{3}$.	**32.** $52.60\frac{4}{5}$.

Notation of Decimals.

163. It has been shown [§ 158] that the denomination of a decimal is the same as the denomination of the place occupied by its right-hand figure; therefore, in writing a decimal, the right-hand figure must be written in the place which has the same denomination as the decimal.

Thus, 325 thousandths must be written so that the 5 stands in the thousandths' place ; as, .325.

Therefore, a rule for writing decimals may be stated as follows :

164. RULE. *Write the number of decimal units and so place the decimal point that the right-hand figure will stand in the place that has the same denomination as the decimal.*

EXERCISE 67.

Write in figures :

1. 4 tenths.	**3.** 67 hundredths.
2. 5 hundredths.	**4.** 8 thousandths.

5. 12 thousandths.
6. 132 thousandths.
7. 6 ten-thousandths.
8. 408 ten-thousandths.
9. 4756 ten-thousandths.
10. 40 hundred-thousandths.
11. 302 hundred-thousandths.
12. 84 millionths.
13. 15 tenths.
14. 212 hundredths.
15. $22\frac{2}{9}$ hundredths.
16. 600 and 6 thousandths.
17. 404 hundredths.
18. 400 and 4 hundredths.

Reduction of Decimals.

EXERCISE 68.

Reduce to fractions or mixed numbers:

1. .325.

SOLUTION. $.325 = \frac{325}{1000} = \frac{3}{8}$.

2. .8.
3. .06.
4. .12.
5. .005.
6. .248.
7. .305.
8. .450.
9. 4.25.
10. 36.5.
11. 1.05.
12. .0008.
13. .4005.

14. $.16\frac{2}{3}$.

SUG. $.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{3 \times 16\frac{2}{3}}{3 \times 100}$.

15. $.8\frac{1}{3}$.
16. $.6\frac{1}{4}$.
17. $.18\frac{3}{4}$.
18. $.14\frac{2}{7}$.
19. $.37\frac{1}{2}$.
20. $.66\frac{2}{3}$.
21. $.62\frac{1}{2}$.
22. $.87\frac{1}{2}$.
23. $.83\frac{1}{3}$.
24. $.22\frac{2}{9}$.
25. $4.44\frac{4}{9}$.
26. $6.33\frac{1}{3}$.

EXERCISE 69.

Reduce to decimals:

1. $\frac{7}{8}$.

FORM.

$$8\overline{)7.000}$$
$$.875$$

SOLUTION. $\frac{7}{8} = \frac{1}{8}$ of $7 = \frac{1}{8}$ of 70 tenths. $\frac{1}{8}$ of 70 tenths = 8 tenths, and 6 tenths over; write 8 in tenths' place. 6 tenths = 60 hundredths. $\frac{1}{8}$ of 60 hundredths = 7 hundredths, and 4 hundredths over; write 7 in hundredths' place. 4 hundredths = 40 thousandths. $\frac{1}{8}$ of 40 thousandths = 5 thousandths; write 5 in thousandths' place.

Note. In practice, annex ciphers to the numerator, divide by the denominator, and point off in the quotient as many decimal places as there are ciphers annexed to the numerator.

2. $\frac{3}{4}$.
3. $\frac{3}{8}$.
4. $\frac{3}{5}$.
5. $\frac{3}{16}$.
6. $\frac{5}{32}$.
7. $\frac{3}{40}$.
8. $\frac{6}{25}$.
9. $\frac{5}{16}$.

Reduce to hundredths:

10. $\frac{3}{7}$.

FORM.

$$7\overline{)3.00}$$
$$.42\frac{6}{7}$$

SOLUTION. $\frac{3}{7} = \frac{1}{7}$ of $3 = \frac{1}{7}$ of 300 hundredths $= 42\frac{6}{7}$ hundredths, or $.42\frac{6}{7}$.

11. $\frac{5}{6}$.
12. $\frac{5}{9}$.
13. $\frac{4}{7}$.
14. $\frac{5}{12}$.

Reduce to thousandths:

15. $\frac{4}{7}$
16. $\frac{5}{9}$.
17. $\frac{1}{6}$.
18. $\frac{1}{12}$.

EXERCISE 70.

Reduce:

1. $.6\frac{2}{3}$ to thousandths.

SOLUTION. $.6\frac{2}{3} = .6 + .0\frac{2}{3} = .6 + \frac{1}{3}$ of $.2 = .6 + \frac{1}{3}$ of $.200 = .6 + .066\frac{2}{3} = .666\frac{2}{3}$.

2. $.4\frac{1}{6}$ to hundredths.
3. $.8\frac{1}{3}$ to hundredths.
4. $.16\frac{2}{3}$ to thousandths.
5. $.14\frac{2}{7}$ to thousandths.
6. $.5\frac{5}{8}$ to thousandths.
7. $.28\frac{4}{7}$ to ten-thousandths.
8. $.666\frac{2}{3}$ to tenths.

SOLUTION. $.666\frac{2}{3} = .6\frac{66\frac{2}{3}}{100} = .6\frac{200}{300} = .6\frac{2}{3}$.

9. .125 to hundredths.
10. .6375 to hundredths.
11. .5625 to tenths.
12. $.333\frac{1}{3}$ to hundredths.
13. $.1666\frac{2}{3}$ to tenths.
14. $.005\frac{5}{8}$ to hundredths.

Addition and Subtraction of Decimals.

EXERCISE 71.

1. Find the sum of .225, 3.14, and 5.6.

FORM.

$$\begin{array}{ll} .225 & = .225 \\ 3.14 & = 3.140 \\ \underline{5.6} & = \underline{5.600} \\ \text{Sum} & = 8.965 \end{array}$$

SOLUTION. .225 = 225 thousandths, 3.14 = 3140 thousandths, and 5.6 = 5600 thousandths. Therefore, their sum = 8965 thousandths = 8.965.

165. RULE. *Reduce the decimals to the same denomination, add as in whole numbers, and give the sum the same denomination as the decimals.*

Find the sum of:

2.	3.	4.
4.56	84.008	47.4904
2.146	.76	2.8
15.8	7.8	7.0098
7.5675	325.972	.00456

5. .7, .045, 2.07, and 54.375.

6. 45, .04, 1.278, and .0004.

7. .27, .270, 2.55, and .0175.

8. 9.98, .875, 10.0016, and .0705.

9. 7 tenths, 8 hundredths, 87 thousandths, and 450 thousandths.

10. 75 tenths, 865 hundredths, and 44 thousandths.

11. Express in hundredths the value of $.6\frac{2}{3} + .18\frac{3}{4} + 1.188\frac{8}{9}$.

FORM.

$$\begin{array}{ll} .6\frac{2}{3} & = .66\frac{2}{3} \\ .18\frac{3}{4} & = .18\frac{3}{4} \\ \underline{1.188\frac{8}{9}} & = \underline{1.18\frac{8}{9}} \\ \text{Sum} & = 2.04\frac{11}{36} \end{array}$$

SOLUTION. $.6\frac{2}{3} = 66\frac{2}{3}$ hundredths, $.18\frac{3}{4} = 18\frac{3}{4}$ hundredths, and $1.188\frac{8}{9} = 118\frac{8}{9}$ hundredths. Therefore, their sum = $204\frac{11}{36}$ hundredths = $2.04\frac{11}{36}$.

Express in tenths the value of:

12. $.75 + .675 + .8125$.

13. $1.125 + .16\frac{2}{3} + 4.814\frac{2}{7}$.

Express in hundredths the value of:

14. $.2 + 1.125 + .6215$.

15. $.000\frac{7}{8} + \frac{2}{3} + .375$.

16. From 13.25 take 6.875.

FORM.

$13.25 = 13.250$
$\underline{6.875} = \underline{6.875}$
$\text{Diff.} = 6.375$

SOLUTION. 13.25 = 13250 thousandths, and 6.875 = 6875 thousandths. Hence, their difference = 6375 thousandths = 6.375.

166. RULE. *Reduce the decimals to the same denomination, subtract as in whole numbers, and give the difference the same denomination as the decimals:*

	17.	**18.**	**19.**
From	8.602	7.50	37.005
Take	4.896	2.375	18.8975

Find the value of:

20. $.8 - .3875$. **23.** $2 - .00375$.
21. $25.6 - 8.9875$. **24.** 5 tenths — 450 thousandths.
22. $42.006 - 1.2958$. **25.** 356 hundredths — 9 thousandths.

26. Find the value of $12.4\frac{1}{6} - 7.1875$ in hundredths.

FORM.

$12.4\frac{1}{6} = 12.41\frac{2}{3}$
$\underline{7.1875} = \underline{7.18\frac{3}{4}}$
$\text{Diff.} = 5.22\frac{11}{12}$

SOLUTION. $12.4\frac{1}{6} = 1241\frac{2}{3}$ hundredths and $7.1875 = 718\frac{3}{4}$ hundredths. Hence, their difference $= 522\frac{11}{12}$ hundredths $= 5.22\frac{11}{12}$.

Express in tenths the value of:

27. $.42 - .375$. **28.** $.2 - .18\frac{3}{4}$. **29.** $2\frac{3}{4} - .16\frac{2}{3}$.

Express in hundredths the value of:

30. $.2 - .125$. **31.** $3 - \frac{7}{8}$. **32.** $\frac{1}{3} - .2$.

33. Express as a mixed number the value of $33\frac{3}{8} + .625 - .0875 + 2.37\frac{1}{2} - .06\frac{1}{4}$.

34. From 1 ten-thousandth take 1 millionth.

35. A meter is about 3.37079 inches longer than a yard, which is 36 inches; find the length of a meter.

36. What decimal must be added to 10 millionths to make .10?

37. The Cuban peso is worth $.926; how much is that less than a dollar?

38. M bought a horse for $175.75, paid $2.50 for his feed, and then sold him for $190. Find the gain.

39. $3.75 is lost by selling an article for $120; what would have been the selling price had the gain been $3.75?

40. A merchant had $275.95 in bank at the opening of business July 1; during the day he drew two checks of $127.80 and $25.75. At the close of the day's business he deposited $35.75. What was then his balance in bank?

41. Add four and eight thousandths, eighteen and four hundred ten-thousandths, six hundred six hundredths, six hundred and six hundredths, six thousand thirty-four and four tenths, and nine thousand nine thousandths.

42. A river rose at a certain point to within 3.7 feet of high water mark, which was 12.1 feet above low water mark; how high above low water mark was the river at that time?

Multiplication of Decimals.

Multiply 4.756:

(1) By 10.

Solution. 10 × 4.756 = 10 × 4756 thousandths = 47560 thousandths = 47.560 = 47.56.

(2) By 100.

Solution. 100 × 4.756 = 100 × 4756 thousandths = 475600 thousandths = 475.600 = 475.6.

(3) By 1000.

Solution. 1000 × 4.756 = 1000 × 4756 thousandths = 4756000 thousandths = 4756.000 = 4756.

167. Observe in the preceding solutions that the product is the multiplicand with the decimal point moved one place rightward when the multiplier is 10, two places rightward when the multiplier is 100, and three places rightward when the multiplier is 1000. Therefore,

PRIN. *Moving the decimal point one place rightward multiplies a number by 10; two places rightward, by 100; three places rightward, by 1000; and so on.*

Divide 847.5:

(1) By 10.

SOLUTION. $\frac{1}{10}$ of $847.5 = \frac{1}{10}$ of $\frac{8475}{10} = \frac{8475}{100} = 84.75$.

(2) By 100.

SOLUTION. $\frac{1}{100}$ of $847.5 = \frac{1}{100}$ of $\frac{8475}{10} = \frac{8475}{1000} = 8.475$.

(3) By 1000.

SOLUTION. $\frac{1}{1000}$ of $847.5 = \frac{1}{1000}$ of $\frac{8475}{10} = \frac{8475}{10000} = .8475$.

168. Observe in the above solutions that the quotient is the dividend with the decimal point moved one place leftward when the divisor is 10, two places leftward when the divisor is 100, and three places leftward when the divisor is 1000. Therefore,

PRIN. *Moving the decimal point one place leftward divides a number by 10; two places leftward, by 100; three places leftward, by 1000; and so on.*

EXERCISE 72.

Multiply:

1. 3.756 by 10; by 100; by 1000; by 10000.

SOLUTION (1) $10 \times 3.756 = 37.56$ [§ 167, Prin.].

2. .0375 by 10; by 10000; by 100; by 1000.
3. 12.5 by 10; by 1000; by 10000; by 100.

4. 274.8 by 20; by 200; by 2000; by 20000.

SOLUTION (1) 20 × 274.8 = 2 × 10 × 274.8 = 2 × 2748 = 5496.

5. .375 by 30; by 300; by 3000; by 30000.
6. .00034 by 40; by 400; by 4000; by 40000.
7. 4.25 by 37, 8.37 by 2400, and 2.86 by 7.2.

FORM.	FORM.	FORM.
(1) 4.25	(2) 8.37	(3) 2.86
37	2400	7.2
2975	3348	572
1275	1674	2002
157.25	20088.00	20.592

SOLUTION (1) 37 × 4.25 = 37 × 425 hundredths = 15725 hundredths = 157.25.

SOLUTION (2) 2400 × 8.37 = 2400 × 837 hundredths = 2008800 hundredths = 20088.00.

SOLUTION (3) 7.2 × 2.86 = $\frac{1}{10}$ of 72 × 2.86 = $\frac{1}{10}$ of 205.92 = 20.592.

From the above solutions we have the following:

169. RULE. *Multiply as in whole numbers and point off as many decimal figures in the product as there are in the multiplicand and the multiplier together.*

8. .5 by 5; .12 by .3; .036 by .5; .005 by 50; 3.5 by .02.

9. 2.35 by 12.
10. .0125 by 48.
11. .705 by 245.
12. .0004 by 375.
13. 6.75 by 40.
14. 18.7 by 1800.
15. .654 by 25000.
16. 8.75 by 2.5.
17. .396 by .0004.
18. 2.12 by .209.
19. .0005 by .005.
20. 1000.01 by 25000.

Express in hundredths:

21. 8 × 1.6$\frac{2}{3}$.

SOLUTION. 8 × 1.6$\frac{2}{3}$ = 13.3$\frac{1}{3}$ = 13.33$\frac{1}{3}$.

22. 12 × 2.2$\frac{7}{9}$.
23. 5$\frac{1}{2}$ × .037$\frac{1}{2}$.
24. .16 × 4.0$\frac{1}{3}$.
25. .0$\frac{5}{6}$ × 3.24.

26. At .25 of a cent apiece what will 25 envelopes cost?

27. At .5 of a cent a pound, how much will an old stove bring, if it weighs 250 pounds?

28. There are 16.5 feet in a rod; how many feet are there in 6.25 rods?

29. There are 7.92 inches in a link of a surveyor's chain; how many inches are there in a chain of 100 links?

30. If a man drives 4.25 miles an hour, how far will he drive in 3.4 hours?

31. A man hauled 12 loads of coal, averaging 38.33⅓ hundredweight per load; how much coal did he haul?

32. How many decimal places has the product when tenths are multiplied by tenths? hundredths by tenths? thousandths by thousandths? hundredths by thousandths?

33. At $4.875 a day, how much will a man earn in 8⅓ days?

34. Find to the nearest cent the value of 125.6 bushels of wheat at $6.25 a bushel.

35. A cubic inch of wrought iron weighs .26 lb.; find the weight of 1728 cubic inches, or a cubic foot.

36. A cubic foot of water weighs 62.5 pounds and oak is 1.17 times as heavy as water; find the weight of an oak plank containing 2 cubic feet.

37. A borrowed $600 and agreed to pay .06 of this sum for its use each year; how much did he pay for its use, if he kept it 6⅔ years?

38. The circumference of a wheel is about 3.1416 times its diameter; how far will a bicycle move forward when the wheels have turned 100 times, if the diameter of the wheels is 2⅓ ft.?

Division of Decimals.

EXERCISE 73.

Divide:

1. 3758.6 by 100.

SOLUTION. 3758.6 ÷ 100 = 37.586 [§ 168, Prin.].

2. 85.67 by 100; by 1000; by 10000.

3. 0.75 by 10; by 100; by 10000.

4. 3.075 by 1000; by 100; by 10.

5. 2000 by 10; by 1000; by 10000.

6. 4.75 by 25.

Form.

$$\begin{array}{l}25\,)\,4.75\,(.19\\ \underline{25}\\ 225\\ \underline{225}\end{array}$$

Solution. $\frac{1}{25}$ of 4.75 = $\frac{1}{25}$ of 475 hundredths = 19 hundredths = .19.

7. 4.75 by 2500.

Solution. 4.75 ÷ 2500 = (4.75 ÷ 100) ÷ (2500 ÷ 100) [§ 101] = .0475 ÷ 25 = .0019.

8. .075 by .25.

Solution. .075 ÷ .25 = (100 × .075) ÷ (100 × .25) [§ 99] = 7.5 ÷ 25 = .3.

9. 7.5 by .25.

Solution. 7.5 ÷ .25 = (100 × 7.5) ÷ (100 × .25) = 750 ÷ 25 = 30.

170. Observe:

First, in the solution of Example 6, that when the divisor is a whole number without terminal ciphers the number of decimal places in the quotient is the same as in the dividend.

Second, in the solutions of Examples 7, 8, and 9, that when the divisor is not a whole number without terminal ciphers, both dividend and divisor may be divided or multiplied by a number that will make it such. Therefore,

Rule. *Multiply or divide both dividend and divisor by a number that will make the divisor a whole number without terminal ciphers, then divide and point off as many decimal figures in the quotient as there are in the dividend.*

Find the value of:

10.	.05 ÷ 5 ; .01 ÷ 2.	**20.**	5 ÷ .5 ; 5 ÷ .005.
11.	.325 ÷ 25.	**21.**	.5 ÷ .05 ; .005 ÷ .5.
12.	3.075 ÷ 75.	**22.**	8.64 ÷ 2.4.
13.	.00252 ÷ 42.	**23.**	.6048 ÷ .48.
14.	461.5 ÷ 142.	**24.**	.1728 ÷ .0144.
15.	98.1 ÷ 3000.	**25.**	4840 ÷ .16.
16.	191.52 ÷ 4200.	**26.**	.9 ÷ .0036.
17.	180 ÷ 50.	**27.**	1 ÷ .125.
18.	.0045 ÷ 1500.	**28.**	15.39 ÷ 4.75.
19.	3.6 ÷ 1500.	**29.**	.00576 ÷ 2.4.

Find in integral, fractional, or mixed form the value of:

30. $.08\frac{2}{3} \div .012\frac{2}{9}$.

SOLUTION. $.08\frac{2}{3} \div .012\frac{2}{9} = .086\frac{2}{3} \div .012\frac{2}{9} = 86\frac{2}{3} \div 12\frac{2}{9} = 780 \div 110 \simeq 78 \div 11 = 7\frac{1}{11}$.

31. $.2\frac{2}{9} \div 5$. **32.** $.066\frac{2}{3} \div 16$. **33.** $1 \div .33\frac{1}{3}$.

Express in hundredths the value of:

34. 2 ÷ 75. **36.** 1 ÷ 400. **38.** $3\frac{3}{4} \div 2\frac{1}{8}$.

35. 6.5 ÷ 8. **37.** $2\frac{2}{3} \div 11$. **39.** $2\frac{2}{3} \div 20$.

40. If 2.25 pounds of sugar cost 13.5 cents, what is the price per pound?

41. If 10.25 yards of calico are valued at 51.25 cents, what is the value per yard?

42. If a horse travels 20.1875 miles in 4.75 hours, what is his average speed per hour?

43. If 82 bushels of wheat weigh $4783.33\frac{1}{3}$ pounds, what is the average weight per bushel?

44. A man bought .125 of a bushel of clover seed for $1; what was the price per bushel?

45. If $.66\frac{2}{3}$ of a man's crop of wheat is 400 bushels, what is his crop?

46. The increase in population in New York City for the 10 years between 1890 and 1900 was 944,611; what was the average annual increase to the nearest unit?

47. A farmer sold 50 bushels of wheat which weighed 2932 pounds; the standard weight of a bushel of wheat being 60 pounds, find the loss in weight per bushel.

48. At what price was dressed beef selling per hundredweight, when 3.85 hundredweight sold for $26.95?

49. When $12 is paid for the loan of $100 for 2 years, how much is paid for the loan of $1 for 1 year?

50. If a certain line were divided into 25 equal parts, the length of each part would be 7.25 inches; what would be the length of each part if the line were divided into 30 equal parts?

51. A dealer offered $25.50 for 5 lambs, but finally agreed to pay $2.75 more for them; what did each lamb cost him?

52. 10 men together contribute $25 to buy fuel and clothing for a poor family, but they find it will require $30 to supply the needed amount; how much more contributed by each man will supply this last amount, if they contribute equal sums?

53. A cubic foot of water weighs 62.5 pounds, and water is .4 as heavy as graphite; what is the weight of a cubic foot of graphite?

Mensuration of Decimals.

EXERCISE 74.

Measure:

1. 75 by 1000.

SOLUTION. 75 : 1000 = 75 ÷ 1000 = .075 [§ 168 Prin.].

2. 478 by 10; by 100; by 1000.

3. 0.275 by 1000; by 10; by 100.

4. 250 by 100; by 10000; by 1000.

5. .24 by 1200.

SOLUTION. .24 : 1200 = .24 ÷ 1200 = .0024 ÷ 12 = .0002

6. 4.9 by 7; by 49; by 14.

7. .85 by 34; by 68; by 51.

8. .42 by 120; by 1800; by 150.

9. .096 by 240; by 3600; by 480.

10. 7.5 by .15; by 1.5; by .015.

11. .027 by 9; by 1.8; by .27; by .00018.

Express in integers, fractions, or mixed numbers the value of:

12. $.016\frac{2}{3} : .02\frac{2}{9}$.

SOLUTION. $.016\frac{2}{3} : .02\frac{2}{9} = .016\frac{2}{3} \div .02\frac{2}{9} = .016\frac{2}{3} \div .022\frac{2}{9} = 16\frac{2}{3} \div 22\frac{2}{9}$

$$= \frac{50}{3} \div \frac{200}{9} = \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{200}}} \times \frac{\cancel{50}}{\cancel{3}} = \frac{3}{4}.$$

13. $.02\frac{3}{7} : 4$.

14. $75 : .2\frac{1}{3}$.

15. $.2 : 133\frac{1}{3}$.

16. $.42\frac{6}{7} : .8$.

17. $.00\frac{8}{9} : .11\frac{1}{9}$.

18. $\frac{2}{3} : 6.6\frac{2}{3}$.

Express in hundredths the value of:

19. $1 : \frac{3}{2}$. **20.** $50 : 11\frac{1}{9}$. **21.** $.375 : \frac{2}{3}$. **22.** $6.2\frac{1}{2} : 9\frac{1}{11}$.

23. At 2.5 cents a pound, how many pounds of old iron must a boy sell to get 50 cents?

24. If a train runs $16.66\frac{2}{3}$ miles per hour, in what time will it run 100 miles?

25. If there are 5.5 yards in a rod, how many rods are there in 36.25 yards?

26. How many hundredths of 5 yards is 4 yards?

27. How many hundredths of 6 feet is $3\frac{1}{3}$ feet?

28. How many hundredths of $83\frac{1}{3}$ pounds is $66\frac{2}{3}$ pounds?

29. A cubic foot of granite weighs 173.75 pounds, and a cubic foot of water 62.5 pounds. How many times as heavy as water is granite?

30. A receives $\$1.87\frac{1}{2}$ a day, and B $\$1.37\frac{1}{2}$; what is the ratio of A's daily wages to B's?

31. A man earns $8 a week and spends $5.75 a week; what part of his earnings does he spend, and what part does he save?

32. A sailing vessel that sails 10 miles an hour requires how many hours to sail 95 miles?

33. A lot is $16\frac{2}{3}$ rods long and 12.5 rods wide; what is the ratio of the length to the width? What is the ratio of the width to the length?

34. When sugar can be bought at $\$.06\frac{1}{4}$ a pound, how many pounds can be bought for $12.50?

35. How many dollars will earn $12.18 in a year, if each dollar earns $.06?

36. How many cubic feet of platinum will weigh 4031.25 pounds, if a cubic foot of water weighs 62.5 pounds and platinum is 21.5 times as heavy as water?

EXERCISE 75.

Miscellaneous Problems.

1. Read .375 in hundredths.

Solution. .375 = *37.5 hundredths.*

2. Read .7846 in tenths; in hundredths; in thousandths.

3. In 4875 how many tens? how many hundreds? how many thousands?

4. In 375.6 how many tenths? how many tens? how many hundreds? how many hundredths?

5. Read .388 to the nearest hundredth.

Solution. .388 lies between .38 and .39 and is nearer .39; hence, read to the nearest hundredth, it is *39 hundredths.*

6. Read .476 to the nearest tenth; to the nearest hundredth.

7. Read .7847 to the nearest hundredth; to the nearest thousandth.

8. Read 3745 to the nearest hundred; to the nearest thousand.

9. Express to the nearest thousandth the value of 375.0875 + .035 − .10028.

10. Find to the nearest millionth the product of 375.358 by .00854.

11. Find to the nearest thousandth the value of 1 ÷ 375.

12. At $12 per 100 find the cost of 1545 rails.

Sug. 1545 = 15.45 hundred.

13. At $24 per 1000 feet, find the cost of 3748 feet of boards.

14. If 1 barrel of good lime is used in laying 1000 bricks, how many barrels must be used in laying 8750 bricks?

15. If 1.5 bushels of hair are used in 100 square yards of plastering, how many bushels of hair are required for 1764 square yards of plastering?

16. A gallon contains 231 cubic inches, and a bushel 2150.42 cubic inches; find to the nearest hundredth the number of gallons in a bushel.

17. The composition of cream is water .66, fat .267, sugar .028, salts .018, nitrogenous matter .027; find the weight of these various substances in 100 pounds of cream.

18. Of what denomination is the product when tenths are multiplied by tenths? hundredths by hundredths? tenths by hundredths? thousandths by hundredths?

19. Of what denomination is the quotient when tenths are divided by tenths? hundredths by tenths? thousandths by hundredths?

20. Good wheat will produce about .42 of its weight of finest flour, and in addition thereto .18 of its weight of seconds and .09 of its weight of biscuit flour; how many pounds of each of these grades of flour are produced from a bushel of wheat weighing 60 pounds?

21. Cast iron is 7.2 times as heavy as water, which weighs 62.5 pounds to the cubic foot; how many cubic feet of cast iron in a ton of 2000 pounds?

22. A gallon contains 231 cubic inches, a bushel 2150.42 cubic inches, and a barrel 31½ gallons; find to the nearest hundredth the number of bushels in a barrel.

23. A has in bank at the beginning of a certain day $262.75; during the day he draws checks amounting to $108.50 and deposits in bank during the day $62.80; what is his balance in bank at the close of the day?

24. A cubic foot of sea-water weighs about 64.3 pounds, and a cubic foot of fresh water 62.5 pounds; find to the nearest hundredth the ratio of the weight of the sea-water to that of the fresh water.

25. A merchant had on hand at the beginning of a day's sales $62.25; during the day he took in $128.92 cash and paid out for produce $26.72 cash; how much money had he on hand at the close of the day?

26. Roofing tin of a certain kind costs $5.46 per 100 square feet; what would the tin cost for a roof containing 1648 square feet?

27. 1000 plastering laths will cover 70 square yards of surface, and 11 pounds of lath nails will nail them on; if the laths cost $4 per 1000 and the nails $.04 a pound, find the cost of the laths and nails required for 770 square yards of surface.

28. Which is nearer to .4085 and by how much, .5 or .405?

29. The quart of Great Britain holds about 2.5 pounds of water, and is equivalent to 1.20032 quarts of the United States standard; find to the nearest ten-thousandth the weight of water that a quart of the latter kind will hold.

30. The increase in population of Baltimore from 1890 to 1900 was .1715 of the population in 1890; if the increase for the 10 years was 74,518, what was the population to the nearest unit in 1890 and in 1900?

31. What was charged for 2240 pounds of coal, if $0.28 was charged for 100 pounds?

32. The average age of the 24 girls in a certain school is 13.55 years; what will be the average age to the nearest hundredth, of the girls in this school, if 6 more enter whose ages average 14.62 years?

33. Simplify $\dfrac{.006 \div .02 \times .02 \div .006 + .002 \times 60 \div .12}{\frac{1}{2} \times \frac{1}{3} \div \frac{1}{4} + \frac{1}{4} \div \frac{1}{2} \times \frac{1}{3} - \frac{1}{6} + \frac{1}{2} \text{ of } 1\frac{1}{2}}$

CHAPTER IV.

DENOMINATE AMOUNTS.

171. Denominate Units are units established by custom or law to measure such quantities as *value*, *weight*, *time*, *length*, *surface*, *solids*, *capacity*, etc.

Thus, *a dollar*, *a pound*, *a day*, *a yard*, etc., are denominate units.

172. A **Simple Denominate Amount** is any number of like denominate units.

Thus, *25 dollars*, $2\frac{1}{2}$ *pounds*, $\frac{3}{4}$ *of a yard* are simple denominate amounts.

United States Money.

173. The standard unit of value in the United States is the DOLLAR.

TABLE.

10 mills (m.) = 1 cent (ct.).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

Since a dime equals $\frac{1}{10}$ of a dollar, a cent $\frac{1}{100}$ of a dollar, and a mill $\frac{1}{1000}$ of a dollar, any sum may be expressed in dollars without reduction by placing the number of dimes in the tenths' place, the number of cents in the hundredths' place, and the number of mills in the thousandths' place.

Thus, 25 dollars 4 dimes 6 cents 5 mills may be expressed $25.465, and read *25.465 dollars*, or *25 dollars 46 cents 5 mills.*

EXERCISE 76.

1. How many mills are there in 3 ct.? 5 ct.? 7 ct.? 1 d.?

2. How many mills are there in $\frac{1}{2}$ ct.? $\frac{1}{5}$ ct.? $\frac{1}{4}$ ct.? $2\frac{1}{2}$ ct.?

3. How many cents are there in 3 d.? 5 d.? $\frac{1}{2}$ d.? $2\frac{1}{2}$ d.?

4. How many cents are there in $2? $4? $8? $10?

5. How many cents are there in $\$\frac{1}{2}$? $\$\frac{1}{4}$? $\$\frac{3}{4}$? $\$\frac{1}{8}$? $\$\frac{3}{8}$? $\$\frac{5}{8}$? $\$\frac{7}{8}$?

6. How many cents are there in $\$\frac{1}{3}$? $\$\frac{2}{3}$? $\$\frac{1}{6}$? $\$\frac{5}{6}$? $\$\frac{1}{12}$? $\$\frac{1}{16}$?

7. How many cents are there in 20 m.? 30 m.? 45 m.? 42 m.?

8. How many dollars are there in 200 ct.? 150 ct.? 450 ct.?

9. What decimal part of a dollar is 5 ct.? 25 ct.? 70 ct.?

10. What decimal part of a dollar is $12\frac{1}{2}$ ct.? $16\frac{2}{3}$ ct.? $33\frac{1}{3}$ ct.?

11. What decimal part of a dollar is 5 m.? 18 m.? 50 m.?

12. What decimal part of a dollar is 18 ct. 5 m.? 20 ct. 8 m.?

Express as dollars:

13. 3 dollars 25 cents.

14. 16 dollars 8 cents.

15. 35 cents.

16. $18\frac{3}{4}$ cents.

17. 4 dollars 16 cents 4 mills.

18. 12 dollars 6 cents 2 mills.

19. 20 dollars 4 mills.

20. 25 cents 3 mills.

21. Find the cost of 500 rails at $4.75 per C.

Note. C stands for hundred and M for thousand [§ 24].

22. Find the cost of 6000 shingles at $18.75 per M.

23. Find the cost of 5000 cigars at $30 per M.

24. Find the cost of 27564 feet of boards at $24 per M.

25. Find the cost of 250 pounds of flour at $6.50 per C.

26. At $.15 per C find the cost of shipping a machine weighing 2748 pounds.

27. Find the cost of 240 rails at $4 per C.

COUNTING.

12 units = 1 dozen (doz.).
12 dozen = 1 gross (gro.).
12 gross = 1 great gross (G. gro.).
20 units = 1 score.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

28. How many units in 7 doz.? 4 score? 2 G. gro.?

29. How many sheets of paper in 2 quires? .75 quire?

30. How many dozen in 3 score? in 6 score?

31. How old is a man that is 3 score years and 10?

32. A man bought 20 quires of paper at 15 cents a quire and sold it at 1 cent a sheet; find his gain.

33. A dealer buys a great gross of matches at $0.60 a gross and retails them at 2 boxes for 5 cents; find his gain.

34. Bought ½ great gross of pens at $0.36 a gross and retailed them at 2 for 1 ct.; find the profit.

Bills and Accounts.

174. A **Bill** is a written statement of the names, quantities, and prices, of articles sold or services rendered, together with the names of the parties and the date and terms of the transaction.

175. An **Account** is a record of business transactions between two parties at any given date.

176. A **Debtor** (Dr.) is the party against whom charges are entered in a bill or an account.

177. A **Creditor** (Cr.) is the party in whose favor entries are made in an account.

Note 1. In an account the debtor and creditor is the same party.

Note 2. A **Debit** (Dr.) is an entry against a debtor.

Note 3. A **Credit** (Cr.) is an entry in favor of the creditor.

A BILL.

YONKERS, N. Y., June 15, 1901.

Mr. James Brown,

To JONATHAN HARE, *Dr.*

Terms Cash.

10 lb. Gran. Sugar	$0.10	1	00		
5 lb. Rio Coffee	.30	1	50		
6 lb. Raisins	.08		48		
2 doz. Lemons	.30		60		
4 lb. Prunes	.12		48	4	06

Received payment,

Jonathan Hare.

AN ACCOUNT.

HANOVER, PA., Dec. 1, 1899.

Mr. J. P. Lewis,

In account with F. L. HAMMOND.

	DR.					
10 yd.	Muslin	$0.12	1	20		
24 "	Ingrain Carpet	.40	9	60		
20 lb.	Gran. Sugar	.06	1	20	12	00
	CR.					
10 doz.	Eggs	$0.18	1	80		
16 lb.	Butter	.22	3	52		
24 "	Poultry	.12	2	88	8	20
	Balance,				3	80

EXERCISE 77.

1. Extend the items in the following bill and find the balance:

WOODBURY, N. J., June 15, 1901.

Mr. James Wilson,

To G. G. GREEN, *Dr.*

Terms Cash.

36 yd. Muslin	$0.09	
1 doz. Men's Linen Hdkfs.	1.80	
2 pr. Gloves	.75	
4 pr. Socks	.25	
10 yd. Fancy Ribbons	.12	
2 Silk Waists	6.00	

Received payment,

2. Extend the items in the following account and find the balance:

COLUMBIA, PA., April 1, 1902.

Mr. A. T. High,

In account with M. J. RHODES.

DR.		
2 M ft. Pine Boards	$32.00	
3 " " Scantling	20.00	
1½ " " Boards	24.00	
3 " Shingles	17.50	
CR.		
2 C Rough Locust Posts	8.00	
6 " Chestnut Rails	6.00	
Balance,		

3. George S. Lease presents the following bill to you, Sept. 1, 1901, for labor: 14 days hauling lumber, $1 per day; 4 days threshing grain, $1.50 per day; 8 days making fence, $1.25 per day. Write the bill and find the amount due.

4. Henry Jones bought of Thos. C. Fluke & Co., 2 qt. Cranberries @ 15 c.; 4 lb. French Prunes @ 25 c.; 4 doz. Fla. Oranges @ 50 c.; 4 glasses Guava Jelly @ 35 c.; 6 bottles Grape Juice @ $2.75 per doz.; 3 lb. White Grapes @ 30 c.; 10 lb. Cut Loaf Sugar @ 7 c.; 3 doz. Royalty Corn @ $1.50; 8 lb. Rice @ 10 c. Write the bill and find the amount due.

5. May 18, 1900, Hays & Co., Baltimore, Md., sent Lewis Harman a bill for goods bought as follows: Jan. 2, 25 bu. Wheat @ 65 c.; 40 bu. Oats at 30 c.; Mar. 2, 2 bbl. Fancy Flour @ $4.45; 2 bu. Clover Seed @ $5.00; 60 bu. Oats @ 31 c. The bill was paid and receipted May 21, 1900. Make out the bill and receipt it.

6. Sept. 1, 1900, R. L. Wilson bought of I. H. Raub, Plainfield, N. J., ¼ lb. Nutmegs @ 55 c. per lb.; 6 packages Cornstarch @ 5 c.; 2 gal. Mott's Pure Apple Vinegar @ 13 c.; 6 lb. "Our Choice" Codfish @ 7 c.; 2 bottles Domestic Comb Honey @ 25 c.; 4 lb. Rio Coffee @ 22 c.; 100 lb. Granulated Sugar @ 5 c. At the same time Mr. Raub bought of Mr. Wilson 18 lb. Butter at 18 c.; 20 doz. Eggs @ 22 c. Make out the account and find the amount due.

Measures of Weight.

TROY WEIGHT.

178. Troy weight is used in weighing gold, silver, platinum, and jewels, except precious stones. The standard unit is the TROY POUND.

TABLE.

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).
1 lb. = 12 oz. = 240 pwt. = 5760 gr.

Note. The unit of weight for precious stones is the **Carat,** which equals 4 diamond-grains, or about 3⅛ Troy grains. Pearls are sold by the diamond grain.

AVOIRDUPOIS WEIGHT.

179. Avoirdupois weight is used for weighing all substances except those weighed by Troy weight, medicines in prescription, diamonds, and pearls.

TABLE.

16 ounces (oz.) = 1 pound (lb.).
25 pounds = 1 quarter (qr.).
4 quarters = 1 hundredweight (cwt.).
20 hundredweight = 1 Ton (T.).

Note 1. 1 lb. avoir. = 7000 Troy grains.

Note 2. The long hundredweight, 112 pounds, and the long ton, 2240 pounds, are used in custom houses and in some wholesale and retail transactions.

Thus, coal in Pennsylvania is bought and sold by the long ton.

Note 3. Medicines are sold in prescriptions by apothecaries' weight which is as follows:

1 pound (℔) = 12 ounces (℥) = 96 drachms (ʒ) = 288 scruples (℈) = 5760 (gr.).

WEIGHTS OF GRAINS, SEEDS, AND OTHER PRODUCE.

180. The following table gives the number of pounds in a bushel of various commodities as used by the Boards of Trade in the principal cities of the United States:

TABLE.

Commodities.	Lb.	Commodities.	Lb.	Commodities.	Lb.
Barley	48	Corn (shelled)	56	Peas	60
Beans	60	Corn (on cob)	70	Rye	56
Buckwheat	48	Clover seed	60	Timothy seed	45
Bran	20	Oats	32	Wheat	60

The following units are used in commercial transactions:

1 barrel of flour = 196 lb.
1 barrel of pork or beef = 200 lb.
1 quintal of fish = 100 lb.
1 keg of nails = 100 lb.
1 firkin of butter = 56 lb.
1 cental of grain = 100 lb., or 1 cwt.
1 bushel of coarse salt = 56 lb.

EXERCISE 78

Reduce:

1. $5\frac{1}{2}$ lb. to pwt.

Solution. 1 lb. = 12 oz. = 12 × 20 pwt
∴ $5\frac{1}{2}$ lb. = $\frac{11}{2}$ × 12 × 20 pwt., or 1320 pwt

2. $2\frac{1}{2}$ cwt. to lb.
3. $\frac{4}{5}$ T. to cwt.
4. 4 oz. Troy to gr.
5 2.4 T. to lb.
6. 5 lb. avoir. to gr.
7. 2 long T. to lb.
8. 720 pwt. to lb.

Solution. 1 lb. = 12 oz. = 12 × 20 pwt., or 240 pwt.
∴ 720 pwt. : 240 pwt., or 3 = the number of pounds in 720 pwt.
∴ 720 pwt. = 3 lb.

9. 50 oz. Troy to lb.
10. 12.5 gr. to pwt.
11. 25920 gr. to lb. Troy.
12. 12500 lb. to T.
13. 8400 lb. to long T.
14. 28 lb. to long cwt.
15. $\frac{3}{8}$ oz. Troy to lb.
16. .8 cwt to T.
17. $6\frac{1}{4}$ lb. Troy to gr.
18. Measure a long ton by a short ton.
19. Find the value of 2745 lb. of wheat at $0.70 a bu.

20. A dealer bought flour at $6 a barrel and sold it at $0.80 a quarter; how much did he gain on a barrel?

21. A man had 4 tons of hay and sold 25 cwt.; what part of his hay did he sell?

22. How many sacks holding 25 pounds each can be filled from a ton of sugar?

23. If a horse eats 4 lb. of oats at a meal, in how many days, eating 3 meals a day, will he eat 1 ton?

Measures of Length.

LINEAR MEASURE.

181. The standard unit of length is the Yard.

TABLE.

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards = 1 rod (rd.).
320 rods = 1 statute mile (mi.).

Note 1. 1 mi. = 320 rd. = 1760 yd. = 5280 ft.

Note 2. The Furlong (40 rd.) is now seldom used.

SURVEYORS' LINEAR MEASURE.

182. In measuring land, surveyors use for the principal unit of length a Chain.

TABLE.

7.92 inches = 1 link (li.).
100 links = 1 chain (ch.).
80 chains = 1 statute mile (mi.).
1 ch. = 4 rd. = 22 yd. = 66 ft. = 792 in.

183. OTHER LINEAR UNITS.

(1) A Fathom = 6 feet, used at sea to measure cables, cordage, and the depth of the sea.

(2) A Hand, or hand's breadth = 4 inches, used in measuring a horse's height.

(3) A Geographic Mile, or Knot = 6086 ft. = the length of one minute of the equatorial circumference of the earth.

(4) A League (England and U. S.) = 3 geographic miles, used to measure distances at sea.

(5) A Pace = 2½ feet in ordinary walking, but 3.3 feet, or ⅕ of a rod, in stepping distances.

EXERCISE 79.

1. How many ft. are there in 8 rd.?

SOLUTION. 1 rd. = $5\frac{1}{2}$ yd. = $5\frac{1}{2} \times 3$ ft.
∴ 8 rd. = $8 \times 5\frac{1}{2} \times 3$ ft. = 132 ft.

2. Reduce to inches: 4 yd.; $1\frac{3}{4}$ yd.; $\frac{2}{3}$ yd.; .75 yd.

3. Reduce to links: 1 mi.; 75 ch.

4. Reduce to yards: 1 mi.; $4\frac{1}{2}$ rd.; $\frac{3}{4}$ rd.

5. How many rods in 480 ft.?

SOLUTION. 1 rd. = $5\frac{1}{2}$ yd. = $5\frac{1}{2} \times 3$ ft., or $16\frac{1}{2}$ ft.
∴ 480 ft. : $16\frac{1}{2}$ ft., or 960 : 33, or $29\frac{1}{11}$ = the number of rd. in 480 ft.

6. Reduce 120 yd. to rd.

7. Reduce 880 yd. to mi.

8. Reduce 840 li. to ch.; to mi.

9. What part of a rod is a foot?

10. What part of a yard is 9 in.?

11. What part of a mile is 7.5 ch.?

12. What part of a foot is a link?

13. Show that a mile = 1760 yd. = 5280 ft.

14. Show that a chain = 792 in. = 66 ft. = 22 yd = 4 rd.

15. A horse whose height is $15\frac{1}{2}$ hands is how many feet high?

16. A farmer stepped the length of his field and found it to be 160 paces; find the length in rods.

17. A ship sunk in 20 fathoms of water; how many feet deep was the water?

18. The circumference of a fly-wheel is 8 feet; how many times has the wheel turned when a point on its rim has traveled a mile?

19. A steamer sailed 60 leagues in a day; how many statute miles did it sail?

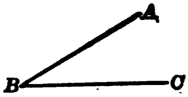
20. A board is 16 ft. long and $4\frac{1}{2}$ in. wide; what part of the length is the width?

Measures of Surfaces.

SQUARE MEASURE.

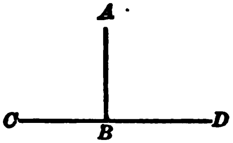
184. An **Angle** is the figure formed by two straight lines meeting at a point.

Thus, figure *A B C* is an angle.



185. If one straight line meets another straight line so as to form two equal angles, the angles are called **Right Angles**.

Thus, if the line *A B* meets the line *C D* so as to make the angle *A B C* equal to the angle *A B D*, each is a right angle.



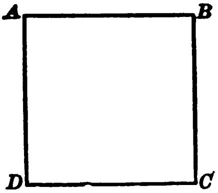
186. When two straight lines make a right angle, they are said to be **Perpendicular** to each other.

Thus, *A B* is perpendicular to *B C* and to *B D*.

187. A **Plane** is a surface on which straight lines may be drawn in different directions.

188. A **Square** is a plane figure having four equal sides and four right angles.

Thus, *A B C D* is a square.

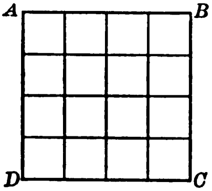


189. A Square Inch is a square an inch long; a Square Foot, a square a foot long; a Square Yard, a square a yard long; a Square Rod, a square a rod long; and a Square Mile, a square a mile long.

Note. The result obtained by using a number twice as a factor is the *square* of the number.

Thus, 3 × 3, or 9, is the square of 3.

190. PRIN. *The number of square units in a square equals the square of the number of units in the length.*



Let *A B C D* be a square whose length is 4 units; then 4 = the number of square units that can be placed in one row; 4 = the number of such rows in the square.

∴ 4 × 4, or 16 = the number of square units in the square.

Note. 4 × 4 may be written 4^2, read *4 squared.*

191. A square foot is a square 12 in. long and 12 in. wide. ∴ 12 × 12, or 144 = the number of square inches in a square foot.

A square yard is a square 3 feet long and 3 feet wide. ∴ 3 × 3, or 9 = the number of square feet in one square yard.

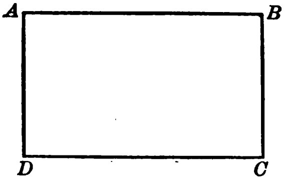
A square rod is a square 5½ yards long and 5½ yards wide. ∴ 5½ × 5½, or 30¼ = the number of square yards in one square rod.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
30¼ square yards	= 1 square rod (sq. rd.).

LAND MEASURE.

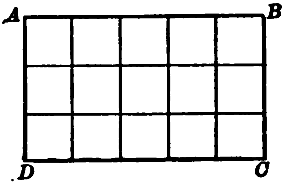
192. **Parallel Lines** are lines which lie in the same plane and cannot meet, however far they may be extended.



193. A **Rectangle** is a plane figure having four sides and four right angles.

The opposite sides are equal and parallel.

Thus, *A B C D* is a rectangle; the side *A B* is equal and parallel to the side *D C* and the side *A D* is equal and parallel to the side *B C.*



194. Prin. *The number of square units in a rectangle equals the number of units in the width times the number of units in the length.*

Let *A B C D* be a rectangle whose length is 5 units, and whose width is 3 units. Then,

5 = the number of square units in 1 row.
3 = the number of rows.
∴ 3 × 5, or 15 = the number of square units in the rectangle.

195. The unit of land measure is the Acre. It may be of any shape, but it is equivalent to a rectangular tract 5 chains, or 20 rods, long and 2 chains, or 8 rods, wide.

TABLE.

10000 square links (sq. li.) } = 1 square chain (sq. ch.).
16 square rods (sq. rd.) }
10 square chains } = 1 acre (A.).
160 square rods }
640 acres = 1 square mile (sq. mi.).

Note. A square rod is also called a Perch.

EXERCISE 80.

1. How many square inches are there in 3 sq. ft.? 9 sq. ft.?
2. How many square feet are there in 5 sq. yd.? 30¼ sq. yd.?
3. Reduce 5 square rods to sq. yd.; to sq. ft.; to sq. in.
4. Reduce 1 acre to sq. yd.; to sq. ft.
5. Reduce 2.5 acres to sq. rd.; to sq. ch.
6. In 2880 square inches, how many sq. yd.?
7. How many acres in 2.75 sq. ch.? 200 sq. rd.?
8. Reduce 15.45 square chains to acres.
9. Reduce 10890 square yards to acres.
10. What part of a square rod is a square foot?

11. What part of an acre is 6 sq. ch.? 50 sq. rd.?

12. Find the number of square inches in a square 6 in. long.

13. Find the number of acres in a square tract of land 8.5 ch. long.

14. Find the number of acres in a tract of land 80 rd. long and 5.5 rd. wide.

15. Find the number of square feet in a floor 14 ft. long and 10 ft. wide.

16. Find the number of square yards in a wall 26 ft. long and 16 ft. high.

17. Show why an acre contains 10 sq. ch. or 160 sq. rd.

18. Show why 16 sq. rd. or 10000 sq. li. equals 1 sq. ch.

19. Show why 640 A. equals 1 sq. mile.

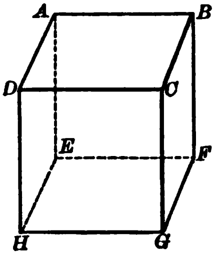
20. How many more square inches are there in a square 4 in. long than in one containing 4 sq. in.?

Measures of Solids.

CUBIC MEASURE.

196. A **Solid** is any bounded portion of space.

197. A **Cube** is a solid bounded by six equal squares.



The **Edges** of a cube are the lines in which the squares meet, and the **Faces** are the equal squares.

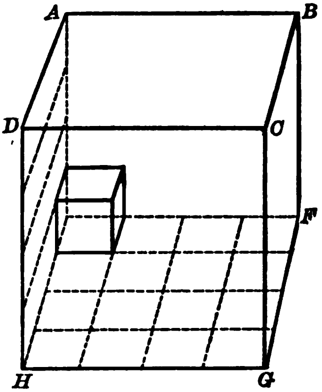
Thus, $ABCD-E$ is a cube; AB, BF, and FG are edges, and $ABCD$, $ABFE$, and $ADHE$ are faces.

A Cubic Inch is a cube an inch long; a Cubic Foot, a cube a foot long; and a Cubic Yard, a cube a yard long.

Note. The result obtained by using a number three times as a factor is the *cube* of the number.

Thus, 3 × 3 × 3, or 27, is the cube of 3.

198. PRIN. *The number of cubic units in a cube equals the cube of the number of units in its length.*



Let $A\,B\,C\,D - G$ be a cube 4 units long.

4 = the number of units in its length.
4 = the number of units in its width.
∴ 4 × 4 = the number of square units in its base.
∴ 4 × 4 = the number of cubic units in 1 layer.
4 = the number of layers.
∴ 4 × 4 × 4 = the number of cubic units in the cube.

Note. 4 × 4 × 4 may be written 4^3, read *4 cubed.*

199. A cubic foot is a cube 12 in. long.
∴ 12 × 12 × 12, or 1728 = the number of cubic inches in 1 cubic foot.

A cubic yard is a cube 3 ft. long.
∴ 3 × 3 × 3, or 27 = the number of cubic feet in 1 cubic yard

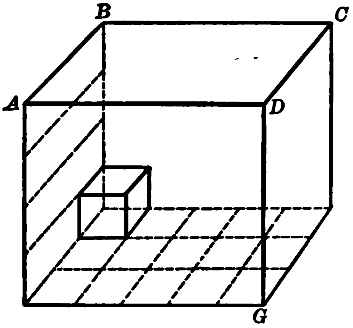
TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

Note 1. A cubic yard of earth is called a LOAD.

Note 2. A cubic foot of water weighs 62½ lb.

WOOD MEASURE.



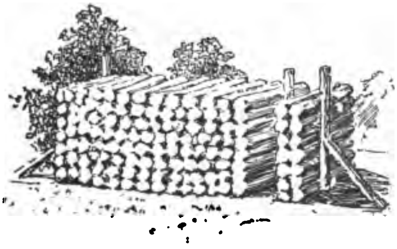
200. A Rectangular Solid is a solid bounded by six rectangles, called *faces*.

Thus, *A B C D* – *G* is a rectangular solid 5 units long, 3 units wide, and 4 units high.

201. PRIN. *The number of cubic units in a rectangular solid equals the number of units in the height, times the number of units in the width, times the number of units in the length.*

5 = the number of units in the length of *A B C D* – *G*.
3 = the number of units in the width.
∴ 3 × 5 = the number of square units in the base.
3 × 5 = the number of cubic units in 1 layer.
4 = the number of layers.
∴ 4 × 3 × 5 = the number of cubic units in the solid.

202. The unit of wood measure is the CORD. It is a rectangular solid 8 feet long, 4 feet wide, and 4 feet high.



The cord is mostly used to measure wood, cut into sticks 4 feet long, and piled up as indicated in the diagram. The cord is also sometimes used to measure other coarse materials, as stone, bark, etc.

A CORD FOOT is a rectangular solid 1 ft. long, 4 ft. wide, and 4 ft. high.

TABLE.

128 cu. ft. = 1 cord (cd.).
16 cu. ft. = 1 cord foot (cd. ft.).
8 cd. ft. = 1 cord.

EXERCISE 81.

1. Reduce 4 cu. yd. to cu. ft.; to cu. in.
2. Reduce .75 of a cu. yd. to cu. ft.; to cu. in.
3. Reduce 4320 cu. in. to cu. ft.; to cu. yd.
4. What part of a cu. yd. is 4½ cu. ft.? 864 cu. in.?
5. Find the number of cu. in. in a cube ⅔ of a foot long.
6. Find the number of cu. ft. in a room 16 ft. long, 12 ft. wide, and 8 ft. high.
7. Show why a cord of wood contains 128 cu. ft.
8. How many cords in a pile of wood 20 ft. long, 4 ft. wide, and 6 ft. high?
9. How many loads of earth must be removed in digging a cellar 36 ft. long, 24 ft. wide, and 8 ft. deep?
10. How many cubical blocks each an inch long can be sawed out of a block 2 ft. long, 8 in. wide, and 2 in. thick, no allowance being made for waste?

Measures of Capacity.

WINE MEASURE.

203. Wine measure is used to measure nearly all kinds of liquids, except milk, ale, beer, and medicines in prescriptions. The unit of wine measure is the GALLON OF 231 CUBIC INCHES.

TABLE.

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
31½ gallons = 1 barrel (bbl.).
2 barrels = 1 hogshead (hhd.) = 63 gal.

ALE AND BEER MEASURE.

204. The unit of ale or beer measure, used also in measuring milk, is the GALLON OF 282 CUBIC INCHES.

TABLE.

2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

Note 1. In mixing liquid medicines druggists use APOTHECARIES' FLUID MEASURE, which is as follows:
60 minims (♏) = 1 fluid dram (ƒʒ); 8 fluid drams = 1 fluid ounce (ƒ℥); 16 fluid ounces = 1 pint (O); 8 pints = 1 gallon (Cong.). A minim is about a drop.

Note 2. An ounce bottle holds a fluid ounce, or $\frac{1}{16}$ of a pint; a 2-ounce bottle, $\frac{1}{8}$ of a pint; a 3-ounce bottle, $\frac{3}{16}$ of a pint, and so on.

DRY MEASURE.

205. The standard unit of dry measure is the BUSHEL, which contains 2150.42 cubic inches.

TABLE.

2 pints (pt.) = 1 quart (qt.).
2 quarts = 1 small measure.
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.) stricken measure.
5 pecks = 1 heaped bushel, nearly.

Note 1. The heaped bushel which contains about 5 pecks, or $\frac{5}{4}$ of a stricken bushel, is used in measuring corn in the ear, potatoes, apples, lime, turnips, and various other substances.

Note 2. A stricken bushel equals about $\frac{5}{4}$ of a cubic foot, and a cubic foot equals about $\frac{4}{5}$ of a stricken bushel.

Note 3. A heaped bushel is about $\frac{25}{16}$ of a cubic foot, and a cubic foot is about $\frac{16}{25}$ of a heaped bushel.

Note 4. Two bushels of corn in the ear are considered the equivalent of one bushel of shelled corn.

EXERCISE 82.

1. How many pints are there in $2\frac{1}{2}$ qt.? $1\frac{3}{4}$ gal.? 6 gal.?

2. How many quarts are there in $2\frac{1}{4}$ pk.? $1\frac{1}{4}$ bu.? $\frac{3}{16}$ bu.?

3. What part of a gallon is 2 qt.? $2\frac{1}{2}$ qt.? $\frac{1}{4}$ qt.? 3 pt.? $1\frac{1}{2}$ pt.?

4. What part of a bushel is 3 pk.? $2\frac{1}{2}$ pk.? 10 qt.? 16 qt.?

5. How many cubic inches in $2\frac{2}{3}$ gal.? 2 pk.?

6. Find to the nearest hundredth the number of gallons in a cubic foot; in a bushel.

7. Show why a heaped bushel equals about $\frac{5}{4}$ of a stricken bushel. [See Note 1, § 205].

8. Show why a cubic foot equals about $\frac{4}{5}$ of a stricken bushel.

9. How many bushels of wheat will a bin hold that is 8 feet long, 4 feet wide, and 3 feet deep? [See Note 2, § 205.]

10. How many bushels of potatoes can be shipped in a closed box $4\frac{1}{2}$ feet long, 3 feet wide, and 2 feet deep? [See Note 3, § 205].

11. How many cubic feet in a bin that holds 240 bushels of rye?

12. How many cubic feet in a wagon-bed that holds, when even full, 48 bushels of potatoes?

13. How many hogsheads of water will a cistern hold that is $5\frac{1}{4}$ feet long, $5\frac{1}{4}$ feet wide, and 10 feet deep?

14. A crib 30 feet long, 4 feet wide, and 10 feet high is filled with corn in the ear; how many bushels of shelled corn will this make? [See Notes 3 and 4, § 205.]

Measures of Time.

206. There are two principal units of time, the DAY and the YEAR. The day is about equal to the time required for the earth to rotate on its axis, and the year, the time required for it to make a revolution in its orbit around the sun.

Note. The exact length of the year is 365 days 5 hours 48 minutes 49.7 seconds. But, as it is not convenient to have other than a whole number of days in a year, it has been decreed that:

(1) Every year whose number is not divisible by 4 shall consist of 365 days, called a COMMON YEAR.

(2) Every year whose number is divisible by 4 shall consist of 366 days, called a LEAP YEAR, except when it is a centennial year.

(3) A centennial year whose number is divisible by 400 shall be a leap year, and when not so divisible, a common year.

207. The year has been divided into 12 calendar months, as follows:

Names.	Number of Days.	Names.	Number of Days.
January	31	July	31
February	28 or 29	August	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

Note. The week consists of 7 days whose names are as follows: *Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday.*

208. TABLE.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
52 weeks = 1 year (yr.), nearly.
365 days = 1 common year.
366 days = 1 leap year.
100 years = 1 century.

Note. A.M. stands for *forenoon;* P.M. stands for *afternoon;* 6.40 A.M. means *40 minutes past 6 A.M.*

209. The following table shows the number of days from any day of one month to the same day of any other month in the same year:

TABLE.

From any Day of	To the same Day of											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August . . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

Angular Measure.

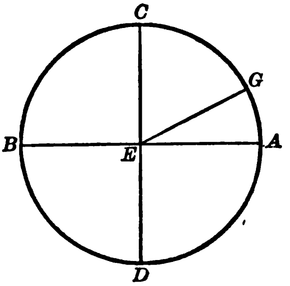
210. An angle equal to $\frac{1}{90}$ of a right angle is an angle of 1 **Degree**, which is the unit of angles; $\frac{1}{60}$ of an angle of 1 degree is an angle of 1 **Minute**; and $\frac{1}{60}$ of an angle of 1 minute, an angle of 1 **Second**.

TABLE.

60 seconds ($''$) = 1 minute ($'$).
60 minutes = 1 degree (deg. or $^\circ$).
90 degrees = 1 right angle (rt. $\angle$).

Circular Measure.

211. A **Circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the **Center.** The bounding line is the **Circumference.** Any part of the circumference is an **Arc**; as, *AG*. A straight line joining the center and any point on the circumference is a **Radius**; as, *EG*. A straight line drawn through the center and terminating in the circumference is a **Diameter**; as, *A B*.



212. Two perpendicular diameters divide the circumference into four equal parts called **Quadrants.** $\frac{1}{90}$ of a quadrant is an arc of 1 degree (1°); $\frac{1}{60}$ of an arc of 1° is an arc of 1 minute (1′), and $\frac{1}{60}$ of an arc of 1′ is an arc of 1 second (1″).

TABLE.

60″ = 1′.
60′ = 1°.
90° = 1 quadrant.
360° = 1 circumference.

EXERCISE 83.

1. How many minutes are there in a day?
2. How many seconds are there in an hour?
3. Reduce $\frac{2}{5}$ of a common year to days.
4. Reduce $\frac{5}{12}$ of a common year to days (to the nearest day).
5. What part of a common year is 30 da.? 60 da.? 90 da.?
6. How many days are there from June 1 to Dec. 1?

7. How many days are there from Dec. 1 to Feb. 21 following?

8. How many days are there from Feb. 15, 1900, to Aug. 1, 1900?

9. How many days are there from Nov. 30, 1900, to Mar. 31, 1901?

10. How many years and days are there from Jan. 15, 1900, to June 1, 1903? From Jan. 1, 1901, to Mar. 4, 1904?

11. What part of a common year is the time from July 15 to Oct. 1?

12. How many leap years were there in the 19th century?

13. Which of the following centennial years are leap years: 1600, 1700, 1800, 1900, 2000?

14. Find by the table (§ 209) the number of days from Jan. 1 to Dec. 1; from June 15 to Nov. 15; from Aug. 20 to Jan. 20.

15. What date is 30 days after Mar. 31?

16. What date is 90 da. after Sept. 1?

17. What date is 3 months after Sept. 1?

18. What date is 2 months after Jan. 30?

19. What date is 3 months after Jan. 31?

20. What date is 60 days prior to Jan. 1?

21. What date is 3 months prior to Feb. 28?

22. How many minutes are there in $\frac{2}{3}°$? $\frac{7}{10}°$? .08°?

23. How many seconds are there in $\frac{2}{9}'$? $\frac{3}{16}'$? .75′?

24. Reduce $\frac{9}{16}$ of a degree to minutes; to seconds.

25. Find the number of degrees in $\frac{1}{60}$ of a right angle.

26. An arc of $22\frac{1}{2}°$ is what part of a circumference?

27. A man who was born 14 B. C. and died 44 A. D., lived to what age?

SUG. 14 B. C. means 14 years before Christ, and 44 A. D. means 44 years after Christ.

28. A man who was born 204 B. C. and died 164 B. C., lived to what age?

Foreign Money.

213. TABLE OF ENGLISH MONEY.

4 farthings (far.) = 1 penny (d.).
12 pence = 1 shilling (s.).
20 shillings = 1 pound (£).
1 guinea = 21s.; 1 crown = 5s.; 1 florin = 2s.
£1 = $4.8665.

Note. In Canada the units of money are the same as in the United States.

214. The standard unit of value in France is the FRANC; its value in U. S. money is $0.193, or $1 = about 5.18 francs.

215. The standard unit of value in Germany is the REICHSMARK, or the MARK; its value in U. S. money is $0.2385, or 4 marks equals $0.945.

EXERCISE 84.

Reduce:

1. £4 to pence.

SOLUTION. £1 = 20s. = 20 × 12d., or 240d.
∴ £4 = 4 × 240d., or 960d.

2. 8s. to pence.
3. £14 to pence.
4. £$\frac{2}{3}$ to shillings.
5. £0.7 to pence.
6. 275d. to pounds.

SOLUTION. £1 = 20s. = 20 × 12d. = 240d.
∴ 275d. : 240d., or $1\frac{7}{48}$ = the number of £ in 275d.
∴ 275d. = £$1\frac{7}{48}$.

7. 8s. to pounds.
8. 75s. to pounds.
9. .6d. to shillings.
10. 2758d. to pounds.
11. £18 to dollars.
12. 1s. to cents.
13. 1 crown to cents.
14. 1 florin to cents.
15. 125 francs to dollars.
16. 75.2 marks to dollars.
17. $100 to pounds.
18. $1 to shillings.
19. $80 to francs.
20. $94.50 to marks.

EXERCISE 85. ORAL.

1. A peck of chestnuts were retailed at 10 ct. a pint; what sum was received for them?

2. What sum should be paid for a quarter of flour at $4.60 per cwt.?

3. If the edge of a cube is 6 in., how many feet are there in the combined length of all the edges?

4. A school that opens at 8.30 A.M. and closes at 2 P.M. is in session how many hours?

5. At 5 ct. per ounce what should be charged for ¼ lb. of cloves?

6. A boy buys ¼ of a gross of matches at $1 a gross, and retails them at 4 boxes for 5 ct.; find his gain.

7. What should it cost to send a letter weighing 2½ oz., if the rate of postage is 2 ct. per ounce or fraction thereof?

8. What should be charged for 750 lb. of feed at $16 per ton?

9. A lady bought 4 lemons at the rate of 15 ct. per dozen; how much should she pay?

10. How many 2-grain pills can be made from an ounce of quinine?

11. How many quarter-gill bottles are required to hold a pint of ink?

12. A dealer wishes to sell apples that cost $2 a bushel so as to gain $1.20 a bushel; how much should he charge for a peck?

13. If the sun rises at 6 A.M. and sets at 5.28 P.M., how long is the day?

14. A trucker put 1¼ bu. of berries into quart boxes; how many boxes did he require?

15. How many 2-inch squares can be cut out of a piece of cardboard 6 inches square?

16. How many ½-inch cubes can be placed in a box 4 in. long, 3 in. wide, and 2 in. high?

17. If 30 one-inch cubes are arranged so as to form a rectangular solid, find its length, width, and height.

18. A rectangular pile of wood is 4 ft. high and the sticks are 4 ft. long; how many cords in the pile if it is 30 ft. long?

19. How many rods long is a square mile of land in the form of a square?

20. Name the months that contain 30 da.; 31 da.

21. Name the leap years between 1900 and 1930.

22. If oats sell for 32 ct. a bushel when they are full weight, what should they sell for when they weigh but 30 lb. to the bushel?

23. When potatoes are selling for 15 ct. per half peck, how much is that a bushel?

24. How many quarts of ice cream must be bought for a company of 20, if one pint is sufficient for 2 persons?

25. The seventh day from to-day comes on what day of the week?

26. A rectangular yard 10 rd. long requires 32 rd. of fence to enclose it; find its width.

27. A farmer who wishes to use 400 lb. of fertilizer to the acre must purchase how many tons for 10 A.?

28. When eggs are selling at 40 ct. per score, how much should be charged for a dozen?

29. Name the centennial years since 1500 A. D. that have been leap years.

30. A laborer who works 10 hr. a day begins work at 6.30 A. M.; at what time should he quit work, if he takes one hour for dinner?

Compound Amounts.

216. A **Compound Amount** is a combination of two or more simple denominate amounts of the same kind of quantity, but of different denominations.

Thus, 3 yd. 2 ft. 4 in. is a compound amount.

REDUCTION.

EXERCISE 86.

1. Reduce 5 rd. 2 yd. 2 ft. to feet.

FORM.

$$\begin{array}{l} 5 \ - 2 - 2 \\ 5\frac{1}{2} \\ \hline 27\frac{1}{2} \\ \ \ 2 \\ \hline 29\frac{1}{2} \\ \ \ 3 \\ \hline 88\frac{1}{2} \\ \ \ 2 \\ \hline 90\frac{1}{2} \end{array}$$

SOLUTION. $1 \text{ rd.} = 5\frac{1}{2} \text{ yd.}$
$\therefore 5 \text{ rd.} = 5 \times 5\frac{1}{2} \text{ yd., or } 27\frac{1}{2} \text{ yd.}$
$27\frac{1}{2} \text{ yd.} + 2 \text{ yd.} = 29\frac{1}{2} \text{ yd.}$
$1 \text{ yd.} = 3 \text{ ft.}$
$\therefore 29\frac{1}{2} \text{ yd.} = 29\frac{1}{2} \times 3 \text{ ft., or } 88\frac{1}{2} \text{ ft.}$
$88\frac{1}{2} \text{ ft.} + 2 \text{ ft.} = 90\frac{1}{2} \text{ ft.}$
$\therefore 5 \text{ rd. } 2 \text{ yd. } 2 \text{ ft.} = 90\frac{1}{2} \text{ ft.}$

2. Reduce 4 T. 8 cwt. 20 lb. to pounds.
3. Reduce $4° \ 25' \ 12''$ to seconds.
4. Reduce £4 8s. 12d. to pence.
5. Reduce 2 sq. rd. 25 sq. yd. 2 sq. ft. to square feet.
6. Reduce 6 oz. 12 pwt. 10 gr. to grains.
7. Reduce 5 sq. rd. 12 sq. yd. 2 sq. ft. to square rods.

SOLUTION. 9 sq. ft. = 1 sq. yd.
$\therefore$ 2 sq. ft. : 9 sq. ft., or $\frac{2}{9}$ = the part of a sq. yd. in 2 sq. ft.
Whence, 12 sq. yd. 2 sq. ft. = $12\frac{2}{9}$ sq. yd.
$30\frac{1}{4}$ sq. yd. = 1 sq. rd.
$\therefore$ $12\frac{2}{9}$ sq. yd. : $30\frac{1}{4}$ sq. yd., or $\frac{40}{99}$ = the part of a sq. rd. in $12\frac{2}{9}$ sq. yd.
Whence, 5 sq. rd. 12 sq. yd. 2 sq. ft. = $5\frac{40}{99}$ sq. rd.

8. Reduce $3° \ 36'$ to the decimal part of a right angle.
9. Reduce £4 8s. 2d. to pounds.
10. Reduce 1 yr. 30 da. to years.
11. Reduce 5 bu. 2 pk. 1 qt. to bu.
12. Reduce 2 cwt. 40 lb. to the decimal of a ton.
13. Reduce 5 yd. $2\frac{1}{2}$ ft. to the fraction of a rod.
14. Express 2 ch. 64 li. as a decimal of a chain; of a mile.

15. Reduce 4 T. 12 cwt. 25 lb. to hundredweight.

Solution. 1 lb. = .01 cwt.
∴ 25 lb. = .25 cwt.
1 T. = 20 cwt.
∴ 4 T. = 80 cwt.

Whence, 4 T. 12 cwt. 25 lb. = 80 cwt. + 12 cwt. + .25 cwt., or 92.25 cwt.

16. Reduce 2 rd. 12 ft. 6 in. to yards.

17. Reduce 12 hhd. 20 gal. 2 qt. to gallons.

18. Reduce 1 da. 10 hr. 15 min. 15 sec. to hours.

19. Reduce 8 sq. yd. 25 sq. ft. 120 sq. in. to square feet.

20. Reduce 10 cu. yd. 4 cu. ft. 864 cu. in. to cubic feet.

Reduce to a compound amount:

21. 2567d.

Form.

12)2567
20)213^{11}
10^{13}

Solution. $\frac{1}{12}$ of the number of d. = the number of s.
∴ 2567d. = 213s. 11d.
$\frac{1}{20}$ of the number of s. = the number of £.
∴ 213s. = £10 13s.
∴ 2567d. = £10 13s. 11d.

22. 1560 min.
23. 6000 gr.
24. 189 in.
25. 47 ft.
26. 821 sq. ft.
27. 1800 yd.

28. $\frac{661}{2880}$ lb. Troy.

Solution. 1 lb. = 12 oz.
∴ $\frac{661}{2880}$ lb. = $\frac{661}{2880}$ of 12 oz., or $2\frac{181}{240}$ oz.
1 oz. = 20 pwt.
∴ $\frac{181}{240}$ oz. = $\frac{181}{240}$ of 20 pwt., or $15\frac{1}{12}$ pwt.
1 pwt. = 24 gr.
∴ $\frac{1}{12}$ pwt. = $\frac{1}{12}$ of 24 gr., or 2 gr.
Whence, $\frac{661}{2880}$ lb. Troy = 2 oz. 15 pwt. 2 gr.

29. $\frac{3}{4}$ rd.
30. $\frac{3}{5}$ da.
31. £$\frac{2}{7}$.
32. $\frac{3}{4}$ sq. rd.
33. $4\frac{2}{3}$ A.
34. $2\frac{1}{11}$ mi.

35. .4 rd.

Solution. 1 rd. = 5.5 yd.
∴ .4 rd. = .4 of 5.5 yd., or 2.2 yd.
1 yd. = 3 ft.
∴ .2 yd. = .2 of 3 ft., or .6 ft.
1 ft. = 12 in.
∴ .6 ft. = .6 of 12 in., or 7.2 in.
∴ .4 rd. = 2 yd. 7.2 in.

36. 1.4 A.
37. .6 sq. rd.
38. 2.07°
39. .75 rd.
40. .045 yr.
41. .65 lb. Troy.

Remark. Compound amounts are rapidly going out of use, except in the measurement of *time, angles,* and *arcs.*

Note. Compound amounts may be added, subtracted, multiplied, divided, or measured, by first reducing them to simple amounts; but for different methods see supplement to this chapter.

Metric System.

217. The METRIC SYSTEM of weights and measures has been adopted by Mexico, Brazil, Chile, Peru, and some other South American countries, and by all European nations, except Great Britain and Russia, where its use is permitted; it was legalized by the United States Government in 1866.

218. The *unit of length* is the METER.

219. The *unit of weight* is the GRAM, which is the weight of a cube of pure water, at greatest density, whose edge is $\frac{1}{100}$ of a meter.

220. The *unit of capacity* is the LITER, which is a cube whose edge is $\frac{1}{10}$ of a meter.

221. The *unit of square measure* is the ARE, which is a square 10 meters long.

222. The *unit of cubic measure* is the CUBIC METER, which is a cube a meter long.

223. The *unit of wood measure* is the STERE, which is a cubic meter.

224. The terms *Deci, Centi,* and *Mille,* denote respectively $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1000}$; and the terms *Deka, Hecto, Kilo,* and *Myria,* 10, 100, 1000, and 10000.

TABLE.

225. LINEAR MEASURE.

10 millimeters ($^{\text{mm}}$)	= 1 centimeter ($^{\text{cm}}$) =	0.3937 in.
10 centimeters	= 1 decimeter ($^{\text{dm}}$) =	3.937 in.
10 decimeters	= **1 Meter ($^{\text{M}}$)** =	**39.37 in.**
10 meters	= 1 dekameter ($^{\text{Dm}}$) =	393.7 in.
10 dekameters	= 1 hectometer ($^{\text{Hm}}$) =	328.08 ft.
10 hectometers	= 1 kilometer ($^{\text{Km}}$) =	0.62137 mi.
10 kilometers	= 1 myriameter ($^{\text{Mm}}$) =	6.2137 mi.

Note. 1$^{\text{mm}}$ = 0.0394 in.

226. MEASURES OF WEIGHT.

10 milligrams ($^{\text{mg}}$)	= 1 centigram ($^{\text{cg}}$) =	0.1543 gr.
10 centigrams	= 1 decigram ($^{\text{dg}}$) =	1.5432 gr.
10 decigrams	= **1 Gram ($^{\text{G}}$)** =	**15.432 gr.**
10 grams	= 1 dekagram ($^{\text{Dg}}$) =	0.3527 oz. av.
10 dekagrams	= 1 hectogram ($^{\text{Hg}}$) =	3.5274 oz. av.
10 hectograms	= **1 Kilogram ($^{\text{Kg}}$)** =	**2.2046 lb. av.**
10 kilograms	= 1 myriagram ($^{\text{Mg}}$) =	22.046 lb. av.
10 myriagrams	= 1 quintal ($^{\text{q}}$) =	220.46 lb. av.
10 quintals	= **1 Metric Ton ($^{\text{MT}}$)** =	**2204.6 lb. av.**

Note. 1$^{\text{mg}}$ = 0.0154 gr.

227. MEASURES OF CAPACITY.

10 milliliters ($^{\text{ml}}$)	= 1 centiliter ($^{\text{cl}}$) =	0.338 *f*℥.
10 centiliters	= 1 deciliter ($^{\text{dl}}$) =	0.845 gi.
10 deciliters	= **1 Liter ($^{\text{L}}$)** =	**0.908 qt. (dry).** **1.0567 qt. (liq.).**
10 liters	= 1 dekaliter ($^{\text{Dl}}$) =	9.08 qt. (dry). 2.6417 gal. (liq.).
10 dekaliters	= 1 hectoliter ($^{\text{Hl}}$) =	**2.8375 bu.** 26.417 gal.
10 hectoliters	= 1 kiloliter ($^{\text{Kl}}$) =	264.17 gal.

Note. 1$^{\text{ml}}$ = 0.27 *f*ʒ.

228. SQUARE MEASURE.

100 sq. millimeters (^{qmm}) = 1 sq. centimeter (^{qcm}) = 0.155 sq. in.
100 sq. centimeters = 1 sq. decimeter (^{qdm}) = 15.5 sq. in.
100 sq. decimeters = $\left\{\begin{array}{l}\textbf{1 Square Meter } (^{QM}) \\ \textbf{1 Centare } (^{CA})\end{array}\right\}$ = **10.764 sq. ft.**
100 centares = **1 Are** (^{A}) = **119.6 sq. yd.**
100 ares = **1 Hectare** (^{Ha}) = **2.471 A.**

229. CUBIC MEASURE.

1000 cu. millimeters (^{cmm}) = 1 cu. centimeter (^{ccm}).
1000 cu. centimeters = 1 cu. decimeter (^{cdm}).
1000 cu. decim. = **1 Cu. Meter** (^{cbm}), or **Stere** (^{s}) = **35.315 cu. ft.**

Note. The Metric System of Weights and Measures was first adopted in France in 1799. The system is almost universally used for scientific measurements. The following objections have been seriously urged against its general use, but as the public is becoming more and more familiar with it, these objections are disappearing.

It has been urged that:

1. The unit base, the meter, is too large to be clearly apprehended by the young and uninstructed.
2. The decimal division is too difficult, and is unsuited to practical purposes.
3. The unit of length should be some dimension of the human body.
4. It is inconvenient to think in one system and deal in another.
5. The decimal unit has failed when applied to the circle.
6. The adoption of the system would invalidate land titles.
7. The meter has never been correctly determined and never will be.

Some advantages that cannot be gainsaid are as follows:

1. Easiness of apprehension by pupils old enough to understand the decimal system of notation.

2. The facility with which the tables may be learned owing to the uniformity of scale.

3. The facility of reduction from one denomination to another of the same kind of quantity.

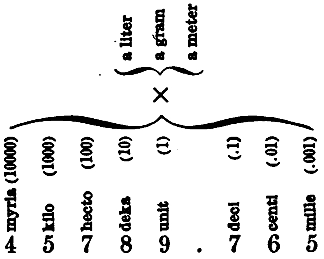
4. The simplicity of the relations of the units of surface, solids, capacity, and weight, to the units of length.

5. Its adoption would simplify international communication, since it is so generally used by other nations.

It is but a matter of time when its use will become universal. In the meantime it should be taught in all public and private institutions of learning.

EXERCISE 87.

SCALE OF METRIC SYSTEM.



1. Read 45789.765^g in kilograms; in hectograms; in dekagrams; in milligrams; in kilograms, grams, and milligrams.

SOLUTION. 45789.765 = 45.789765 thousands.
∴ $45789.765^g = 45.789765^{Kg}$.

2. Read 45789.765^m in kilometers; in dekameters; in centimeters: in millimeters; in kilometers, meters, and millimeters.

3. Read 45789.765^l in kiloliters; in hectoliters; in dekaliters; in centiliters; in kiloliters, liters, and milliliters.

Express:

4. 4500^g in kilograms.

5. $.45^g$, $.7^g$, and $.005^g$ in milligrams.

6. 4756845^g in kilograms; in metric tons.

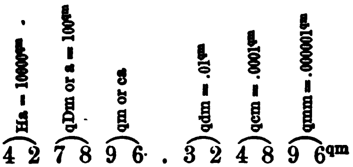
7. 8796.8^{m} in kilometers; in millimeters.
8. 4.5675^{l} in deciliters; in milliliters; in centiliters.
9. $.4^{l}$, $.04^{l}$, and $.005^{l}$ in centiliters; in milliliters.
10. $.6^{m}$, $.75^{m}$, and $.075^{m}$ in centimeters; in millimeters.
11. 8^{l}, 2^{dl}, and 6.1^{cl} in milliliters.
12. 2^{dg}, 4^{cg}, and 7.5^{g} in milligrams.
13. 4^{dg}, 7^{cg}, and 254^{mg} in grams.
14. 22.5^{mg}, 218.75^{mg}, and $7^{g}\ 456^{mg}$ in grams.
15. $18^{Kg}\ 40^{g}\ 60^{mg}$ in grams.

Solution. $18^{Kg}\ 40^{g}\ 60^{mg} = 18000^{g} + 40^{g} + .060^{g} = 18040.060^{g}$.

16. $6^{Km}\ 5^{Dm}\ 7^{m}\ 6^{dm}$ as meters.
17. $2^{Kg}\ 247^{g}\ 84^{mg}$ as grams.
18. $4^{MT}\ 7^{Kg}\ 975^{g}$ as grams.
19. 2756.456^{g} as kilograms, grams, and milligrams.
20. 1425.62^{m} as kilometers, meters, and millimeters.
21. 65.3274^{Kl} as kiloliters, liters, and milliliters.
22. 4278564^{g} as metric tons, kilograms, and grams.

EXERCISE 88.

SCALE OF SURFACE MEASURE.



1. Read and express 427896^{qm} in ares; in hectares; in centares.

Solution. $427896 = 4278.96$ hundred.
$\therefore 427896^{qm} = 4278.96^{a}$.

2. Read and express $.324896^{qm}$ in square decimeters; in square centimeters; in square millimeters.

Solution. $.324896 = 32.4896$ hundredths.
$\therefore 324896^{qm} = 32.4896^{qdm}$.

Read and express:

3. 145.75^{a} in hectares; in centares.

4. $.75^{qm}$ in square decimeters; in square centimeters.

5. 1.750^{qm} in square decimeters; square millimeters; centares.

6. $.4^{qdm}$ in square meters; in square centimeters; in square millimeters.

7. $5^{Ha}\ 2^{a}\ 7^{ca}$ in square meters.

Solution. $5^{Ha}\ 2^{a}\ 7^{ca} = 50000^{qm} + 200^{qm} + 7^{qm} = 50207^{qm}$.

8. $45^{Ha}\ 2^{a}\ 70^{ca}$ in square meters.

9. $4^{qdm}\ 25^{qcm}\ 8^{qmm}$ in square meters.

Solution. $4^{qdm}\ 25^{qcm}\ 8^{qmm} = .04^{qm} + .0025^{qm} + .000008^{qm} = .042508^{qm}$.

10. $4^{qdm}\ 6^{qcm}$ in square meters; in square decimeters.

11. $4^{Ha}\ 75^{ca}$ in square meters; in ares.

12. 1.5^{qcm} in square meters; in square millimeters.

13. $14^{a}\ 7^{qm}$ in ares; in hectares.

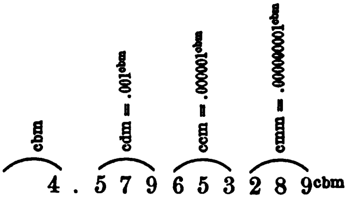
14. 4567.65^{qm} in square dekameters, square meters, and square decimeters.

15. 75832.4^{qm} in hectares, ares, and centares.

16. 45.674^{Ha} in hectares, ares, and centares.

17. 456.78^{a} in hectares, ares, and centares.

SCALE OF CUBIC MEASURE.



Read and express:

18. 4.579^{cbm} in cubic decimeters.

Solution. $4.579 = 4579$ thousandths.
$\therefore 4.579^{cbm} = 4579^{cdm}$.

19. 5.4^{cbm} in cubic decimeters; in cubic centimeters; in cubic millimeters.

20. $1^{\text{ccm}}\ 10^{\text{cmm}}$ in cubic meters.

SUG. $1^{\text{ccm}}\ 10^{\text{cmm}} = .000001^{\text{cbm}} + .000000010^{\text{cbm}}$.

21. 16^{cdm} in cubic meters; in cubic centimeters; in cubic millimeters.

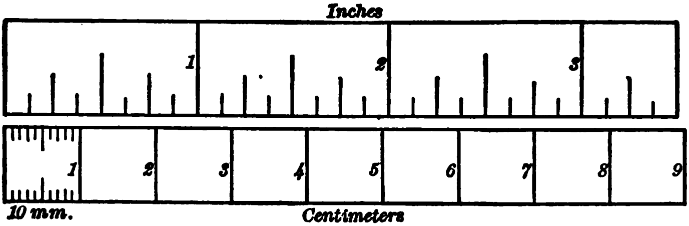
22. 250^{ccm} in cubic meters; in cubic decimeters; in cubic millimeters.

23. 7.5^{ccm} in cubic meters; in cubic millimeters.

24. 146.750^{cbm} in cubic meters and cubic decimeters.

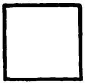
25. 4462.15^{cdm} in cubic meters, cubic decimeters, and cubic centimeters.

EXERCISE 89.

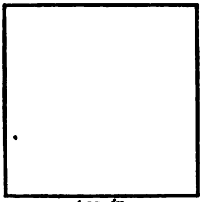


1. Find to the nearest hundredth the number of centimeters in an inch.

2. Find to the nearest hundredth the number of rods in a dekameter.



1 qcm.

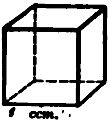


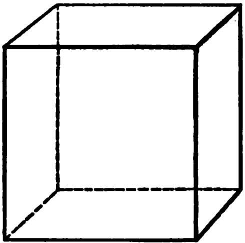
1 sq. in.

3. Find to the nearest hundredth the number of square centimeters in a square inch.

4. Find to the nearest hundredth the number of square yards in 10^{qm}.

5. Reduce 25^{Ha} to acres; 1 A to ares.


1 ccm.


1 cu. in.

6. Find to the nearest hundredth the number of cubic centimeters in a cubic inch.

7. Find to the nearest hundredth the number of cubic yards in a stere.

8. Reduce 25^{Kg} to pounds; 1 pound Troy to grams.

9. Reduce 20^{Hl} to bushels; to gallons.

10. Reduce 4 qt. (dry) to liters; 1 gal. (liquid) to liters.

11. A cubic centimeter of water weighs a gram. Show that a cubic decimeter, or a liter, weighs a kilogram, and that a cubic meter weighs a metric ton.

12. Find the weight of a liter of milk, if it is 1.03 times as heavy as water.

13. Find the weight of 2^{ccm} of silver, if it is 10.47 times as heavy as water.

14. A piece of land 45^{m} long and 20.5^{m} wide contains how many ares?

15. How many cubic centimeters does a flask hold that has a capacity of 5^{cl}?

16. Find the weight of a milliliter of water.

17. How many hectoliters will a bin hold that is 4^{m} long, 2.5^{m} wide, and 1.75^{m} deep?

18. A flask holds 2.5^{Kg} of water; how many liters does it hold? how many cubic decimeters?

19. If a farmer sows 1^{Hl} 4^{Dl} of wheat on a hectare of land, how much seed would it require for a field 45^{Dm} long and 42^{Dm} wide?

20. How many steres of wood in a pile 15^{m} long, 1.25^{m} wide, and 1.25^{m} high?

21. Find in milligrams the weight of a centiliter of alcohol, which is .792 times as heavy as water.

22. If a field yielded on an average 15^{Hl} of wheat per hectare and each hectoliter of wheat yielded 55^{Kg} of flour, how many metric tons of flour did the field yield, if it is 50.5^{Dm} long and 24^{Dm} wide?

23. The weight of the olive oil in a flask is 36^{g} 600^{mg}; how many cubic centimeters of this oil are there, if it is .915 times as heavy as water?

24. A dealer bought a square piece of land 36^{Dm} long at $200 per hectare. He divided it into lots containing 72^{a} each, which he sold at $350 per lot; find his profit.

25. A piece of steel bar 1^{m} long, 8^{cm} wide, and 5^{cm} thick weighs how many kilograms, if steel is 7.81 times as heavy as water?

26. How many kiloliters of water will a tank hold that is 4.5^{m} long, 2.4^{m} wide, and 4^{m} deep? Find the weight of the water in metric tons.

27. A vessel weighs 4.8^{Kg} empty, and full of water 26.75^{Kg}; how many liters does it hold?

28. A liter jar weighs 700^{g}; what would it weigh when filled with alcohol, which is .792 times as heavy as water?

Practical Measurements.

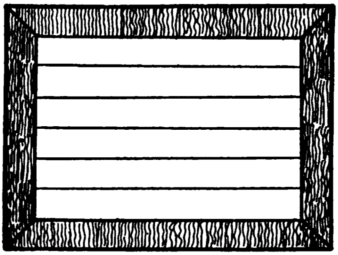
CARPETING.

230. Carpets are usually $\frac{3}{4}$ of a yard wide. Some cheaper grades are 1 yard wide. Matting and oilcloth are usually sold by the square yard.

231. In determining the number of yards of carpet required for a floor, the number of strips needed should be found; a fractional part of a strip should be regarded as a full strip.

Thus, if a floor is 10 ft. 6 in. wide and carpet is ¾ of a yd., or 2¼ ft., wide, then

10½ ft. : 2¼ ft., or 4⅔ = the number of strips that will cover the floor, and since a part of a strip cannot be bought, *5 strips must be bought.*



232. Sometimes the carpet of a room is surrounded by a border, which should be cut at each corner as in the diagram; the length of the border required is then the entire distance around the room.

Note. The standard widths of borders are ⅜ yd., ⅝ yd., and ¾ yd.

EXERCISE 90.

1. How many strips of carpet ¾ yd. wide must be bought for a room 10 ft. wide, the strips to be laid lengthwise?

2. How many yards of carpet ¾ yd. wide must be bought for a floor 18 ft. long, 16 ft. wide, if the strips are to be laid lengthwise?

3. Find the cost of carpeting a room 16 ft. long, 14 ft. wide, with carpet ¾ yd. wide, costing $1.50 a yard, if the strips run lengthwise, and ⅛ yd. is allowed on each strip for waste in matching the pattern.

4. Find the number of yards of carpet ¾ yd. wide that must be bought for a 2d story front room 15 ft. long, 22 ft. wide, the strips to run lengthwise and 2½ yd. to be allowed for waste.

5. Find the cost of the oilcloth required for a hall 12 ft. long, $3\frac{1}{2}$ ft. wide, at 40 cents per square yard.

6. Find the cost of carpeting a room 20 ft. long, 16 ft. wide, with carpet $\frac{3}{4}$ yd. wide, costing \$0.90 per yard. The strips are to be laid lengthwise, and the carpet is to be surrounded by a border $\frac{5}{8}$ yd. wide, costing \$1.20 per yard.

7. Let the student make a drawing of a floor 18 ft. long, 16 ft. wide, on a scale of 1 in. to $1\frac{1}{3}$ ft., showing strips of carpet $\frac{3}{4}$ yd. wide running lengthwise on this floor, and surrounded by a border $\frac{3}{4}$ yd. wide. How many yards of carpet and of border are required, no allowance being made for waste?

PAPERING.

233. Wall paper is generally 18 in. wide, and is sold by the roll. A single roll is 8 yd. long, and a double roll 16 yd. long. Border paper is sold by the yard, and is of various widths.

234. No definite rule can be given for determining the exact amount of paper required to paper a room, because of the allowance that must be made for windows, doors, matching of pattern, and width of border.

Some paperhangers deduct from the entire surface of the side walls above the baseboard, the surfaces of the doors and windows, while others deduct one half of a single roll for each door or window.

No deduction is usually made for border, thereby making sufficient allowance for waste in matching.

EXERCISE 91.

1. Find the cost of the paper and the border required for the walls of a room 18 ft. long, 12 ft. wide, and 9 ft. high above the baseboard, if the room contains 2 windows 6 ft. × 3 ft. and a door $7\frac{1}{2}$ ft. × 4 ft.; the paper to cost \$0.22 per piece and the border \$0.15 per yd.

SOLUTION. 2 × 18 ft. + 2 × 12 ft., or 60 ft. = the distance around the room.

9 ft. = the height of the room.

∴ 9 × 60, or 540 = the number of sq. ft. in the surface of the walls.

2 × (3 × 6) + (4 × 7½), or 66 = the number of sq. ft. to be deducted.

∴ 540 − 66, or 474 = the number of sq. ft. to be papered.

24 × 1½, or 36 = the number of sq. ft. in 1 roll.

474 : 36 = 13⅙.

∴ 14 = the number of rolls required.

60 ft., or 20 yd. = the distance around the room.

∴ 20 yd. = the length of the border required.

$0.22 = the cost of a roll, and $0.15 = the cost of 1 yd. of border.

∴ 14 × $0.22 + 20 × $0.15, or $6.08 = the required cost.

2. Find the cost of the paper required for the walls of a room 18 ft. long, 14 ft. wide, and 9 ft. high above the baseboard, allowing for 1 door 7½ ft. × 4 ft., and 4 windows, each 6 ft. × 4 ft., the paper to cost $0.35 per piece and the border $0.20 per yard.

3. Find the cost of the paper required for the walls and ceiling of a room 20 ft. long, 16 ft. wide, and 9 ft. high above the baseboard, allowing for 2 doors and 4 windows, for each of which ½ of a single roll is deducted; the paper to cost $0.22 per piece and the border 15 cents per yd.

Note. Since side wall patterns differ from ceiling patterns, the number of rolls required for them must be reckoned separately.

4. Find the cost of the paper required for the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 10 ft. high, allowing for 2 doors, each 7½ ft. × 4 ft., 4 windows, each 6 ft. × 4 ft., and a baseboard 9 in. wide; the paper to cost $0.18 per piece and the border $0.10 per yd.

PLASTERING, PAINTING, AND PAVING.

235. In estimating labor, the square yard is taken as the unit in plastering, painting, and paving.

In making estimates, half of the area of the doors, windows, and other openings is deducted from the total area and the result expressed to the nearest square yard.

EXERCISE 92.

1. Estimating that a pound of pure ground white lead is required to give three coats of paint to 2 sq. yd. of surface, how many pounds of this lead would be required to paint with three coats a surface 20 ft. long and 18 ft. wide?

2. 1 bu. of cement and 2 bu. of sand will about cover $3\frac{1}{2}$ sq. yd. of surface 1 in. thick; how many bushels of each would be required to cement the floor of a cellar 24 ft. × 22 ft. to the depth of 1 in.?

3. At $0.15 per sq. yd. find the cost of plastering the walls and ceiling of a room 20 ft. long, 18 ft. wide, and 10 ft. high, allowing for a door $7\frac{1}{2}$ ft. × 4 ft., and 3 windows 6 ft. × 4 ft.

4. Find the number of gallons of paint required to give two coats to both sides of a tight board fence 6 ft. high, erected around a tennis lawn 20 rd. long and 16 rd. wide, if about one gallon of paint is required to give 100 sq. yd. of surface two coats.

5. 1000 plastering laths will cover about 70 sq. yd. of surface, and 11 lb. of nails will nail them on; 8 bu. of good lime, 16 bu. of sand, and 1 bu. of hair will make enough mortar to plaster 100 sq. yd. of surface. Find the amount of each of these substances used in plastering the walls and ceiling of a room 18 ft. long, 16 ft. wide, and 10 ft. high, allowing for 2 doors, each $7\frac{1}{2}$ ft. × 4 ft., and 2 windows, each 6 ft. × 4 ft.

LUMBER MEASURE.*

236. Lumber is generally bought and sold by *board measure.* The unit of board measure is the Board Foot, which is equivalent to the contents of a board a foot square and an inch thick.

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* See also Timber Measure, sections 424 and following.

EXERCISE 93.

1. Find the contents of a board 18 ft. long, 9 in. wide, and 1 in. thick.

SOLUTION. 1 = the number of board feet in a board 1 ft. long, 1 ft. wide, and 1 in. thick.

∴ 18 × 1, or 18 = the number of bd. ft. in a board 18 feet long, 1 ft wide, and 1 in. thick.

∴ $\frac{9}{12} \times 18 = \frac{9 \times 18}{12}$ = the number of bd. ft. in a board 18 ft. long, 9 in. wide, and 1 in. thick. Therefore,

237. PRIN. *The number of board feet in a board one inch thick equals the number of inches in the width times the number of feet in the length, divided by 12.*

Note. All boards less than one inch thick are bought and sold by surface measure, the thickness not being considered.

2. How many board feet in a plank 16 ft. long, 6 in. wide, 2 in. thick?

SOLUTION. $\frac{6 \times 16}{12}$ = the number of bd. ft. in the plank if 1 in thick [§ 237].

∴ $\frac{2 \times 6 \times 16}{12}$ = the number of bd. ft. in the plank when 2 in. thick. Therefore,

238. PRIN. *The number of board feet in a piece of lumber more than an inch thick equals the number of inches in the thickness, times the number of inches in the width, times the number of feet in the length, divided by 12.*

3. Find the cost of 10 boards 16 ft. by 9 in. at $30 per M.

4. Find the number of bd. ft. in a beam 24 ft. long, 10 in. wide, and 6 in. thick.

5. How many bd. ft. in a square stick of timber 50 ft. long, $1\frac{1}{2}$ ft. wide, and 10 in. thick?

6. At $24 per M, what is the value of a pile of timber 16 ft. long, 8 ft. wide, and 6 ft. high?

7. Find the cost of 60 flooring boards 3½ in. wide and 16 ft. long at $24 per M.

8. At $14 per M, find the cost of the boards that must be bought to make a wagon bed 14 ft. long, 4 ft. wide, and 3 ft. high, the boards being 14 ft. long and 12 in. wide.

Note. There will be a waste in cutting the boards for the ends.

9. Find the cost of the following bill of lumber:

10 pieces of scantling 2 × 3, 18 ft. long, at $18 per M.
4 planks 1½ × 6, 14 ft. long, at $27.50 per M.
6 yellow pine beams 4 × 4, 20 ft. long, at $35 per M.
40 pine boards 8 in. wide, 16 ft. long, at $35 per M.

10. How many flooring boards 16 ft. long, 3 in. wide, must be purchased to lay a floor of a house 24 ft. long, 22 ft. wide?

11. How many shingles 4 in. wide, and laid 6 in. to the weather, are necessary for 100 sq. ft. of roof?

12. How many square feet of roof will 1000 shingles cover, if they are 6 in. wide and laid 6 in. to the weather?

13. A bale of shingles is estimated to contain 250, averaging 4 in. wide; how many square feet of roof will a bale cover, if the shingles are laid 4½ in. to the weather?

14. Show that it requires 720 shingles for 100 sq. ft. of roof, if the shingles are 4 in. wide and laid 5 in. to the weather.

15. Find the number of shingles 4 in. wide, laid 5 in. to the weather, required for a roof 38 feet long and the rafters on each side 14 ft. long, the first course being double.

SUG. The length of the rafters is the width of the roof.

MASONRY AND BRICKWORK.

239. The unit of measure of ordinary stone walls is the PERCH. It is a wall 16½ feet long, 1½ feet wide, and 1 foot high, and contains 24¾ cubic feet.

When stone is laid in mortar, 22 cubic feet, or $\frac{8}{9}$ of a perch, are required to make a perch of wall, the remaining 2¾ cubic feet being allowed for mortar.

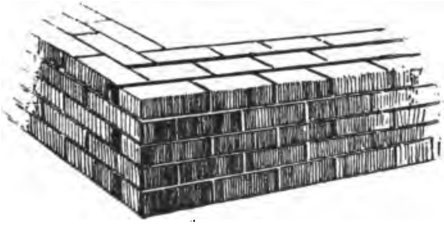
In making estimates for the cost of labor and material for rough stone walls, the distance around the walls on the outside is measured, the corners being thus counted twice, and no allowance is made for openings in the walls.

240. Brickwork is usually estimated by the THOUSAND BRICKS.

241. The unit of measure for walls built of cut stone is usually the SQUARE FOOT.

In making estimates for the cost of labor and material for walls built of cut stone or brick, allowance is made for corners and all openings, if no special contract is made to the contrary.

242. The dimensions of a common brick are 8 in. × 4 in. × 2 in. Philadelphia and Baltimore bricks are 8¼ in. × 4⅛ in. × 2⅜ in.



Bricks are laid in walls as indicated in the diagram, the courses of mortar being ¼ of an inch thick.

243. It requires for each square foot of wall:

7 bricks if the wall is 1 brick, or 4 in. thick.
14 bricks if the wall is 2 bricks, or 9 in. thick.
21 bricks if the wall is 3 bricks, or 13 in. thick.
28 bricks if the wall is 4 bricks, or 18 in thick.

244. From the above we have the following:

RULE. *To find the number of bricks required for a wall, multiply the number of square feet by 7, 14, 21, or 28, according as the wall is 1, 2, 3, or 4 bricks thick.*

EXERCISE 94.

1. Find the number of bricks required to build to the height of 24 ft. the walls of a house 36 ft. long, 30 ft. wide, outside measurement, the walls being 3 bricks, or 13 in. thick, and an allowance of 312 sq. ft being made for doors and windows.

Solution. 2 (36 + 30) × 24, or 3168 = the number of sq. ft. of exposed surface in the walls.

$4 \times \frac{13}{12} \times 24$, or 104 = the number of sq. ft. to be deducted for corners.

312 = the number of sq. ft. to be deducted for openings.
∴ 3168 − (104 + 312), or 2752 = the estimated number of sq. ft.
21 = the number of bricks required for each sq. ft.
∴ 2752 × 21, or 57792 = the number of bricks required.

2. How many perch of stone in a pile 33 ft. × 4 ft. × 3 ft.?

3. How many perch of stone must be bought for a mortar wall 40 ft. long, 4 ft. high, and $1\frac{1}{2}$ ft. thick?

4. At $0.75 a perch, what will a mason charge to build a wall 18 in. thick for a cellar 40 ft. long, 36 ft. wide, and 10 ft. deep, outside measurement?

5. How many perch of mortar wall can be built with 24 cu. ft. of stone?

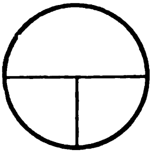
6. It is estimated that the cost of the material and labor for each 1000 bricks in a wall is about $9. Find the cost, at this rate, of building to the height of 24 ft. the four walls of a house 36 ft. long, 30 ft. wide, outside measurement, the walls being 2 bricks, or 9 in., thick, and there being 4 doors $7\frac{1}{2}$ ft. × 4 ft., and 24 windows 6 ft. × 4 ft.

7. Estimating that it costs $0.20 a load to remove the earth, and $1.20 to build each perch of wall, find the cost of digging and walling the cellar for a house 49 ft. 6 in. long and 33 ft. wide, the cellar to be dug to a depth of 8 ft., and the walls, $1\frac{1}{2}$ ft. thick, to extend 2 ft. above the ground.

8. Show from data given in § 239 that a perch contains $24\frac{3}{4}$ cu. ft.

THE CIRCLE. [See § 211, p. 154.]

245. *IMPORTANT TRUTHS.*



1. *The diameter equals twice the radius.*

2. *The circumference equals about* $\frac{22}{7}$ *times the diameter.*

3. *The number of square units in a circle equals one-half the number of units in the radius times the number of units in the circumference.*

4. *The number of square units in a circle equals* $\frac{22}{7}$ *times the square of the number of units in the radius.*

Note. For fuller treatment of circles see p. 301.

EXERCISE 95.

1. Find the circumference of a circle whose radius is 14 ft.

Sug. Circum. = $\frac{22}{7} \times 2 \times 14$ ft. [§ 245, 1.]

2. Find the number of square feet in a circle whose diameter is 2 ft., and whose circumference is $6\frac{2}{7}$ ft.

Sug. The number of sq. ft. = $\frac{1}{2}$ of $1 \times 6\frac{2}{7}$.

3. Find the number of square yards in a circle whose radius is 7 yd.

Sug. The number of sq. yd. = $\frac{22}{7} \times 7^2$.

4. Find the circumference of a circle whose diameter is 2.5 ft.

5. Find the number of square inches in a circle whose diameter is $\frac{3}{4}$ in.

6. Find the radius of a circle whose circumference is 176 ft.

7. Find the number of square feet in a circle whose circumference is 88 ft.

8. How many times does the wheel on a carriage turn in going a mile, if the diameter of the wheel is 4 ft.?

9. The diameter of the wheels of a bicycle is 18 in.; how far will it move forward in one turn of the wheels?

10. How far does the point of the minute hand of a clock move in 15 minutes, if the hand is 4 in. long?

11. How many yards from the center must the inner edge of a circular race track be located in order that it may measure a mile?

12. Knowing that a geographic mile is 1′ of the earth's equatorial circumference, find the length in statute miles of an arc of 15°. [§183, 3.]

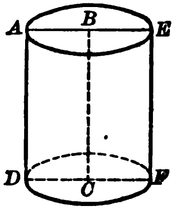
13. How many circular gun wads can be cut out of a piece of pasteboard 18 in. long by 8¼ in. wide, if the radius of each wad is ¼ of an inch?

14. How much would it cost at $1 per square foot to cement the bottom of a cistern, the bottom being a circle having a radius of 4 ft.?

THE CYLINDER.

246. A **Right Cylinder** is a solid formed by revolving a rectangle about one of its sides as an axis.

Thus, the solid formed by revolving the rectangle *A B C D* about the side *B C* as an axis is a right cylinder. The surface formed by the side *A D* is the **Lateral Surface.** The circles formed by the sides *A B* and *D C* are the **Bases.** The axis *B C* is the **Altitude.**



247. *IMPORTANT TRUTHS.*

1. *The number of square units in the lateral surface of a right cylinder equals the number of units in the altitude times the number of units in the circumference of the base.*

2. *The number of cubic units in a right cylinder equals the number of units in the altitude times the number of square units in the base.*

EXERCISE 96.

1. Find the lateral surface of a right cylinder, the diameter of whose base is 16 in. and whose altitude is 8 in.

SUG. $\frac{22}{7} \times 16 =$ the number of inches in the circumference of the base.

Whence, $8 \times \frac{22}{7} \times 16 =$ the number of square inches in the lateral surface. [§ 247, 1.]

2. Find the number of cubic feet in a right cylinder, the radius of whose base is 4 ft. and whose altitude is 14 ft.

SUG. $\frac{22}{7} \times 4^2 =$ the number of square feet in the base.

Whence, $14 \times \frac{22}{7} \times 4^2 =$ the number of cubic feet in the cylinder. [§ 247, 2.]

3. Find the lateral surface of a cylindrical cup, the radius of whose base is 1½ in. and whose altitude is 3 in.

4. Find the number of cubic inches of water the cup in problem 3 will hold.

5. Find the entire surface of a right cylinder, the radius of whose base is 2½ ft. and whose altitude is 8 ft.

6. Find the altitude of a right cylinder, the radius of whose base is 14 in. and whose lateral surface is 880 sq. ft.

7. How deep must a cistern be to hold 40 bbl. of water, if the diameter of its base is 6 ft.?

8. How many cubic centimeters of water will a cylindrical flask hold, the radius of whose base is 2^{cm} and whose altitude is 24^{cm}?

9. Find to the nearest hundredth of a gallon how much water a cylindrical tank will hold, whose diameter is 20 ft. and whose altitude is 16 ft.

10. Find to the nearest gallon how much water will flow in 6 hr. through a cylindrical pipe whose diameter is 9 in., if the water flows at the rate of 2 mi. per hour.

EXERCISE 97.

Miscellaneous Problems.

1. Show that a rod is exactly 25 links.

2. A man has a lot 100 ft. by 80 ft. How many sq. ft. will he have left for a yard after he has built a house 36 ft. by 24 ft.?

3. How many acres of a man's farm must be taken to run a road through it ½ of a mile long and 32 ft. wide?

4. Find to the nearest hundredth the number of gallons in a cubic foot.

5. Find the number of sq. in. in the surface of a common brick.

6. How many cubes, 2 in. on an edge, can be placed on the surface of a table 4 ft. 6½ in. long and 2 ft. 3 in. wide?

7. How many sq. in. of the surface of the table mentioned in example 6 would remain uncovered?

8. How many sq. in. of pasteboard are necessary to make a cube 4 in. long?

9. How many bricks laid on their broadest face are necessary to make a pavement 220 yd. long and 8 ft. wide? (Solve by cancellation.)

10. A certain contractor pays his laborers $0.12½ an hour, and they work 9½ hours per day: how much does a laborer earn in Nov. 1900, deducting election day, four Sundays, and Thanksgiving Day?

11. Slate weighs 170 lb. to the cu. ft. What is the ratio of its weight to the weight of an equal bulk of water?

12. A roof made of slate $\frac{3}{16}$ of an in. thick, laid on sheeting 1 in. thick, weighs about 1000 lb. to 100 sq. ft; find the weight of a roof of this kind on a building 36 ft. long, having rafters 18 ft. long.

13. Slates that are 8 in. by 12 in. will each cover about 36 sq. in. of exposed roof surface; how many slates of these dimensions are needed for a "square," or 100 sq. ft., of roofing?

14. Slates that are 12 in. × 9 in. × $\frac{1}{8}$ in. weigh about 1.33 lb to the slate; how many slates should a man load on his wagon, if he wishes to have a ton? (Answer to the nearest unit.)

15. Find the cost of each of the following:

(*a*) 2785 lb. of straw at $8 per ton.

(*b*) 4545 lb. of wheat at $0.80 per bushel.

(*c*) 275 rails at $6 per C.

(*d*) 285 ft. of boards at $30.75 per M feet.

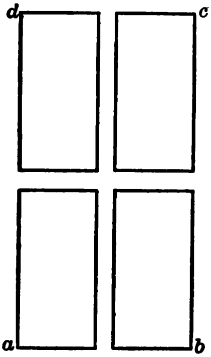
16. Find the cost of each of the following:

(*a*) 2000 lb. of shelled corn at $0.30 per bu.

(*b*) 6780 lb. of hay at $16 per ton.

(*c*) 7656 lb. of cattle at $5 per cwt.

(*d*) 1875 cabbage plants at $8.75 per M.



17. A has a rectangular farm $\frac{1}{2}$ of a mile long and $\frac{1}{4}$ of a mile wide, through which he runs two roads, each 33 ft. wide, parallel to the sides *ab* and *bc*, respectively, in the accompanying diagram.

Find: (1) By how many acres these roads reduce his farm.

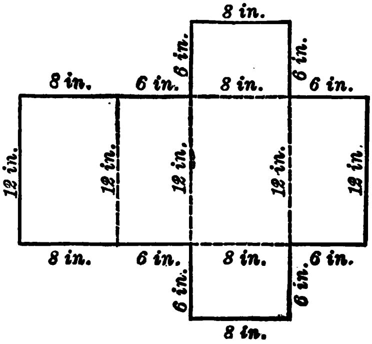
(2) How many acres are left in the farm.

18. One hundred five cubical blocks, each an inch long, can be piled into a rectangular solid of what dimensions?

19. Find the contents of the box that can be made from a piece of pasteboard cut as shown in the accompanying diagram.

20. Find the total length of the edges of a box 4 ft. × 3 ft. × $2\frac{1}{2}$ ft.

21. Find the entire surface of a cord of wood, standard dimensions.



22. Find the integral dimensions of a tin box that will hold just a gallon.

23. How many 4-inch cubes can be cut out of a block 22 in. × 18 in. × 14 in?

24. How many quarter-inch squares can be cut out of a piece of paper 4 in. × 3½ in.?

25. Over how many minute spaces does the minute hand of a clock pass in a day? The hour hand?

26. The depth of the wheat in a bin 8 ft. × 4 ft. is 1 ft. 9 in.; estimating a bushel equal to $\frac{5}{4}$ of a cubic foot, how many bushels of wheat are there in the bin?

27. How much wheat must be sown on 4½ acres, if 2¼ bu. are sown to an acre?

28. What is the area of a square that can be set off with 40 rd. of fencing?

29. A train leaves Philadelphia at 10 A. M. and arrives at Atlantic City at 11.15 A. M.; if the distance is 58.8 miles, find to the nearest hundredth of a mile the average rate of the train per hour.

30. Two trains start from two stations 150 miles apart and run toward each other; if they meet in 2 hr. 40 min. and the faster runs at an average speed of 30 mi. per hour, what is the average speed of the slower train?

31. How many feet of boards 1 in. thick can be sawed from a log 16 ft. × 18 in. × 18 in., allowing ⅛ of an inch waste on each cut?

32. A coal dealer bought 150 tons (2240 lb.) of coal at $3.75 per ton; if he retailed it at $0.28 per 100 lb., find his profit.

33. Susquehanna granite is 2.704 times as heavy as water; what is the weight of a block that is 4 ft. long, 2 ft. 6 in. wide, and 1 ft. 9 in. thick?

34. A farmer drills his corn in rows 2¾ ft. apart; how many rods in the width of a field that contains 160 rows, the outside ones being 1 yd. from the fence?

35. A dealer bought chestnuts at $2.40 a bushel and retailed them at 10 cents a quart; find his gain on a bushel, if there was a loss of one quart in retailing.

36. An article was bought in England for 4 shillings 6 pence; what was its cost in U. S. money?

37. A high wind that has a velocity of 30 miles per hour exerts a force of 4.428 lb. per square foot; what force would such a wind exert on a tight board fence 80 yd. long and 6 feet high?

38. The average annual fall of rain and melted snow in Philadelphia is 39.8 inches; find to the nearest gallon the quantity that falls on an acre.

39. The rate of postage on first class mail matter in the U. S. is two cents per ounce or fraction thereof; find the cost of sending by mail a sealed letter weighing 5½ ounces.

40. The fastest regular trains between Washington and New York on the Baltimore and Ohio R. R. make the run of 225.3 mi. in 5 hours. Deducting 12 minutes for the Jersey City Ferry and 10 minutes for the Canton Ferry, find the average speed of these trains per hour to the nearest tenth of a mile.

41. If it requires 1 lb. of barbed wire of a certain kind for each wire in a rod of fence, how many pounds of wire would be required to build a fence of 3 wires around a field, the sides of which measure 90 rd., 85 rd., 72 rd., and 60 rd.?

42. The quantity of air taken into the lungs at each ordinary inspiration of an adult, is on an average 30 cu. in. Assuming that he takes 17 inspirations per minute, how many cubic feet of air does he breathe in 24 hr.?

43. A dealer who buys thread at 55 ct. per doz. sells it for 5 ct. a spool; how much does he gain on a gross of thread?

44. Third class mail matter in the U. S. includes printed books and pamphlets, and the rate of postage thereon is one cent for each two ounces or fraction thereof; find the cost of sending by mail a book weighing 1 lb. 3 oz.

45. The fastest running train on the P. and R. R. R. made the run from Camden to Atlantic City, a distance of 55.5 miles, in 48 min.; find its average speed in miles per hour.

46. Milk is about 1.03 times as heavy as water; estimate to the nearest hundredth of a pound the weight of a quart of milk (70½ cu. in.), knowing that a cubic foot of water weighs 62.5 lb.

Supplement.

ADDITION AND SUBTRACTION OF COMPOUND AMOUNTS.

Find the sum of 5 rd. 1 yd. 2 ft., 8 rd. 5 yd. 1 ft., and 7 rd. 2 ft.

Form.

	rd.	yd.	ft.	
	5	1	2	
	8	5	1	
	7	0	2	
Sum. =	21	1½	2	=
	21	1	3½	=
	21	2	0½	

Solution. Write the amounts as in the form. The sum of the amounts in the column of feet is 5 ft., which equals 1 yd. 2 ft. Write 2 under the column of feet and add 1 yd. to the yards. The sum of the amounts in the column of yards is 7 yd., which equals 1 rd. 1½ yd. Write 1½ under the column of yards and add 1 rd. to the rods. The sum of the amounts in the column of rods is 21 rd.; write 21 under the column of the rods. 1½ yd. 2 ft = 1 yd. 3½ ft. = 2 yd. 0½ ft.

∴ the sum is 21 rd. 2 yd. ½ ft.

From 4 rd. 2 yd. 1 ft. take 1 rd. 4 yd. 2 ft.

Form.

	rd.	yd.	ft.	
	4	2	1	
	1	4	2	
Diff. =	2	2½	2	=
	2	2	3½	=
	2	3	0½	

Solution. We cannot take 2 ft. from 1 ft., so we take 1 yd. from 2 yd. in the minuend, leaving 1 yd., and add it to the 1 ft. making 1 yd. 1 ft., or 4 ft. 2 ft. from 4 ft. leaves 2 ft. We cannot take 4 yd. from 1 yd., so we take 1 rd. from the 4 rd. in the minuend, leaving 3 rd., and add it to the 1 yd. making 1 rd. 1 yd., or 6½ yd. 4 yd. from 6½ yd. leaves 2½ yd. 1 rd. from 3 rd. leaves 2 rd. 2½ yd. 2 ft. = 2 yd 3½ ft. = 3 yd. 0½ ft.

∴ the difference is 2 rd. 3 yd. ½ ft.

EXERCISE 98.

Add:

1.

bu.	pk.	qt.
2	3	2
5	6	4
3	4	3

2.

yr.	mo.	da.
18	6	18
20	3	16
9	10	12

3.

rd.	yd.	ft.	in.
120	5	2	6
108	3	1	6
72	3	1	4

4.

	yr.	mo.	da.
From	1876	4	18
take	65	8	30

5.

	°	′	″
From	175	40	15
take	90	45	40

6.

	A.	sq. rd.	sq. yd.
From	10	40	4
take	2	80	5

Find the sum of:

7. 8 yr. 2 mo. 10 da., 4 yr. 6 mo. 18 da., and 24 yr. 8 mo. 16 da.

8. 14 ch. 90 li., 4 ch. 85 li., and 30 ch. 40 li.

9. 16 A. 128 sq. rd., 4 A. 10 sq. rd. 16 sq. yd., and 4 A. 8 sq. rd. 20 sq. yd.

10. 24° 15′ 40″, 75° 35′ 18″, and 80° 18′ 45″.

Find the difference of:

11. 1856 yr. 6 mo. 18 da. and 70 yr. 8 mo. 24 da.

12. 170° 15′ 15″ and 90° 30′ 45″.

13. 18 rd. 3 yd. 2 ft. 4 in. and 6 rd. 4 yd. 2 ft. 8 in.

14. £15 8s. 4d. and £5 10s. 8d. 2far.

15. Washington was born Feb. 22, 1732, and died at the age of 67 yr. 9 mo. 22 da. Find the date of his death.

16. The sides of a field measure 16 ch. 40 li., 8 ch. 45 li., 14 ch. 7 li., and 22 ch. 14 li. Find the distance around the field.

17. A merchant received 3 invoices of goods, the first weighing 2 T. 17 cwt. 2 qr. 1 lb., the second 1 T. 8 cwt. 3 qr. 20 lb., and the third 4 T. 6 cwt. 1 qr. 12 lb. Find the weight of the three invoices. [§ 179, Note 2.]

18. May 28, 1900, a total eclipse of the sun began at New Orleans at 7 hr. 37 min. 12 sec. A. M. and ended at 10 hr. 11 min. 42 sec. A. M. What was the duration of the eclipse there?

19. Dec. 21, 1900, the sun rose at 7 hr. 26 min. A. M., and set at 4 hr. 30 min. P. M. How long was it on that day from sunrise to sunset?

20. June 21 the sun set at 7 hr. 40 min. P. M. and rose the following morning at 4 hr. 23 min. A. M. What was the length of the intervening night?

21. A was born July 2, 1856, and died April 5, 1901. What was his age, to the nearest year, when he died?

22. What time elapses from 6.30 P. M. Sunday to 4.15 A. M. Thursday?

23. How much larger is an arc of 180° than one of 45° 15′ 45″?

24. Full moon occurred at 4.56 A. M. June 2, 1901, and again at 6.33 P. M. July 1. Find the length of the intervening lunar month.

25. Queen Victoria died Jan. 22, 1901, aged 81 yr. 7 mo. 28 da.; find the date of her birth, and her age when she began her reign, June 20, 1837.

MULTIPLICATION, DIVISION, AND MENSURATION OF COMPOUND AMOUNTS.

Multiply 12 sq. rd. 10 sq. yd. 8 sq. ft. by 6.

FORM.

	sq. rd.	sq. yd.	sq. ft.
	12	10	8
			6
Product =	74	4½	3 =
	74	4	7½

SOLUTION. 6 × 8 sq. ft. = 48 sq. ft. = 5 sq. yd. 3 sq. ft. Write 3 under sq. ft. and add 5 sq. yd. to the next product. 6 × 10 sq. yd. + 5 sq. yd. = 65 sq. yd. = 2 sq. rd. 4½ sq. yd. Write 4½ under sq. yd. and add 2 sq. rd. to the next product.

6 × 12 sq. rd. + 2 sq. rd. = 74 sq. rd.
4½ sq. yd. 3 sq. ft. = 4 sq. yd. 7½ sq. ft.
∴ the product = 74 sq. rd. 4 sq. yd. 7½ sq. ft.

Divide £21 10s. 6d. by 4.

FORM.

	£	s.	d.
4)	21	10	6
	5	7	7½

SOLUTION. ¼ of £21 = £5$^{\underline{\pounds1}}$. Write 5 under £ and add £1 to 10s., making 30s. ¼ of 30s. = 7s.$^{\underline{2s.}}$ Write 7 under s. and add 2s. to 6d., making 30d. ¼ of 30d. = 7½d. ∴ the quotient = £5 7s. 7½d.

Measure 2 gal. 3 qt. 2 pt. by 2 qt. 1 pt.

SOLUTION. 2 gal. 3 qt. 2 pt. = 24 pt.
2 qt. 1 pt. = 5 pt.
∴ 2 gal. 3 qt. 2 pt. : 2 qt. 1 pt. = 24 pt. : 5 pt. = 24 : 5 = 4⅘.

EXERCISE 99.

Multiply:

1.

£	s.	d.
4	8	4
		6

2.

rd.	yd.	ft.	in.
5	2	2	6
			5

3.

A.	sq. rd.	sq. yd.
5	100	25
		4

Divide:

4.

	°	′	″
8)	45	4	18

5.

	£	s.	d.
6)	8	4	6

6.

	rd.	yd.	ft.	in.
4)	25	4	2	8

7. Multiply £6 4s. 9d. by 5.
8. Multiply 12° 3′ 2″ by 15.
9. Multiply 6 rd. 4 yd. 2 ft. by 3.
10. Multiply 2 gal. 3 qt. 1 pt. by ¾.

SUG. Multiply by 3 and divide the result by 4.

11. Divide £20 4s. 6d. by 5.
12. Divide 2 A. 120 sq. rd. 25 sq. yd. by 4.
13. Divide 6° 45′ 6″ by 15.
14. Divide 3 pk. 6 qt. 1 pt. by ⅔.

SUG. Multiply by $\frac{3}{2}$.

15. Measure 4 bu. 7 qt. by 1 pk. 5 qt. 1 pt.
16. Measure £57 17s. 4d. by £7 4s. 8d.
17. Measure 58 rd. 5 yd. 6 in. by 19 rd. 3 yd. 1 ft. 8 in.

18. A laborer began work at 8.30 A.M. and quit at 5.45 P.M. during the 23 working days of Feb. 1901; allowing him 45 min. each day for dinner, how much did he earn that month if he was paid $12\frac{1}{2}$ ct per hour?

19. The length of an arc of 60° of a certain circle is 4 yd. 2 ft. 3 in. Find the length of the circumference.

20. 7 workmen each worked from 6.30 A.M. to 5 P.M. during the working days of a certain week, stopping each day 40 min. for dinner. How much did they earn that week at 10 ct. per hour?

21. A real estate agent bought a tract of land containing 6 A. 14 sq. rd. and divided it into 10 building lots of equal size. Find the number of square rods in each lot.

22. An arc of 1° measures 4 yd. 2 ft. 6 in. Find the length of an arc of 1′ in the same circle.

23. A banker charged $1 for a draft of 4s. 2d.; how much would he charge for one of £10 4s. 6d.?

24. A farmer sold his crop of wheat at 80 ct. per bu. (60 lb.); how much did he receive for it, if it weighed 2 T. 6 cwt. 40 lb.?

25. A dealer bought a bill of goods for £40 10s. 6d. If £1 is worth $4.86, what was the cost in U. S. money?

26. The "Deutschland" made the trip from Plymouth to New York, a distance of 3044 knots, in 5 da. 15 hr. 46 min. What was her average speed per hour to the nearest hundredth of a knot?

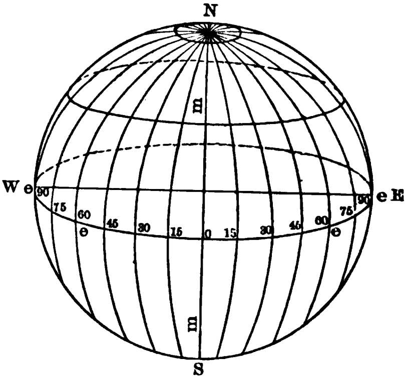
27. There are 13 lunar months in a year, and 365 da. 5 hr. 48 min. 49.7 sec. in a year; find the length of a lunar month.

28. The year consists of 365 da. 5 hr. 48 min. 49.7 sec. How much time is lost in 4 years by counting 365 days to the year?

29. A wagon loaded with hay weighed 42 cwt. 40 lb.; the wagon weighed 14 cwt. 80 lb. Find the value of the hay at $16 per ton.

LATITUDE AND LONGITUDE.

248. To locate places on the surface of the earth a circumference is supposed to be drawn through Greenwich, England, and the north and south poles of the earth. Half-way, or 90°, from either pole another circumference, called the **Equator**, is supposed to be drawn east and west around the earth. These two circumferences divide each other into two equal parts of 180° each.



The **meridian** of a place is the half-circumference supposed to be drawn from pole to pole through the place. It is divided by the equator into two equal parts of 90° each.

The meridian of Greenwich is called the **Prime Meridian**, and its opposite meridian, or the other half of the circumference of which the prime meridian is one half, is the 180th meridian.

249. The **Latitude** of a place is the distance the place is from the equator measured in degrees upon the meridian of the place, and is north or south accordingly as the place is north or south of the equator.

250. The **Longitude** of a place is the distance measured in degrees on the equator from the prime meridian to where the meridian of the place crosses the equator, and is east or west accordingly as the place is east or west of the prime meridian.

251. Parallels of Latitude are circumferences extending east and west around the earth parallel to the equator.

Note. The diagram on p. 190 represents the surface of the earth, on which are drawn a parallel of latitude, and meridians 15° apart.

252. The following truths are evident:

1. *All places on the equator have no latitude.*

2. *All places on the same parallel of latitude have the same latitude.*

3. *All places on the prime meridian have no longitude.*

4. *All places on the same meridian have the same longitude.*

5. *The greatest latitude that a place may have is 90°.*

6. *The greatest longitude that a place may have is 180°, being the distance from the prime meridian to the 180th meridian.*

EXERCISE 100.

How far and in what direction, measured on the same meridian, is:

1. 65° N. from 20° N.?
2. 10° N. from 75° N.?
3. 40° S. from 85° S.?
4. 65° S. from 55° S.?
5. 30° N. from 25° S.?
6. 80° S. from 10° N.?
7. 20° E. from 5° E.?
8. 45° E. from 110° E.?
9. 120° W. from 70° W.?
10. 35° W. from 75° W.?
11. 120° E. from 120° W.?
12. 25° W. from 65° E.?

13. The latitude of A is 60° 55′ N., and of B, on the same meridian, 30° 20′ N.; how far and in what direction is B from A?

14. The latitude of Portland, Me., is 43° 39′ 28″ N., and of Boston 42° 21′ 28″ N.; find their difference of latitude.

15. The latitude of New York is 40° 45′ 23″ N. and Charleston, S. C., is 8° 3′ 39″ farther south than New York; what is the latitude of Charleston?

16. The latitude of San Francisco is 37° 47′ 28″ N. and Chicago is 3° 2′ 3″ farther north than San Francisco; find the latitude of Chicago.

17. The latitude of Washington, D. C., is 38° 55′ 15″ N., and of Valparaiso 33° 1′ 53″ S.; find the difference of their latitudes.

18. The longitude of a vessel was 48° 15′ 16″ W. at noon on Wednesday, and 50° 10′ 12″ W. at noon on Thursday; how far west did it sail in the meantime?

19. One vessel is located at 25° 30′ 6″ E., and another at 10° 5′ 5″ W.; how far is the second west of the first?

20. The longitude of a steamer was 20° 16′ 18″ E. at noon on Thursday, and by noon on Friday it had moved 3° 6′ west; what was then its longitude?

21. A vessel in one day sailed 2° 10′ 5″ east from 6′ 14″ W.; what was then its longitude?

LONGITUDE AND TIME.

253. The earth rotates on its axis from west to east in 24 hours. This makes the sun appear to revolve about the earth from east to west once in the same time.

Since the sun appears to revolve through 360° in 24 hours, its apparent motion is 15° per hour, 15′ per minute, and 15″ per second.

It is noon at any place when the sun is on the meridian of that place, and since the sun apparently moves 15° from east to west in 1 hour, it is then 1 P.M. 15° east of that place, 11 A. M. 15° west of it, 2 P. M. 30° east of it, 10 A. M. 30° west of it, and so on. From this we see that a difference of 15° in the longitude of two places makes a difference of 1 hour in their time. Similarly, a difference of 15′ in their longitude makes a difference of 1 minute in their time, and a difference of 15″ in their longitude makes a difference of 1 second in their time. Hence,

254. 1st, $\frac{1}{15}$ *of the number of degrees, minutes, and seconds in the difference of longitude of two places equals the number of hours, minutes, and seconds in the difference of their time.*

2nd, *15 times the number of hours, minutes, and seconds in the difference of time of two places equals the number of degrees, minutes, and seconds in the difference of their longitude.*

Remark. It is evident from § 253 that places east of any given meridian have later time and those west of it earlier time than places on the meridian.

EXERCISE 101.

1. The longitude of A is 25° 45′ 6″ E. and of B, 17° 10′ 16″ E.; when it is 12.30 P.M. at A, what is the time at B?

FORM.

$$\begin{array}{lll}
25^\circ & 45' & 6'' \\
17^\circ & 10' & 16'' \\
\hline
8^\circ & 34' & 50''
\end{array}$$

$$\begin{array}{r|l}
15 & 8 - 34 - 50 \\
\hline
 & \quad\ 34 \quad 19\tfrac{1}{3}
\end{array}$$

$$\begin{array}{lll}
\text{hr.} & \text{min.} & \text{sec.} \\
12 & 30 & 0 \\
 & 34 & 19\frac{1}{3} \\
\hline
11 & 55 & 40\frac{2}{3}
\end{array}$$

SOLUTION. The difference of longitude is 8° 34′ 50″. Since $\frac{1}{15}$ of the number of degrees, minutes, and seconds in the difference of longitude equals the number of hours, minutes, and seconds in the difference of time, we find by dividing by 15 that the difference of time is 34 min. 19⅓ sec.; and since B is west of A, the time at B is 34 min. 19⅓ sec. earlier than 12.30 P.M. (12 hr. 30 min. after midnight), or 11 hr. 55 min. 40⅔ sec. A. M.

2. When it is 10.30 A.M. at A, long. 10° 4′ 6″ W., the time at B is 1.20 P.M.; find the longitude of B.

FORM.

$$\begin{array}{ll}
\text{hr.} & \text{min.} \\
13 & 20 \\
10 & 30 \\
\hline
2 & 50
\end{array}$$

$$\begin{array}{ll}
2 - & 50 \\
 & 15 \\
\hline
42 & 30
\end{array}$$

$$\begin{array}{lll}
42^\circ & 30' & 0'' \\
10^\circ & 4' & 6'' \\
\hline
32^\circ & 25' & 54''
\end{array}$$

SOLUTION. The difference in time is 2 hr. 50 min. Since 15 times the number of hours and minutes in the difference of time equals the number of degrees and minutes in the difference of longitude, we find by multiplying by 15 that the difference of longitude is 42° 30′; and since the time at B is later than at A, the longitude of B is 42° 30′ east of 10° 4′ 6″ W., or 32° 25′ 54″ E.

3. What is the difference in time between two places whose difference of longitude is 30°? 90°? 45′? 60′? 15″? 120″?

4. What is the difference in time between two places whose difference of longitude is 15° 15′? 30′ 45″? 45° 15′ 15″?

5. What is the difference in longitude between two places whose difference of time is 2 hr.? 6 hr.? 2 min.? 4 min.? 2 sec.? 3 sec.?

6. What is the difference in longitude between two places whose difference in time is 1 hr. 1 min.? 2 min. 3 sec.? 3 hr. 2 min. 3 sec.?

7. Find the difference in time between A and B when the longitude of:

(*a*) A is 60° W. and of B 20° W.
(*b*) A is 40° E. and of B 60° E.
(*c*) A is 20° W. and of B 20° E.

8. Find the time at B when the time at A is 6 A.M., if the longitude of:

(*a*) A is 120° W. and of B 160° W.
(*b*) A is 160° E. and of B 20° E.
(*c*) A is 30° W. and of B 6° E.
(*d*) A is 15° E. and of B 12° W.

9. How far east or west is B from A, if:

(*a*) The time at B is 4 P.M. when the time at A is 10 P.M.?
(*b*) The time at B is 2 A.M. when the time at A is 2 P.M.?
(*c*) The time at B is 6.30 A.M. when the time at A is 6.45 A.M.?
(*d*) The time at B is 9 P.M. Tuesday when the time at A is 2 A.M. Wednesday?

10. Find the longitude of B, if the time at B is 6 A.M. when the time at A is 8.30 A.M.:

(*a*) When the longitude of A is 75° W.
(*b*) When the longitude of A is 87° E.

Note. The longitude of a place is often expressed in time.

Thus, the longitude of Cambridge, Mass., is 4 hr. 44 min. 31 sec. W. This means that Cambridge is west of Greenwich, and that their difference of time is 4 hr. 44 min. 31 sec.

What is the longitude in degrees, minutes, and seconds of:

11. Cambridge, if it is 4 hr. 44 min. 31 sec. W.?

12. Philadelphia, if it is 5 hr. 0 min. 38.5 sec. W.?

13. Washington, D. C., if it is 5 hr. 8 min. 15.7 sec. W.?

What is the longitude expressed in time of:

14. New York, if it is 74° 3″ W.?

15. Paris, if it is 2° 20′ 22″ E.?

16. A ship sails from 40° 8′ 16″ W. to 36° 6′ 40″ E.; what change of time, to the nearest second, should a passenger make in his watch?

17. A ship sailed 5° 15′ 6″ east from noon Wednesday till noon Thursday; how long was the ship sailing this distance?

18. A ship sailed 5° 20′ 15″ west from noon Thursday till noon Friday; how long was the ship sailing this distance?

19. The longitude of Boston is 71° 3′ 30″ W., and of San Francisco 122° 24′ 15″ W.; when it is 9.30 P.M. Wednesday at San Francisco, what time is it at Boston?

20. When it is 1 P.M. at Oxford, Eng., it is 1 hr. 58 min. 45.3 sec. P.M. at Berlin; if the longitude of Oxford is 1° 15′ 6″ W., what is the longitude of Berlin?

21. The time at West Point is 5 A.M. when the time at Providence, R. I., is 5 hr. 10 min. 13.1 sec. A.M.; if the longitude of West Point is 4 hr. 55 min. 50.6 sec. W., what is the longitude of Providence?

22. The longitude of Rome is 12° 27′ 14″ E., and that of New York 74° 3″ W.; when it is 11 P.M. Tuesday at New York, what time is it at Rome?

255. STANDARD TIME. To avoid the confusion that arose from changing the time in traveling, a system of

Standard Time was adopted and put in force on Sunday, Nov. 18, 1883. This system practically divides our country into four belts and adopts the local time of a meridian in each belt as the common time for all places in that belt.

256. The four standards are based on four meridians 15°, or 1 hour, apart. They are as follows:

The Eastern Standard, based on the 75th meridian (about one degree west of New York City's meridian).

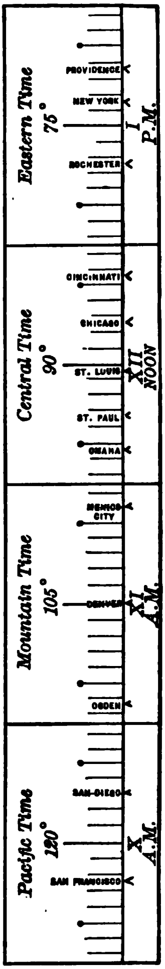
Nearly all cities east of longitude 82° 30′ use Eastern Standard time.

The Central Standard, based on the 90th meridian (2½ degrees west of Chicago's meridian). The western boundary of the Central belt is longitude 97° 30′. Central time is one hour slower than Eastern time.

The Mountain Standard, based on the 105th meridian (near Denver). The Mountain time belt has for its western boundary longitude 112° 30′. Mountain time is one hour slower than Central and two hours slower than Eastern time.

The Pacific Standard, based on the 120th meridian (2⅝ degrees east of San Francisco's meridian). Pacific time is one hour slower than Mountain time.

257. The accompanying diagram* illustrates the above divisions, and the time in each when it is 1 o'clock P. M. in the Eastern belt.



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Note. It was intended that each standard meridian should govern the belt 7½ degrees on each side of it, but there have been some slight variations from this, due to the fact that railroad companies often change time at important railroad centers near a boundary when such centers are not located on the boundary.

SIGHT EXERCISE.

1. When it is noon in the Eastern belt, what is the time in each of the other belts?

2. When it is noon by Mountain time, what is the time in each of the other belts?

3. Find the difference in Standard time between Denver and New York; between San Francisco and New York.

4. In traveling from Philadelphia to San Francisco would a traveler find his watch too slow or too fast, and by how many hours?

5. The longitude of New Haven is 72° 55′ 18″ W.; in what belt is New Haven?

6. When the Standard time at Omaha is 6 P. M., what is the Standard time at New Haven?

7. In traveling from Ogden to New York how would a traveler correct his time?

8. In traveling from St. Louis to San Diego how would a traveler correct his time?

9. How much slower is local, or sun, time than Standard, or railroad, time 77° 30′ W.?

10. When it is noon at the 75th meridian west, is the local time of places that are east of the 75th and in the Eastern belt faster or slower than railroad time?

11. A traveler that left San Francisco at noon Sunday, Pacific time, arrived in New York at noon Friday, Eastern time; how long did it require to make the journey?

CHAPTER V.

PERCENTAGE.

258. Per Cent means *on* or *by the hundred.* Thus,

(1) An agent who charges 5 per cent for collecting a debt charges $5 on every $100 collected.

(2) A man who gains 3 per cent on an investment gains $3 on every $100 invested.

(3) If 6 per cent of an army were killed, 6 of every 100 were killed.

(4) A farmer who sold $2\frac{1}{2}$ per cent of his land sold $2\frac{1}{2}$ acres of every 100 acres.

(5) If $\frac{3}{4}$ per cent of a certain weight is copper, $\frac{3}{4}$ lb. of every 100 lb. is copper.

In general:

6 per cent of any number or amount equals 6 on every 100 units of it, or .06 of it, or $\frac{3}{50}$ of it. Similarly,

4 per cent of a number or amount equals .04 of it, or $\frac{1}{25}$ of it.

$2\frac{1}{2}$ per cent of a number or amount equals $.02\frac{1}{2}$ of it, or $\frac{1}{40}$ of it.

$\frac{2}{3}$ per cent of a number or amount equals $.00\frac{2}{3}$ of it, or $\frac{1}{150}$ of it.

100 per cent of a number or amount equals the number or amount itself.

Note. The symbol % is read *per cent.*

EXERCISE 102.

1. What part of a number is:

(*a*) 1% of it?
(*b*) 3% of it?
(*c*) 5% of it?
(*d*) 10% of it?
(*e*) 15% of it?
(*f*) 20% of it?
(*g*) 60% of it?
(*h*) 35% of it?
(*i*) 72% of it?
(*j*) 85% of it?
(*k*) 48% of it?
(*l*) 64% of it?

2. What part of a number is:

(*a*) 12½% of it? (*c*) 62½% of it? (*e*) 6¼% of it?
(*b*) 37½% of it? (*d*) 87½% of it? (*f*) 18¾% of it?

3. What part of a number is:

(*a*) 2½% of it? .5% of it? 1.5% of it?
(*b*) 6⅔% of it? .75% of it? 7.5% of it?

4. How many times a number is:

(*a*) 100% of it? (*d*) 600% of it? (*g*) 175% of it?
(*b*) 200% of it? (*e*) 125% of it? (*h*) 250% of it?
(*c*) 500% of it? (*f*) 150% of it? (*i*) 1000% of it?

5. Find 5% of $40.

FORM.

$40
.05
$2.00

SOLUTION (1) 5% of $40 = $\frac{5}{100}$, or $\frac{1}{20}$, of $40 = $2.

SOLUTION (2) 5% of $40 = .05 × $40 = $2.

Find:

6. 4% of 50.
7. 25% of $8.
8. 75% of 6 bu.
9. 80% of 15 yd.
10. 15% of 30 da.
11. 30% of 2 gi.
12. 45% of $150.
13. 100% of 6 A.
14. 105% of 40 yr.
15. 125% of $2075.50.
16. 12½% of 16.
17. 37½% of $24.
18. 62½% of 6 A.
19. 87½% of 2 yd.
20. 33⅓% of 5280 ft.
21. 66⅔% of 360°.
22. 16⅔% of $2506.
23. 6¼% of 1525 gal.
24. 18¾% of 1600 sheep.
25. 133⅓% of $175.20.
26. ¾% of 20.
27. ⅘% of 500.
28. ½% of ¼.
29. ¾% of 2½.
30. ⅞% of $2752.
31. .3% of 400 gal.
32. .25% of $10025.
33. .75% of 2567^{cdm}.
34. 1.75% of 144^{g}.
35. 2.5% of 45^{a}.

36. $25 is 2½% of what amount?

SOLUTION (1) \$25 = 2½%, or $\frac{2\frac{1}{2}}{100}$, or $\frac{1}{40}$, of the required amount.

∴ 40 × \$25, or \$1000 = the required amount.

SOLUTION (2) \$25 = 2½%, or .02½, or .025 × the required amount.

∴ \$25 ÷ .025, or \$1000 = the required amount.

FORM.

$$\begin{array}{r} .025\overline{)\$25.000} \\ \hline \$1000 \end{array}$$

Find the number or amount of which:

37. 20 is 25%.
38. 75 is 5%.
39. \$80 is 80%.
40. 2 mi. is 75%.
41. 9045 is 45%.
42. 550 A. is 110%.
43. 5^{m} is 600%.
44. ⅔ is 125%.
45. 60 da. is 150%.
46. 1¾ da. is 150%.
47. 8 gal. is 12½%.
48. 36 yd. is 37½%.
49. 25 in. is 62½%.
50. 9.1° is 87½%.
51. \$275 is ⅘%.
52. ⅚ mi. is 1¼%.
53. \$1755 is 112½%.
54. .024 is .8%.
55. $.5^{\text{a}}$ is .02½%.
56. \$275.50 is 200%.

57. What per cent of a number is:

(*a*) ¼ of it?

SOLUTION. ¼ of a number = ¼ of 100% of it, or 25% of it.

(*b*) ½ of it?
(*c*) ⅛ of it?
(*d*) ⅙ of it?
(*e*) $\frac{1}{12}$ of it?
(*f*) $\frac{1}{20}$ of it?
(*g*) $\frac{1}{25}$ of it?
(*h*) $\frac{1}{50}$ of it?
(*i*) $\frac{1}{30}$ of it?
(*j*) $\frac{1}{100}$ of it?

58. What per cent of a number is:

(*a*) ¾ of it?
(*b*) ⅖ of it?
(*c*) ⅘ of it?
(*d*) $\frac{7}{10}$ of it?
(*e*) $\frac{2}{25}$ of it?
(*f*) $\frac{7}{100}$ of it!

59. What per cent of a number is:

(*a*) ⅜ of it?
(*b*) ⅝ of it?
(*c*) ⅞ of it?
(*d*) $\frac{5}{12}$ of it?
(*e*) $\frac{7}{12}$ of it?
(*f*) $\frac{9}{8}$ of it?

60. What per cent of a number is:

(*a*) 2 times the number?
(*b*) 5 times the number?
(*c*) 1½ times the number?
(*d*) 1¾ times the number?
(*e*) 8 times the number?
(*f*) 10 times the number?
(*g*) 2⅕ times the number?
(*h*) 1⅓ times the number?

61. What per cent of a number is:

(*a*) .5 of it?

Solution. .5 of a number = .50 of it, or 50% of it.

(*b*) .25 of it?
(*c*) .7 of it?
(*d*) .3 of it?
(*e*) 2.56 times the number?
(*f*) 1.1 times the number?
(*g*) $2.45\frac{1}{3}$ times the number?

62. What per cent of a number is:

(*a*) .125 of it?

Solution. 125 of a number = $.12\frac{1}{2}$ of it, or $12\frac{1}{2}$% of it.

(*b*) .375 of it?
(*c*) .005 of it?
(*d*) .0675 of it?
(*e*) $.133\frac{1}{3}$ of it?
(*f*) $.111\frac{1}{9}$ of it?
(*g*) .0002 of it?

63. What per cent of 50 is $12\frac{1}{2}$?

Solution (1) $12\frac{1}{2} = \frac{12\frac{1}{2}}{50}$, or $\frac{1}{4}$, of 50.

$\therefore 12\frac{1}{2} = \frac{1}{4}$ of 100% of 50, or 25% of 50.

Solution (2) 1% of 50 is .5.

Form.

$.5\overline{)12.5}$
25

$\therefore$ 12.5 is as many times 1% of 50 as .5 is contained times in 12.5, or 25.

What per cent of:

64. 5 is 4?
65. 75 is 25?
66. 1 gal. is 1 qt.?
67. 2 is 10?
68. 1 pk. is 1 bu.?
69. $\frac{1}{2}$ is $\frac{1}{4}$?
70. $\frac{2}{3}$ is $\frac{3}{4}$?
71. $10 is $0.10?
72. 1^{g} is 1^{mg}?
73. .5 is .25?
74. 5 is $2\frac{1}{2}$?
75. 10 is $1\frac{1}{4}$?
76. 1 is 5?
77. $2\frac{1}{2}$ is 10?
78. 1 T. is 1 cwt.?
79. 250 bu. is 2 bu.?
80. 50 A. is 50 A.?
81. $4000 is $125?
82. 16 is .5?
83. 112.5 is 18.75?
84. 1760 yd. is $5\frac{1}{2}$ yd.?
85. $12.50 is $2\frac{1}{2}$ mills?
86. $3500 is $10?
87. $\$87\frac{1}{2}$ is $\$37\frac{1}{2}$?
88. $450.75 is $18.03?
89. $20 is $200?
90. $888\frac{8}{9}$ is $2\frac{2}{9}$?
91. 950 is 1.9?
92. 48 is 200?
93. .025 is .0075?

What per cent (correct to the nearest hundredth) of:

94. 2756 is 246?

95. $576.50 is $27.56?

96. 27.275^s is 128^s?

97. $487.55 is $2.75?

98. 27.58 lb. is .45 lb.?

99. 457.5 gr. is 4.8 gr.?

EXERCISE 103. ORAL.

1. A family that uses 1% of a ton (2240 lb.) of coal a day, uses how many pounds?

2. Forty per cent of a class of 20 pupils were promoted; how many were promoted?

3. How many words does a boy miss out of 100, if he spells 95% of them correctly?

4. The population of a town in 1890 was 250. During the next 10 years it increased 2%; what was it then?

5. An agent collected rents amounting to $400 and was paid $\frac{3}{4}$% of the sum collected; how much was he paid?

6. A farmer's oats crop was 160 bu.; he fed 40% of it and sold the remainder. How much did he sell?

7. A piece of cloth $6\frac{1}{4}$ yd. long shrunk 4% when sponged; what was then its length?

8. $12\frac{1}{2}$% of a pole 24 ft. long is in the ground; how far is the top of the pole from the ground?

9. What weight of silver is in a spoon weighing 16 pwt., if it is $37\frac{1}{2}$% pure?

10. If poultry when dressed lose 20% of their weight, what should a turkey weighing 10 lb. dress?

11. How many words did a boy spell correctly in a lesson, if he missed 10 and was given an average of 80%?

12. If the loss from dressing is 40%, find the weight of a calf that dressed 80 lb.

13. 30% of a class failed in examination. If 14 passed, how many were in the class?

14. A newsboy sold the "Bulletin" for 400% of cost. If he charged 2 ct. for it, how much did it cost him?

15. How many yards of cloth must I buy to have 2½ yd. after it is sponged, if it shrinks 16⅔% in sponging?

16. 37½% of the pupils in a school are boys, and there are 35 girls; find the number of pupils.

17. After my house had been damaged to the extent of $200, it was worth 80% of its original value; what was its original value?

18. 2^{Kg} of iron were found in a piece of ore 20% pure; find the weight of the piece of ore.

19. An agent earned $60 during a certain month by collecting rents for 2% of the sum collected; how much did he collect?

20. What per cent should a boy receive who spells 40 words of the 50 given?

21. What per cent pure is milk, when a pint of water is mixed with 3½ qt. of pure milk?

22. In a crate of eggs containing 6 doz., 6 were found broken; what was the per cent of loss from breakage?

23. What is the per cent of yield, when 2 bu. of wheat sowed yields 30 bu.?

24. A laborer's daily wages were increased from $1 to $1.10 a day; find the per cent of increase.

25. A kilogram of ore yielded 250^{g} of copper; what per cent of the ore was copper?

26. What per cent of a gallon is a pint?

27. When 6 cents is paid for the loan of $1, what is the rate per cent paid?

28. If 2 lb. of green tea are mixed with 3 lb. of black, what per cent of the mixture is green tea? What per cent is black tea? What per cent of the black tea is the green tea?

29. A boy attended school 16 days out of 20 during a certain month. Find the per cent of attendance.

30. The standard gold coin of the U. S. is 9 parts pure gold to 1 part alloy; what per cent is alloy? The alloy is what per cent of the pure gold? The pure gold is what per cent of the alloy? The pure gold is what per cent of the coin?

259. The **Percentage** is the result obtained by taking a given per cent of a number or amount.

260. The **Base** is the number or amount of which the given per cent is taken.

261. The **Rate Per Cent**, or the **Rate**, is the number of units taken of each hundred of the base.

Thus, an agent who charges \$5 on every \$100 he collects, charges a rate of 5%.

EXERCISE 104.

1. A lawyer collected \$500, and charged $2\frac{1}{2}\%$ of the sum collected for his services; how much did he hand over to his client?

SOLUTION. \$500 = the sum collected.
$2\frac{1}{2}\%$, or $\frac{1}{40}$, of the sum collected = the lawyer's fee.
$\therefore$ $\frac{1}{40}$ of \$500, or \$12.50 = " " "
And, \$500 − \$12.50, or \$487.50 = the sum paid over.

2. A had \$275 in bank and withdrew 20% of it; how much did he withdraw, and how much had he remaining in bank?

3. 10% of all U. S. gold and silver coin is copper alloy; find the weight of the copper and also of the silver in a silver dollar, if its weight is 412.5 gr.

4. The area of Pennsylvania is 45215 sq. mi.; if .508% of this area is water surface, find the amount of water surface, and also of land surface in the state. (Ans. to the nearest sq. mi.)

5. A merchant agreed to settle some outstanding bills amounting to \$275 for 75% of their full value; how much did he lose?

6. The standard gold dollar of the U. S. weighs 25.8 gr.; if it is 90% pure, find the amount of gold in it, and also the amount of alloy.

7. Pewter is $85\frac{5}{9}\%$ copper and $14\frac{4}{9}\%$ tin; how much copper and how much tin in 36 lb. of pewter?

8. Gunpowder is 75% saltpeter, 10% sulphur, and 15% charcoal; how many pounds of each are there in 250 lb. of powder?

9. Water expands 10% of its volume in freezing; how many cubic feet of ice can be made from 15 cu. ft. of water?

10. A farmer sold 12½% of his wheat crop in July and 10% of what remained in August; how much had he then remaining, if at first he had 280 bu.?

11. A dealer bought 200 bu. of potatoes at 50 ct. per bu.; 10% of them spoiled and the remainder shrunk 10%; if he sold them at 75 cts. per bu., find his gain.

12. The population of Richmond, Va., was 81388 in 1890, and the increase for the next ten years was 4.5%; find the population in 1900. (Ans. correct to the nearest 1.)

13. B bought ¼ of an acre of land, which he divided into 3 lots, the first containing 33⅓% of it, the second 50% of it, and the third the remainder; what part of an acre did each lot contain?

14. Wood ashes contain about 5% of potash, 1.5% of phosphoric acid, 32.5% of lime; how many pounds of each of these substances in a ton of ashes?

15. If a miller takes 10% of each bushel of corn ground for toll, and there is also a loss of about 1% in grinding, how many pounds of feed are returned to a farmer for each bushel of corn (56 lb.) that was ground?

16. Potatoes are about 75% water, and of the remainder about 64.8% is starch; how many pounds of starch in 100 lb. of potatoes?

17. A barn worth $900 was insured for ⅔ of its value at ¾% premium; what was the owner's loss, if the barn was totally destroyed after the premium had been paid?

Note. **Premium** is the amount charged for insuring property and is reckoned on the amount insured.

18. Find the tax paid in one year on a property assessed at $3000, if the rate of school tax for that year was .2¼%, the rate of county tax .1½%, and the rate of road tax .3%.

Note. A **Tax** is a levy for public purposes. It is reckoned on the assessed value of property.

19. A dealer imported 1260 yd. of Axminster carpets, for which he paid $0.80 per yard. He was charged a duty of $0.45 per yard and 40% of the cost of the goods; what duty did he pay?

Note. **Duty** is a tax on imported goods.

20. $375 is 75% more than what amount?

SUG. $375 = 175%, or $\frac{7}{4}$, of the required amount.

21. 12½ is 66⅔% more than what number?

22. When a certain number is diminished by 18¾% of itself, the remainder is 26; find the number.

SUG. 26 = 100% − 18¾%, or 81¼%, or $\frac{13}{16}$, of the required number.

23. When a certain number is diminished by 60% of itself, the remainder is ⅘; find the number.

24. 60 is 33⅓% less than what number?

25. When a certain weight is increased by 10% of itself, the result is 22 lb.; find the weight.

26. When a certain length is diminished by 33⅓% of itself, the remainder is 1 ft.; find the length.

27. At the close of business on Tuesday a merchant deposited in bank $385, which was 87½% of the sum he deposited on Monday; how much did he deposit on Monday?

SOLUTION. 87½%, or ⅞, of the deposit on Mon. = the deposit on Tues.
$385 = " " " "
∴ ⅞ of the deposit on Mon. = $385.
Whence, ⅛ " " " " " = ⅐ of $385, or $55.
And, $\frac{8}{8}$ " " " " " = 8 × $55, or $440.

28. If sweet almonds yield oil to the amount of 47% of their weight, how many pounds of almonds will yield 1 lb. of oil?

29. Milk yields butter to the amount of 4% of its weight; how many pounds of milk will yield 20 lb. of butter?

30. 86.8% of the area of Rhode Island is land; find the entire amount of surface and also the amount of water surface in the state, if the amount of land surface is 1085 sq. mi.

31. A newsboy sold papers for 5 ct. each, which was 200% of what he paid for them; what did he pay for them?

32. After a piece of cloth had shrunk 5% of its length by sponging, it measured $3\frac{4}{5}$ yd.; how many yards did it measure at first?

33. If water expands 10% of its volume in freezing, how much water must be frozen to make 100 cu. ft. of ice?

34. The population of Canton, O., was 30667 in 1900, and the increase from 1890 to 1900 was 17.1%; find the population in 1890.

35. A merchant failing in business settled with his creditors by paying them $62\frac{1}{2}$% of their claims; how much did they lose, if he paid them $10,000?

36. A detachment of soldiers were twice attacked. In the first attack 20% of them were killed, and in the second 10% of the remainder; if the detachment then mustered 72 men, how many were killed?

37. A pays $8 a year premium on his house, which is insured at $\frac{3}{4}$ of its real value. Find the real value of the house, if the rate of insurance is $\frac{4}{5}$%.

38. A farmer's taxes for a certain year were $34.51; what was the assessed value of his property, if the rate of taxation for school tax was $\frac{1}{4}$%; for road tax $\frac{4}{10}$%; and for county tax $\frac{1}{5}$%?

39. A farm of 80 acres was assessed at $30 an acre, which was 80% of its real value; what was its real value? The rate of school tax levied on the assessed value was $2\frac{1}{4}$ mills on $1; find the school tax for that year.

40. From a 36-gallon barrel of vinegar a merchant has sold 12 gal.; what per cent of the barrel has he remaining?

SOLUTION. 36 gal. = the amount the barrel first contained.
12 gal. = the amount sold.
∴ $\frac{1}{3}$ of the barrel = the amount sold.
Whence, $\frac{1}{3}$ of 100%, or $33\frac{1}{3}$%, of the barrel = the amt. sold.
And, 100% − $33\frac{1}{3}$%, or $66\frac{2}{3}$%, of the barrel = the amt. remaining

41. If in 100 lb. of corn there are 5.4 lb. of fat, what is the per cent of fat in corn?

42. There is about .18 lb. of fat in 10 lb. of oats; find the per cent of fat in oats.

43. When 2 lb. of black tea are mixed with 3 lb. of green tea, what per cent of the mixture is black tea and what per cent green tea?

44. The amount of land surface in New Jersey is 7455 sq. mi., and the amount of water surface 360 sq. mi.; what per cent of the area of the state is land and what per cent water? (Ans. correct to nearest .001%.)

45. The nickel 5-cent piece is composed of 57.87 gr. of copper and 19.29 gr. of nickel; what per cent of the coin is copper and what per cent nickel?

46. A clerk who earns a salary of $40 per month pays $180 a year for house rent; what per cent of his salary does he pay for house rent?

47. From three pieces of cloth containing 40 yd., 36 yd., and 30 yd. a dealer sold respectively 20%, 50%, and 90%; what per cent of the whole remained?

48. 20% of any number equals what per cent of one fourth of it?

In the following cities find the increase and the per cent of increase in the population from 1890 to 1900. (Answer to the nearest tenth per cent.)

Cities.		Population.		Increase from 1890-1900.	
		1900.	1890.	Number.	Per Cent.
49.	New York, N. Y.	3,437,202	2,492,591		
50.	Chicago, Ill.	1,698,575	1,099,850		
51.	Philadelphia, Pa.	1,293,697	1,046,964		
52.	St. Louis, Mo.	575,238	451,770		
53.	Boston, Mass.	560,892	448,477		
54.	Baltimore, Md.	508,957	434,439		
55.	Cleveland, Ohio	381,768	261,353		
56.	Buffalo, N. Y.	352,387	255,664		
57.	San Francisco, Cal.	342,782	298,997		
58.	Cincinnati, Ohio	325,902	296,908		

59. One of the principal circulating libraries of New York issued to its readers during the year extending from Nov. 1, 1896, to Nov. 1, 1897, 231,052 volumes; of these 162,093 came under the head of fiction; find, to the nearest hundredth, the per cent of the number of volumes issued that was fiction.

60. A's store valued at $5000 was insured for 1 year at ¾ of its value. If the premium paid was $37.50, what rate of insurance was charged?

61. The assessed value of the property of a town is $2,500,000. What rate of taxation must be laid in order to raise $4,750 for the support of the schools and pay a collector a commission of 5% for collecting?

Sug. 5% of the amount collected = the collector's commission.
Whence, 95% of the amount collected = $4750.

Commercial Discount.

262. Commercial Discount is any deduction made from the *list price*, the *time price*, or the *marked price* of goods.

If but one discount is made on any price, a certain per cent of that price is deducted; if two or more discounts are made, the first discount is reckoned on the price, the second on the remainder, the third on the next remainder, and so on.

Thus, if an article is listed at $0.40 and sold at a discount of 10%, the discount is 10% of $0.40, or $0.04.

If it is listed at $0.40 and sold at 10% and 5% off, the first discount is 10% of $0.40, or $0.04; the second, 5% of $0.36, or $0.018; and the total discount, $0.04 + $0.018, or $0.058.

EXERCISE 105.

1. What must be paid for a safe listed at $200 and bought at 25% and 10% off?

Solution. 25%, or ¼, of $200, or $50 = the first discount.
∴ $200 − $50, or $150 = the first remainder.
10% of $150, or $15 = the second discount.
And, $150 − $15, or $135 = the cost.

2. On a bill of goods amounting to $205.40 a discount of 5% was allowed; find the cost.

3. Find the cost of an autoharp the catalogued price of which is $15, bought subject to a discount of 25%.

4. A merchant reduced Victoria lawn, marked 15 ct. per yd., 16⅔%; what was then the selling price?

5. A refrigerator marked at $60 was sold subject to a discount of 25% and 20%; find the selling price.

6. A dealer bought umbrellas amounting to $2500 subject to a discount of 10% and 10%; find the cost.

7. A piano marked $400 was sold at a discount of 30%, 20%, and 10%; find the cost.

8. A photographer bought cameras amounting to $250, on which he was allowed a discount of 10%, 20%, and 5%; find the cost.

9. A wholesaler sold goods at a discount of 20% and 20% from the marked price and allowed a further discount of 5% for cash; what did he receive for goods marked $0.50 per yd.?

10. March 1, a dealer bought a bill of goods amounting to $350, payable in 60 da., subject to a discount of 1%, if paid within 30 da., or 3% if paid within 10 da. How much would settle this bill Apr. 20? Mar. 8? Mar. 28?

11. After taking off a discount of 7% for cash, a dealer sent $6.51 in payment of a bill of suspenders; what was the bill?

SUG. 100% of the bill − 7% of the bill, or 93% of the bill = the sum sent, or $6.51.

12. A bill of goods was bought Apr. 1, subject to a discount of 5%, if paid in 30 days. Apr. 30 a check for $213.75 was sent in payment of the bill; what was the bill and what the discount?

13. The net cost of a bill of dry goods was $200; what was the list price, if it was bought at a discount of 20% and 25%?

Find the rate of discount on:

14. Tapestry curtains marked down from $5 to $3.50.

15. Towels marked down from 16 ct. to 12½ ct.

16. Napkins marked down from $2 per doz. to $1.65.

17. Wilton carpets marked down from $2.50 per yd. to $2.

18. Men's worsted suits marked down from $25 to $18.

19. Which is better for the buyer, and by what per cent of the list price, to purchase goods at a discount of 40% and 10%, or at a discount of 20% and 20%?

20. Show that a discount of 20% and 10% equals a discount of 10% and 20%.

21. Extend the items in the following bill and find the amount due Aug. 15, 1900:

STEELTON, PA., Aug. 6, 1900.

Mr. B. B. Witmer,

To CAPITAL SHIRT CO., *Dr.*

Manufacturers of shirts.

Terms, 30 da., 5%; 10 da., 6%.

1 doz.	#407 shirts	4	75	.	..		
1/2 "	354 "	6	50	.	..		
1/2 "	453 "	9	00	.	..		
1/2 "	0/7 "	4	50	.	..		
1 1/6 "	641 "	4	50	.	..		
2/3 "	13625 "	4	50	.	..		
1 "	693 "	4	75	.	..	27	75
	Less 6%					.	..
						26	09

Note. # denotes *number*. "Terms 30 da., 5%; 10 da., 6%," denotes that if the bill is paid within 10 days from date, 6% of it will be deducted, or if paid after 10 days and before the expiration of 30 days, 5% will be deducted.

Make out the following bills, and find the amounts due on them:

22. J. S. Peters, Harrisburg, Pa., bought of The Central News Company, Philadelphia, 36 boxes paper #3403 @ 16 c.; 24 boxes #246 @ 15 c.; 24 boxes #930 @ 9 c.; 36 boxes #366 @ 18 c.; 24 boxes #948 @ 14 c.; 1 quire Blotting Paper @ 60 c. Discount allowed, 20%.

23. L. I. Crawford, Norristown, Pa., bought of A. S. Downie, 417 Market St., Philadelphia, on Mar. 30, 1900, 3 lb. Erasers #60 @ 75 c.; 3 gr. Isaac Stub Pens #7 @ $1.00; 2 doz. Eye Shades @ 90 c.; 1 M. Envelopes #1562–4 @ $1.50. Terms, 30 da.; 1% cash in 10 da. The bill was paid Apr. 6.

Profit and Loss.

263. The **Cost** of an article is the amount paid for it. The **Selling Price** is the amount received for it. The **Profit** is the amount that the selling price exceeds the cost and the expenses. The **Loss** equals the amount that the cost and the expenses exceed the selling price. When there are no expenses, the profit equals the selling price less the cost, and the loss equals the cost less the selling price.

264. *Gain or loss per cent is always reckoned on the cost.*

EXERCISE 106.

1. Gingham was sold at $0.18 per yd., which was at a gain of $12\frac{1}{2}\%$; find the gain and the cost per yard.

SOLUTION. $12\frac{1}{2}\%$, or $\frac{1}{8}$, of cost = gain.
∴ $\frac{9}{8}$ of cost = s. p.
$0.18 = s. p.
∴ $\frac{9}{8}$ of cost = $0.18.
$\frac{1}{8}$ of cost = $0.02 = the gain.
$\frac{8}{8}$ of cost = $0.16.

2. A dealer sold some damaged goods for $24, which was at a loss of $11\frac{1}{9}\%$; find the cost and the loss.

SUG. $11\frac{1}{9}\%$, or $\frac{1}{9}$, of cost = loss.
∴ $\frac{8}{9}$ of cost = s. p., or $24.

3. An article was bought for $24 and sold for $40; find the gain per cent.

Solution 1. $40 = s. p.
$24 = cost.
∴ $40 − $24, or $16 = gain.
Whence, $\frac{16}{24}$, or ⅔ of cost = gain.
Or, ⅔ of 100%, or 66⅔%, of cost = gain.
Whence, 66⅔ = the gain per cent.

Solution 2. $40 = s. p.
$24 = cost.
∴ $40 − $24, or $16 = gain.
1% of $24, or $0.24 = gain at 1%.
∴ $16 : $0.24, or 66⅔ = the number of per cent gained.

Note. Solution 2 can be used to advantage when the numbers are large or involve decimals.

Find the gain and selling price, if the cost is:

4. $200, and the rate of gain 20%.

5. $32, and the rate of gain 12½%.

6. $0.60, and the rate of gain 66⅔%.

7. $800, and the rate of gain ⅜%.

8. $0.30, and the rate of gain 200%.

9. $0.01, and the rate of gain 100%.

Find the marked prices of:

10. Ribbon costing $0.10 per yd. and marked at an advance of 20%.

11. Gloves costing $0.75 per pair and marked at an advance of 25%.

12. Seal coats costing $30 and marked at an advance of 16⅔%.

13. French flannel waists costing $2.50 and marked at an advance of 60%.

14. Golf capes costing $8 and marked at an advance of 37½%.

Find the loss and selling price, if the cost is:

15. $3, and the rate of loss 30%.

16. $24, and the rate of loss 16⅔%.

17. $0.75, and the rate of loss 6⅔%.
18. $40, and the rate of loss ¾%.
19. $0.02, and the rate of loss 50%.
20. $1, and the rate of loss 100%.
21. $2,000, and the rate of loss .4%.

Find the cost and the gain or loss, if the selling price is:

22. $50, and the rate of gain 25%.
23. $72, and the rate of gain 12½%.
24. $0.06, and the rate of gain 200%.
25. $0.40, and the rate of loss 20%.
26. $1.50, and the rate of loss 50%.
27. $1.10, and the rate of gain 10%.
28. $1, and the rate of loss 33⅓%.

Find the selling price when:

29. The gain is $20 and the rate of gain 10%.
30. The gain is $0.06 and the rate of gain 66⅔%.
31. The gain is $2.50 and the rate of gain 400%.
32. The loss is $500 and the rate of loss 20%.
33. The loss is $0.08 and the rate of loss 33⅓%.
34. The loss is $1 and the rate of loss 1%.

Find the rate of gain or loss when the cost is:

35. $6, and the selling price $8.
36. $0.05, and the selling price $0.04.
37. $0.02, and the selling price $0.06.
38. $75, and the selling price $25.
39. $1, and the selling price $2.
40. $200, and the selling price $201.

Find the gain or loss on goods listed at:

41. $3, bought at a discount of 20%, and sold at a gain of 20%.

42. $0.25, bought at a discount of 25%, and sold at a gain of 12½%.

43. $45, bought at a discount of $11\frac{1}{9}\%$, and sold at a loss of $6\frac{1}{4}\%$.

44. $100, bought at a discount of 20% and 10%, and sold at a loss of 5%.

Find the gain or loss per cent on goods listed at:

45. $6, bought at a discount of $33\frac{1}{3}\%$, and sold for $6.

46. $0.40, bought at a discount of 10%, and sold for $0.48.

47. $0.04, bought at a discount of 25%, and sold for $0.02.

48. $25, bought at a discount of 25%, and sold for $20.

49. Thread costing 52 ct. per doz. was sold at 5 ct. a spool; find the gain per cent.

50. A wholesale dealer marked shoes at $1.50 a pair. He sold them at a discount of 20% to a retailer, who sold them at a profit of 40%. Find the retail price.

51. What per cent is gained on goods that are sold for double their cost?

52. What per cent is lost on goods that are sold for half their cost?

53. By selling a coat for $6, I lose 25%; find the loss.

54. A dealer was offered $180 for a horse, which was 20% more than it cost him; he afterward sold it for $125; find the gain or loss per cent.

55. Crochet hooks were sold for 2 ct. each, which was at a gain of 44%; what did they cost per gross?

56. At what price per piece must leather belts be sold that the gain may be 20%, if they cost $1.50 a dozen?

57. Mohair shoe laces listed at 30 ct. per doz. were bought at 20% off and sold at 3 ct. a pair; find the gain per cent.

58. An overcoat that cost $5 was marked to gain 50%, but it was sold out of season at a discount of 30% from marked price; find the gain or loss per cent.

59. A shoe dealer marked a pair of gum boots to gain 20%, but he finally sold them for $2.40, which was at a loss of 25%; find the marked price.

60. A newsboy buys papers at 1 ct. each and sells them at 2 ct. each; find his gain per cent.

61. A retailer buys oilcloth at 60 ct. per sq. yd., subject to a discount of 16⅔% and 4%, and sells it at 60 ct. per sq. yd.; find his gain per cent.

62. A hardware dealer bought nails at $2.50 per keg of 100 lb.; what per cent does he gain by selling them at 4 ct. a pound, if they lose 5% in weighing?

63. The loss on an article sold for $6 was 25%; what would have been the gain or loss per cent, if it had been sold for $7.50?

64. A dealer bought English cheese at $0.20 per lb.; how many ounces should he give for 10 ct. in order to gain 20%?

65. A dealer bought oranges at 20 ct. per doz.; what should he charge for 3 in order that his gain may be 50 per cent?

66. A suit was sold for 94% of cost; if this was at a loss of 36 ct., find the selling price.

67. Steamer rugs were sold for 125% of cost; if the gain on them was $1 each, find the selling price.

68. A music dealer bought a secondhand organ for $75 and asked such a price for it that he could reduce it $10 and still make a profit of 20%; what did he ask for it, and what rate of discount did he offer?

69. A woman gains 50% by selling 6 boxes of "Blue Hen" matches for 10 ct.; what would she gain per cent by selling 2 boxes for 5 ct.?

70. A dealer bought hams for 12 ct. per pound; at what price must he sell them to gain 20%, if they lose 20% of their weight in drying?

71. A commission man bought live turkeys at 10 ct. per pound; what must he sell them for per pound to gain 20%, if they lose 20% of their weight in dressing?

72. A coal dealer buys coal at $4 per ton (2240 lb.) and retails it at 28 ct. per 100 lb.; find his gain per cent.

Commission.

265. Commission is the sum charged by an agent for transacting business for another person, who is called his **Principal.** It is a certain per cent of the money involved in the transaction.

Thus, if an agent sells goods amounting to $200 on a commission of 5%, his commission is 5% of $200, or $10.

266. When a *sale* is made, *commission is reckoned on the amount of sales,* and when a *purchase* is made, *on the amount of the purchase.*

267. A **Consignment** is the goods sent by one person to another for sale. The party who sends the goods is the **Consignor,** and the party to whom they are sent is the **Consignee.**

268. The **Net Proceeds** of a consignment is the sum that is left after all expenses are paid.

EXERCISE 107.

1. An agent sold farming implements to the amount of $2045; he paid freight charges of $25, and, after deducting a commission of 20%, remitted the net proceeds. Find his commission and the net proceeds.

SOLUTION. $2045 = the amount of sales.
20%, or $\frac{1}{5}$, of the amt. of sales = the commission.
∴ $\frac{1}{5}$ of $2045, or $409 = the commission.
$25 = the freight charges.
∴ $409 + $25, or $434 = the total expenses.
And, $2045 − $434, or $1611 = the net proceeds.

2. My agent purchased for me 1000 bu. of wheat at $0.72 per bu., prepaid the freight, amounting to $20, and charged me a commission of 2½%; find entire cost.

SOLUTION. 1000 × $0.72, or $720 = the cost of the wheat.
2½% of $720, or $18 = the commission.
$20 = the freight charges.
∴ $720 + $18 + $20, or $758 = the total cost.

3. An agent sold $210 worth of books and was paid a commission of 25%; find his commission.

4. A commission merchant sold 520 lb. of dressed turkey at $0.15 per pound, charging a commission of 5%; what sum should he remit?

5. A commission merchant sold 800 doz. of eggs at $0.18 per doz. After deducting a commission of 4% and paying $1.50 for delivery, how much should he remit?

6. A real estate agent sold a house for B, receiving for it $3750; he charged a commission of 4% and $7.60 for additional expenses. Find the net proceeds.

7. An agent sold a sewing machine for $30 and charged a commission of 5%. If he paid $2.50 to have it delivered, how much should he remit?

8. My agent bought 20 T. of bran at $18 per ton; how much money should I send him in settlement, if his commission is 4%?

9. I purchased a house for $4000 through a real estate agent, who charged me a commission of 4% and $4.50 additional for having the deed recorded; how much did the property cost me?

10. My agent purchased a quantity of rye at $0.40 per bushel, charging a commission of 2%; how many bushels did he buy, if the total cost was $816?

Sug. Find the commission on the cost of 1 bu.

11. A lawyer collected for his client 60% of a debt of $1200; if the lawyer charged a commission of 20%, how much should he hand over to his client?

12 An administrator is usually paid a commission of 5% on all personal property sold, and $2\frac{1}{2}$% on all real estate; what commission should be paid an administrator of an estate, the personal property of which sold for $625 and the real estate for $3020?

13. An agent received $2000 with which to buy flour; if he charged a commission of 5%, how many barrels at $4 per barrel did he buy, and what was the surplus?

14. A real estate agent sold for B a farm, receiving for it $3500; he charged a commission of 3%, and the transfer of the property cost $25.60 additional; how much should B receive?

15. What amount must be sent an agent so that after taking out a commission of 2½% and prepaying the freight amounting to $18.50, he may have sufficient to purchase 600 bu. of wheat at $0.70 a bu.?

16. An agent received $210 with which to purchase produce after deducting his commission of 5%; find the amount expended for produce and the commission.

SUG. 5% of the amt. expended = the commission.
∴ 105% of the amt. expended = the amt. received, or $210.

17. I sent my agent $512.50 with which to purchase leaf tobacco after deducting his commission of 2½%; how much commission should he deduct?

18. An agent sold 60 tons of coal at $5 per ton and charged a commission of 2%. He invested the net proceeds in corn at $0.50 per bu., charging a commission of 2%; how many bushels of corn did he purchase, and what was the surplus?

19. A lawyer charged me a commission of 15% for collecting a debt overdue; if he remitted to me $850, what commission did he charge?

20. An agent sent a piano dealer $450 as the net proceeds of the sale of a piano, after deducting a commission of $50; what rate of commission was charged?

21. A dealer in farming implements received from his agent $520 as the net proceeds of a sale, after the agent had deducted a commission of $60 and paid $20 freight charges; find the rate of commission charged.

22. I paid $336 for 400 bu. of wheat purchased through an agent at $0.80 per bushel; what rate of commission did the agent charge?

23. The entire cost of a carload of horses, including $200 freight charges and the agent's commission amounting to $400, was $4600; what rate of commission did the agent charge?

Simple Interest.

269. **Interest** is money paid for the use of money.

270. The **Principal** is the sum on which interest is paid.

271. Interest for 1 year is reckoned by taking a given *rate per cent* of the principal.

272. The time for which interest is paid is usually reckoned in *years and days when it exceeds a year*, and *in days only when it is less than a year, the exact number of days being counted.*

Thus, if a sum of money is on interest from June 1, 1899, to Oct. 15, 1901, the time is counted as follows: from June 1, 1899, to June 1, 1901, is 2 yr.; and from June 1, 1901, to Oct. 15, 1901, is 136 da., making in all 2 yr. 136 da. Again, if the sum is on interest from May 1, 1900, to Dec. 1, 1900, the time is 214 da.

273. When interest is payable by the month, or when a debt is payable at the expiration of a certain number of months, *calendar months* are meant.

Thus, A sum loaned March 1, payable in 3 mo., is due June 1.
A sum loaned Jan. 31, payable in 3 mo., is due Apr. 30.
A sum loaned Apr. 30, payable in 3 mo., is due July 30.

If these sums bore interest at $\frac{3}{4}\%$ a month, the interest in each case would be $2\frac{1}{4}\%$ of the sum.

274. Interest for time less than a year is usually reckoned on the basis of 360 days to the year; but sometimes it is reckoned on the basis of 365 days to the year, in which case it is called **Exact Interest**.

Note. In this book, unless otherwise specified, interest will be reckoned on the basis of 360 da. to the year; that is, 1 da. will be considered $\frac{1}{360}$ of a year.

275. The **Amount** is the sum of the interest and the principal.

276. Each state has established a **Legal Rate** of interest for the settlement of interest accounts when no rate has been agreed upon by the parties concerned.

277. **Usury** is the amount of interest charged above what is legally allowed; it is unlawful interest, and there are various penalties in the different states for receiving it.

Thus, in Delaware the penalty is forfeiture of the money loaned; in New Jersey, forfeiture of interest; in Pennsylvania, forfeiture of the usurious interest, if sued for within six months.

278. PRIN. *The interest on any principal for 60 days at 6% equals .01 of the principal.*

For, the interest on any principal for 360 da. at 6% = .06 of the principal.

∴ The interest on any principal for 60 da. at 6% = .01 of the principal.

279. PRIN. *One sixth of the number of days equals the number of mills in the interest on $1 at 6%.*

For, 6 ct., or 60 mills = the interest on $1 for 360 da. Here we see that 60, the number of mills, is $\frac{1}{6}$ of 360, the number of days.

EXERCISE 108.

1. Find the interest on $200 from Jan. 1, 1900, to June 1, 1902, at 4%.

FORM.

$$\begin{array}{r} \$200 \\ .04 \\ \hline \$8.00 \\ 2\frac{151}{360} \\ \hline \$16.00 \\ 3.35\frac{5}{9} \\ \hline \$19.35\frac{5}{9} \end{array}$$

SOLUTION BY THE GENERAL METHOD.

The time from Jan. 1, 1900, to Jan. 1, 1902 = 2 yr.

The time from Jan. 1, 1902, to June 1, 1902 =
(30 + 28 + 31 + 30 + 31 + 1) da. = 151 da.

∴ The time the prin. is on int. = 2 yr. 151 da. = $2\frac{151}{360}$ yr.

.04 of $200, or $8 = the int. for 1 yr.

∴ $2\frac{151}{360}$ × $8, or 19.35\frac{5}{9}$, or $19.36 = the required int.

SOLUTION BY THE SIX PER CENT METHOD.

The time the prin. is on int. = 2 yr. 151 da.
$25\frac{1}{6}$ mills, or $\$0.025\frac{1}{6}$ = the int. on \$1 for 151 da. at 6% [§ 279].
$\therefore$ $\$0.004\frac{7}{36}$ = “ “ “ \$1 “ 151 “ “ 1%.
And, $\$0.016\frac{7}{9}$ = “ “ “ \$1 “ 151 “ “ 4%.
\$0.08 = “ “ “ \$1 “ 2 yr. “ 4%.
$\therefore$ $\$0.096\frac{7}{9}$ = “ “ “ \$1 “ the given time and rate.
Whence, 200 × $\$0.096\frac{7}{9}$, or $\$19.355\frac{5}{9}$, or \$19.36 = the required int.

SOLUTION BY THE 60-DAY METHOD.

The time the prin. is on int. = 2 yr. 151 da. = 14 × 60 da. + 31 da.
\$2.00 = the int. for 60 da. at 6% [§ 278].
14
\$28.00 = “ “ “ 2 yr. 120 da. (14 × 60 da.) at 6%.
1.00 = “ “ “ 30 “ ($\frac{1}{2}$ of 60 “) “ 6%.
$0.03\frac{1}{3}$ = “ “ “ 1 “ ($\frac{1}{30}$ of 30 “) “ 6%.
$\$29.03\frac{1}{3}$ = “ “ “ the required time at 6%.
$9.67\frac{7}{9}$ = “ “ “ “ “ “ “ 2%.
$\$19.35\frac{5}{9}$ = “ “ “ “ “ “ “ 4%.
$\therefore$ \$19.36 = the required interest.

Note. To find the amount, add the interest to the principal.

To find the interest at
2%, take $\frac{1}{3}$ of the interest at 6%.
3%, take $\frac{1}{2}$ of the interest at 6%.
4%, take $\frac{2}{3}$ of the interest at 6% (subtract $\frac{1}{3}$ of the interest at 6%).
$4\frac{1}{2}$%, take $\frac{3}{4}$ of the interest at 6% (subtract $\frac{1}{4}$ of the interest at 6%).
5%, take $\frac{5}{6}$ of the interest at 6% (subtract $\frac{1}{6}$ of the interest at 6%).
7%, take $\frac{7}{6}$ of the interest at 6% (add $\frac{1}{6}$ of the interest at 6%).
$7\frac{1}{2}$%, take $\frac{5}{4}$ of the interest at 6% (add $\frac{1}{4}$ of the interest at 6%).

2. Find the exact interest on \$1200 from Mar. 1 to Aug. 1 at $4\frac{1}{2}$%.

FORM.
\$1200
$.04\frac{1}{2}$
\$54.00
153
365) \$8262.00 (\$22.64

SOLUTION. The time from Mar. 1 to Aug. 1 = (30 + 30 + 31 + 30 + 31 + 1) da. = 153 da. = $\frac{153}{365}$ yr.

$.04\frac{1}{2}$ of \$1200, or \$54 = the int. for 1 yr.
153 da., or $\frac{153}{365}$ yr. = the time [§ 274].
$\therefore$ $\frac{153}{365}$ of \$54, or \$22.64 = the required interest.

Note. Since by the six per cent method and 60-day method interest is reckoned on the basis of 360 da. to the year, it is evident that neither of these methods can be employed in finding *exact* interest.

Find by the general method the interest of:

3. \$60 for 2 yr. at 6%.

4. \$100 for $1\frac{1}{2}$ yr. at 4%.

5. \$50 for $2\frac{3}{4}$ yr. at 4%.

6. \$375 for 3 yr. 90 da. at 5%.

7. \$378 from May 1 to Dec. 31 at 4%.

8. \$1000 from Jan. 1, 1900, to Sept. 21, 1902, at $4\frac{1}{2}$%.

9. \$50.75 from Oct. 15, 1899, to Jan. 1, 1901, at $4\frac{4}{5}$%.

10. \$225 from Apr. 1, 1899, to Mar. 1, 1900, at $4\frac{1}{2}$%.

11. \$278 from June 8, 1900, to Sept. 4, 1901, at 2% semiannually.

Sug. The time = 2 half-years 88 da. = $2\frac{88}{180}$ half-years.

12. \$75.85 from June 18, 1900, to Apr. 1, 1901, at $1\frac{1}{2}$% per quarter.

Sug. The time = 3 quarters 14 da. = $3\frac{14}{90}$ quarters.

13. \$200 from May 1 to Dec. 18 at $\frac{3}{4}$% per month.

Sug. The time = 7 mo. 17 da. = $7\frac{17}{30}$ mo.

Find by the six per cent method the interest of:

14. \$225 for 246 da. at 6%.

15. \$450 for 2 yr. 18 da. at 6%.

16. \$480 from Dec. 1, 1900, to July 1, 1901, at 6%.

17. \$2000 from May 1, 1899, to July 2, 1901, at 4%.

18. \$820 from Sept. 15, 1900, to Dec. 15, 1901, at $4\frac{1}{2}$%.

19. \$28.80 from Oct. 15, 1900, to Mar. 1, 1902, at $7\frac{1}{2}$%.

20. \$627 from May 1, 1898, to Sept. 1, 1900 at 3%.

21. \$528.60 from Mar. 31, 1900, to May 20, 1901, at 5%.

22. \$420 from Jan. 1, 1902, to Mar. 1, 1904, at 3%.

Sug. The time = 2 yr. 60 da.

23. \$680 from Jan. 2, 1904, to Mar. 2, 1905, at $4\frac{1}{8}$%.

Find by the 60-day method the interest and the amount of:

24. $240 for 60 da. at 6%.

25. $250 for 180 da. at 6%.

26. $82.50 for 90 da. at 6%.

27. $48.90 for 75 da. at 6%.

28. $325.60 for 2 yr. 105 da. at 6%.

29. $927.50 for 100 da. at 6%.

30. $278 for 240 da. at 4%.

31. $39.60 for 300 da. at $4\frac{1}{2}$%.

32. $82.94 for 126 da. at 3%.

33. $100 for 200 da. at 4%.

34. $98 from Oct. 1, 1900, to May 24, 1901, at 6%.

35. $207.85 from Dec. 12, 1901, to Sept. 5, 1902, at $3\frac{3}{4}$%.

Find the exact interest of:

36. $200 for 120 da. at 4%.

37. $678 from June 9 to Dec. 15 at 5%.

38. $828 from Sept. 15 to Dec. 31 at 6%.

39. $48.25 from Dec. 1, 1900, to June 15, 1901, at $4\frac{1}{2}$%.

40. $128 from Mar. 14, 1900, to Aug. 30, 1901, at $4\frac{3}{4}$%.

SUG. The time = 1 yr. 169 da., or $1\frac{169}{365}$ yr.

41. $718 from Dec. 18, 1899, to Oct. 18, 1903, at $4\frac{4}{5}$%.

EXERCISE 109.

1. What principal will amount to $595.20 in 2 yr. 200 da. at 4%?

FORM.

$$\begin{array}{l}
\$0.04 \\
\quad\ \ 2\frac{5}{9} \\
\hline
\$0.10\frac{2}{9} \\
\ \ 1.00 \\
\hline
\$1.10\frac{2}{9})\,\$595.20 \\
\quad\ \ 9 \qquad\qquad\ 9 \\
\hline
\$9.92)\,\$5356.80\,(\,540
\end{array}$$

SOLUTION.

$0.04 = the int. on $1 for 1 year.

2 yr. 200 da., or $2\frac{5}{9}$ yr. = the time.

∴ $2\frac{5}{9} \times$ $0.04, or $0 $10\frac{2}{9}$ = the int. on $1 for the time.

And, $$1.10\frac{2}{9}$ = the amt. of $1 for the time.

$595.20 = the entire amount.

∴ $595.20 : $$1.10\frac{2}{9}$, or 540 = the number of dollars in the principal.

∴ $540 = the principal.

280. The **Present Worth** of a debt due at some future time is such a sum as, put on interest, will amount to the debt when due.

281. The **True Discount** of a debt due at some future time is the difference between the debt and the present worth.

282. *To find the present worth of a debt is to find the principal.*

Thus, problem 1 might be stated: Find the *present worth* of $595.20 due 2 yr. 200 da. hence, at 4%. The debt would then be $595.20, the present worth $540, and the true discount $595.20 – $540, or $55.20.

What principal will produce:

2. $32 interest in 2 yr. 240 da. at 6%?

SUG. Find the int. on $1 for 2 yr. 240 da. at 6%, as in example 1, and measure the given interest by it.

3. $11.60 interest from Feb. 1, 1900, to July 1, 1902, at 4⅘%?

4. $24.30 interest from Dec. 20, 1901, to Mar. 20, 1904, at 3%?

5. $18.60 interest from Sept. 20, 1900, to Dec. 1, 1901, at 4½%?

6. $20 exact interest from May 1 to Dec. 15 at 4%?

What principal will amount to:

7. $695.50 from Feb. 20, 1900, to June 20, 1902, at 3%?

8. $1000 from June 1, 1899, to Sept. 1, 1901, at 6%?

9. $368.64 from Oct. 20, to May 27 at 4%, exact interest?

10. $1000 from May 1 to Dec. 2 at 6%, exact interest?

Find the present worth and true discount of:

11. $262.40 due 2 yr. 120 da. hence, at 4%.

12. $481.50 due 1 yr. 144 da. hence, at 5%.

13. $1170.50 at 6% on Sept. 15, 1900, if it is due July 15, 1902.

14. $2000 at 6% on Apr. 1, 1901, if it is due Sept. 1, 1902.

EXERCISE 110.

1. At what rate will $200 earn $18 in 2 yr. 90 da.?

FORM.

$200
.01
$2.00
2¼
$4.50)$18.00(4

SOLUTION.

.01 of $200, or $2 = the int. for 1 yr. at 1%.
2 yr. 90 da., or 2¼ yr. = the time.
∴ 2¼ × $2, or $4.50 = the int. for the time at 1%.
$18 = the given int.
∴ $18 : $4.50, or 4 = the required per cent.
That is, 4% = the rate.

At what rate will:

2. $320 earn $17.28 in 1 yr. 72 da?

3. $1000 earn $138 in 2 yr. 315 da?

4. $400 earn $44.60 from Sept. 1, 1900, to July 7, 1902?

5. 456 earn $4.56 from July 1, to Sept. 11?

6. $278 earn $19.46 from May 1, 1900, to Sept. 22, 1901?

7. $100 amount to $104.60 from June 20, 1901, to Aug 13, 1902?

SUG. $104.60 − $100, or $4.60 = the interest.

8. $425 amount to $446.25 from Dec 1, 1901, to Mar. 1, 1902?

9. $400 amount to $416.80 from Jan. 8 to Nov. 19?

10. $500 amount to $508 at exact interest, from Sept. 1 to Jan. 25?

11. $240 amount to $244.32 at exact interest, from Feb. 15 to Sept. 22?

12. Any sum double itself in 20 yr.?

SOLUTION. 200% of the prin. = the required amt.
100% " " " = " prin.
∴ 100% " " " = " int.
20% " " " = " " for 20 yr. at 1%.
∴ 100% of the prin. : 20% of the prin., or 5 = the required per cent
That is, 5% = the rate.

13. Any sum double itself in 15 yr?

14. Any sum earn twice itself in 50 yr.?

15. Any sum amount to 2½ times itself in 25 yr.?

EXERCISE III.

1. In what time will $400 amount to $438.40 at 4%?

FORM.

$$\begin{array}{r|l} \$400 & \$438.40 \\ .04 & 400.00 \\ \hline \$16.00\,) & \$38.400\,(2.4 \end{array}$$

SOLUTION.

.04 of $400, or $16 = the int. for 1 yr.

$438.40 − $400, or $38.40 = the entire interest.

∴ $38.40 : $16, or 2.4 = the number of years.

.4 yr. = .4 of 360 da., or 144 da.

∴ 2 yr. 144 da. = the required time.

In what time will:

2. $500 earn $72 interest at 4½%?

3. $1000 earn $138 interest at 4⅘%?

4. $200 amount to $208 at 4%?

5. $425 amount to 446.25 at 4%?

6. $250 amount to $300 at 6%?

7. $200 amount to $300 at 4½%?

On what date will:

8. $320 have amounted to $337.28, if loaned Mar. 1, 1906 at 4½%?

9. $400 have amounted to $416.80, if loaned Jan. 2 at 4⅘%?

10. $1000 have amounted to $1060, if loaned May 1, 1901, at 4%?

11. $500 have earned $8, if loaned Mar. 1 at 4%, exact interest?

12. In what time will any sum double itself at 4%?

SOLUTION. 200% of the prin. = the required amt.
100% " " " = the prin.
∴ 100% " " " = " int.
4% " " " = " " for 1 yr. at 4%.
∴ 100% of the prin. : 4% of the prin., or 25 = the number of years.
∴ 25 yr. = the required time.

13. In what time will any sum double itself at 3½%?

14. In what time will any sum earn 1½ times itself at 4½%?

15. In what time will any sum amount to 1¾ times itself at 6%?

Stocks and Bonds.

283. A **Corporation** is a body doing business as a single person under a general law or a special charter.

284. The capital of a corporation is called **Stock**.

285. The stock of a corporation is divided into a number of equal parts called **Shares.**

286. The face or original value of a share is its **Par Value.**

Note. The par value of a share of stock is usually $100, and it will be so understood in this book [See example 3, Ex. 112] unless a different par value is expressed [See example 1, Ex. 112].

287. A **Certificate of Stock** is a document specifying the number of shares to which the holder is entitled, and the par value of each.

288. The price at which stock sells in the market is its **Market Value.**

289. The market price of stock is usually quoted at a certain per cent of its par value; but in Philadelphia quotations the selling price of a share is given.

Thus, on a certain day Pennsylvania R. R. stock, par $50, was quoted in the New York market at 148½, which means that it was sold on that day at 148½ % of $50, or $74¼ per share. The same stock selling at the same price was quoted in the Philadelphia market at 74¼, meaning that it sold at $74¼ per share.

290. A **Dividend** is that part of the profits of a corporation which is divided among the stockholders. It is usually a certain per cent of the par value of the stock.

Thus, if a 5 % dividend is declared on stock, par value $50, the owners of the stock are paid 5% of $50, or $2.50 on each share owned.

291. A **Bond** is a written obligation of a corporation or a government to pay a specified sum of money at a certain time, together with interest at regular intervals at a fixed rate per cent per annum.

Bonds may sell above or below their face, or par, value and are usually quoted in the market at a certain per cent of their face value.

Bonds are usually named from the rate of interest they pay and the year in which they are payable.

Thus, U. S. 4s, 1907 means U. S. Government bonds bearing 4% interest, and payable in 1907.

292. **Brokerage** is the commission charged by a stockbroker for buying or selling stocks or bonds for another person. It is reckoned on the par value, and the usual rate is ⅛% or ¼%.

EXERCISE 112.

1. Find the cost of 4 shares of P. R. R. stock ($50) at $71½ a share, brokerage ¼%.

Solution. $50 = the par value of 1 share.
¼% of the par value = the brokerage.
∴ ¼% of $50, or $0.125 = the brokerage on 1 share.
$71.50 = the market value of 1 share.
∴ $71.50 + $0.125, or $71.625 = the entire cost of 1 share.
And, 4 × $71.625, or $286.50 = the entire cost of 4 shares.

2. A sold 20 shares of railroad stock ($50) at 98¾%; how much did he realize, brokerage ¼%?

Solution. $50 = the par value of 1 share.
¼% of the par value = the brokerage.
∴ ¼% of $50, or $0.125 = the brokerage on 1 share.
98¾% of the par value = the market value.
∴ 98¾% of $50, or $49⅜ = the market value of 1 share.
And, $49.375 − $0.125, or $49.25 = the net proceeds of 1 share.
∴ 20 × $49.25, or $985 = the net proceeds of 20 shares.
That is, $985 = the sum realized.

3. How many shares of American Cotton Oil stock, selling at 30¼%, can be bought for $1518.75, brokerage ⅛%, and what is its par value?

Solution. $100 = the par value of 1 share.
30¼% of the par value = the market value.
∴ $30.25 = the market value of 1 share.
$0.125 = the brokerage on 1 share.
∴ $30.375 = the entire cost of 1 share.
$1518.75 = the entire cost of the stock.
∴ $1518.75 : $30.375, or 50 = the number of shares.
And, 50 × $100, or $5000 = the par value.

4. Find the yearly income on 10 shares of stock ($50) paying a semi-annual dividend of 3½%.

Solution. $50 = the par value of 1 share.
3½% of the par value = the semi-annual dividend.
∴ 3½% of $50, or $1.75 = the semi-annual dividend on 1 share.
And, 2 × $1.75, or $3.50 = the annual dividend on 1 share.
Whence, 10 × $3.50, or $35 = the annual dividend on 10 shares.

5. What rate of interest, to the nearest hundredth per cent, does an investor receive who buys 4% stock, market value $110½ per share, brokerage ¼%?

Solution. $100 = the par value of 1 share.
4% of the par value = the dividend.
∴ 4% of $100, or $4 = the dividend on 1 share.
$110.50 = the market value of 1 share.
¼% of $100, or $0.25 = the brokerage on 1 share.
∴ $110.75 = the entire cost of 1 share.
.01 × $110.75, or $1.1075 = the income, if the rate is 1%.
∴ $4 : $1.1075, or 3.61 = the required per cent.
That is, 3.61% = the required rate of interest.

6. Find the cost of 10 shares of American Refining Co. stock at 136⅜%, brokerage ⅛%.

7. Find the cost of 15 shares of Delaware & Hudson Canal stock at 158½%, brokerage ⅛%.

8. Find the cost of 25 shares of Delaware, Lackawanna & Western R. R. stock ($50) at 197%, brokerage ⅛%.

9. Find the cost of 20 shares of Lehigh Valley R. R. stock ($50) at $34⅝ per share, brokerage ¼%.

10. Find the cost of 4 U. S. 4% bo

11. Find the cost of 3 $50 U. S. 113.

12. Find the proceeds from ount and the compound interest
more & Ohio R. R. stock at

13. Find the proceeds the compound amount of $1 for 3 yr. at 3%.
Southern R. R. stock [See Table, p. 234].

14. Find the pr or $546.36 = the compound amount of $500.
tock ($50) at $6 00, or $46.36 = the compound interest.

15. Find the ompound amount and the compound interest of
Co. stock ($50 mo. at 6%, payable semi-annually.

16. Find mo. = 3 semi-annual periods. The semi-annual rate =
at 111¾%, br

17. How Table the compound amount of $1 for 3 periods at 3%.
for $1126. compound amount and the compound interest of:

18. H for 4 yr. at 6%. 5. $375 for 3 yr. at 3½%.
bought fo for 3 yr. at 4½%. 6. $2000 for 5 yr. at 2½%.

19. F 0 for 2 yr. at 5%, payable semi-annually.
be boug 00 for 2½ yr. at 6%, payable semi-annually.
surplus 00 for 1 yr. at 5%, payable quarterly.

20. 1 yr. = 4 quarterly periods. The quarterly rate is 1¼%.
$10,00 500 for 9 mo. at 6%, payable quarterly.

21. 1000 for 1 yr. 3 mo. at 8%, payable quarterly.
be sold $3000 for 4 yr. 6 mo. at 5%, payable semi-annually.

22. $840 for 4 yr. 6 mo. at 5%, payable semi-annually.
broker 2000 for 4 yr. 6 mo. at 4%, payable quarterly.

23. the principal that will amount to:
⅛%, to
and wh 11098.45 in 3 yr. 6 mo. at 3%, payable semi-annually.

24. 3 yr. 6 mo. = 7 semi-annual periods. The semi-annual rate =

25. 845 = the compound amt. of $1 for 7 periods at 1½%.

26. 098.45 : $1.109845 = the number of dollars in the principal.

27. $4020.288 in 6 yr. at 5%.
paying $7593.196 in 11 yr. at 6%.

28. $29717.15 in 3 yr. 6 mo. at 5%, payable semi-annually.
North $48406683.78 in 4 yr. 9 mo. at 3%, payable quarterly

CHAPTER VI.

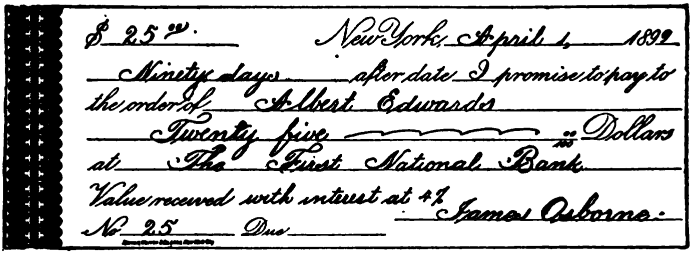
NEGOTIABLE PAPERS.

Promissory Notes.

295. A **Promissory Note** is a written promise to pay a specified sum of money to a certain party at a certain time.

FORM 1.

INDIVIDUAL NOTE.



$\$25^{00}$ New York, April 1, 1892

Ninety days after date I promise to pay to the order of Albert Edwards

Twenty five $\frac{00}{100}$ Dollars

at The First National Bank

Value received with interest at 4% James Osborne.

No 25 Due

Thus, the above is a form of promissory note.

296. The **Maker** of a promissory note is the party who signs it and thereby promises to pay. The **Payee** is the party whose name is written in the body of the note and to whom the promise to pay is made.

297. The **Face** is the amount named in the note.

Thus, in the above note, *James Osborne* is the *maker*, *Albert Edwards* the *payee*, and *twenty-five dollars* the *face*.

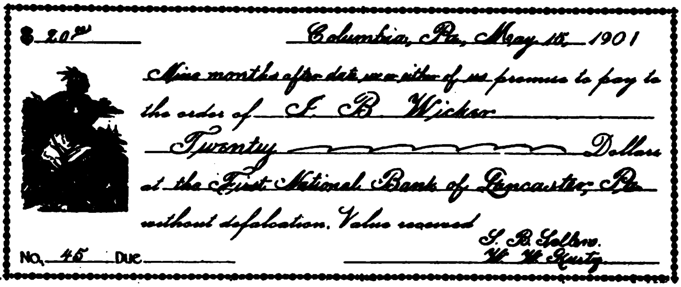
298. An **Individual Note** is a note signed by but one person.

Thus, Form 1 is an individual note.

299. A **Joint and Several Note** is a note signed by two or more persons, each making himself responsible for the payment thereof.

FORM 2.

JOINT AND SEVERAL NOTE.



$ 20 00 Columbia, Pa., May 15 1901

Nine months after date we or either of us promise to pay to the order of J. B. Wicker

Twenty Dollars

at the First National Bank of Lancaster, Pa.

without defalcation. Value received

S. B. Selbro.

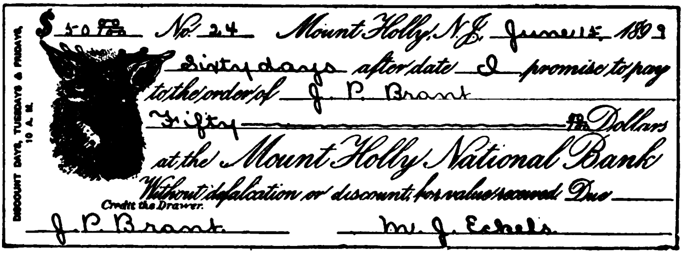
No. 45 Due W. W. Kuntz

If a note is written " I promise to pay " or " We or either of us promise to pay," and signed by two or more persons, each is responsible for the payment thereof: but if it is written " We jointly promise to pay " or " We promise to pay," and signed by two or more persons, they are *jointly* responsible for the payment thereof.

300. An **Accommodation Note** is a note used in borrowing money from a bank.

FORM 3.

ACCOMMODATION NOTE.



DISCOUNT DAYS, TUESDAYS & FRIDAYS, 10 A. M.

$ 50 00 No. 24 Mount Holly, N. J. June 15 1899

Sixty days after date I promise to pay

to the order of J. P. Brant

Fifty 00 Dollars

at the Mount Holly National Bank

Without defalcation or discount, for value received. Due

Credit the Drawer.

J. P. Brant M. J. Eckels

Banks usually require from a borrower, besides his own name, the name of some other responsible person as security. The note is made payable to the security, who writes his name across the back of it, and also under the phrase "Credit the Drawer."

Thus, if M. J. Eckels desires to borrow fifty dollars from the Mount Holly National Bank and have J. P. Brant for his security, they will write the note as in Form 3, and J. P. Brant will sign his name on the back. M. J. Eckels may then present the note at bank and receive the face less the interest charged by the bank.

301. *IMPORTANT FACTS.*

1. *A note may be made payable at a certain place.* [See Form 1.]

Note 1. If a note is made payable at a particular place, as at a bank, it should be presented at that place for payment when due, during business hours.

Note 2. When no place of payment is designated in a note, it should be presented for payment at the maker's place of business during business hours or at his residence.

2. *A note may be written so as to include the payment of interest.* [See Form 1, p. 236.]

Note 1. When no rate is mentioned in an interest-bearing note, it bears interest at the legal rate of the state in which it is payable.

Note 2. When a non-interest-bearing note is not paid when due, it will bear interest at the legal rate from the day it is due to the day it is paid.

Note 3. Some interest-bearing notes have a condition stated in them that, if not paid when due, they will thereafter bear a higher specified rate than the one named in the main note; when no such condition is stated, they will continue to bear interest at the original rate until paid.

3. A note may be made payable on demand, a certain number of days after date, a certain number of months after date, or upon a certain date.

Note 1. The day on which a note becomes due is called the **Day of Maturity.**

Note 2. A note payable on demand is due whenever the holder demands payment.

Note 3. A note payable a certain number of days after date is due at the expiration of that number of days, not counting the date.

Thus, the note in Form 1, dated April 1, 1899, is due June 30.

Note 4. A note payable a certain number of months after date is due so many calendar months after date.

Thus, the note in Form 2, dated May 15, 1901, is payable Feb. 15, 1902.

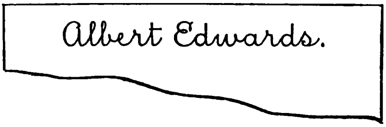
Note 5. In some states 3 days, called **Days of Grace,** are allowed for the payment of a note after the expiration of the time specified in the note. In such case a note dated May 15, 1901, and payable 9 months after date, would fall due Feb. 18, 1902.

4. A note may be made payable to the order of the *payee*, to the *payee or bearer*, or to the *payee only.*

Note 1. If a note is made payable to the order of the payee [See Form 1], it may be transferred by him to another. This is done by the payee's writing his own name across the back of the note, which process is called **Indorsing** the note. The person receiving the note then becomes the holder, and may again transfer it by indorsement to another person and so on.

Note 2. An indorsement containing the name of the indorser only, is called an **Indorsement in Blank.**

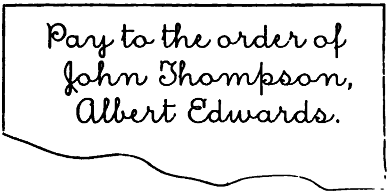
INDORSEMENT IN BLANK.



Albert Edwards.

Note 3. An indorsement directing payment to be made to the order of a certain person, is an **Indorsement in Full**, or a **Special Indorsement.**

SPECIAL INDORSEMENT.



Pay to the order of
John Thompson,
Albert Edwards.

Note 4. If a note is made payable to bearer, it is transferable without indorsement.

Note 5. If a note is made payable to the payee only, it is not transferable.

Note 6. Any note transferable to another is said to be **Negotiable.**

EXERCISE 114.

1. Name the maker and the payee in each of the notes on pp. 236 and 237.

2. Who pays the notes pp. 236 and 237 when due?

3. When is the note p. 236 due?

4. When is the note p. 237, Form 2, due? How many days does it run?

5. When is the note p. 237, Form 3, due?

6. Write a promissory note for $10 with yourself as maker, and L. B. Wells as payee, dating the note June 3, 1901, and making it payable in 60 days.

7. If L. B. Wells wishes to transfer the note in example 6 to R. M. Sampson, in what two ways may he write the indorsement?

8. Suppose you buy a horse from S. L. Jamison for $90, payable in 9 months; write the joint and several note that you could give him in payment, if R. M. White signs the note with you. Date the note Sept. 20, 1901, and make it payable at the nearest bank.

9. When is the note in example 8 due? If S. L. Jamison has it cashed at bank, how will he indorse it? Where will you pay the note when due? When is it worth $90?

10. Jan. 16, 1901, S. R. Miller accommodates you by becoming your security on a 90-day note for $50 to enable you to borrow from the nearest bank; write the note.

11.

No. 95. LANCASTER, PA., Nov. 1, 1901.

Ninety days after date I promise to pay to the order of WILLIAM P. DAVIS, at

The Farmers' National Bank of Lancaster,

Two hundred twenty .... Dollars.

Without defalcation, value received, with interest at 4½%.

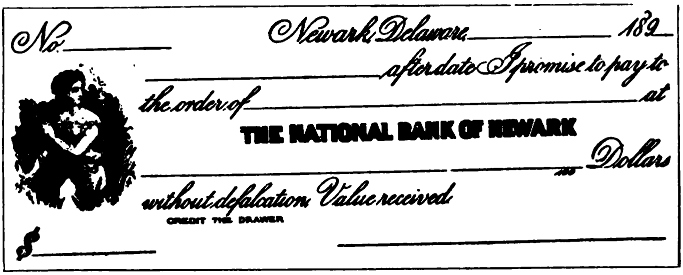
L. J. DRIGGS.

Name the parties to the above note. When is it due? How much is due at maturity? Who pays the note when due? Who receives the note when paid?

12. How much interest is due at maturity on a note for $240 dated Chester, Pa., April 1, 1901, payable in 6 months, if it bears interest at 4½%?

13. How much is due at maturity on a note for $200 dated Dover, Del., May 1, 1900, payable in 90 days with interest?

14. Fill out the following note:



No. Newark, Delaware 189
after date I promise to pay to
the order of at
THE NATIONAL BANK OF NEWARK
Dollars
without defalcation, Value received
CREDIT THE DRAWER
$

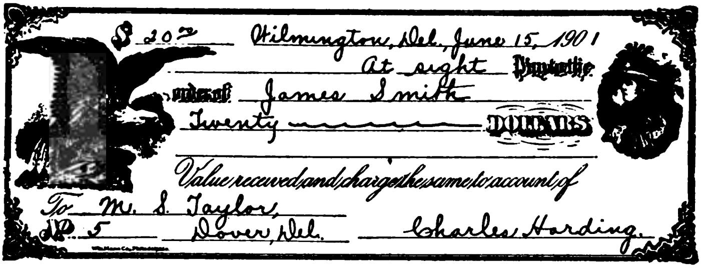
After filling out the above note, name the parties. Tell when the note is due. What indorsements are necessary in order that the maker may have the note cashed at the National Bank of Newark?

Drafts.

302. A **Draft,** or **Bill of Exchange,** is a written order whereby one party directs another to pay a specified sum of money to the order of a certain person at a certain time.

FORM 4.

SIGHT DRAFT.



$ 20 00 Wilmington, Del., June 15, 1901

At sight Pay to the

order of James Smith

Twenty Dollars

Value received and charge the same to account of

To M. S. Taylor,

No. 5 Dover, Del.

Charles Harding.

Thus, the above is a form of draft.

303. The **Drawer** of a draft is the party who signs it and thereby orders its payment. The **Drawee** is the party who is ordered to pay. The **Payee** is the party to whom it is made payable.

304. The **Face** is the amount named in the draft.

Thus, in the above draft, *Charles Harding* is the *drawer, M. S. Taylor* the *drawee*, and *James Smith* the *payee. Twenty dollars* is the *face.*

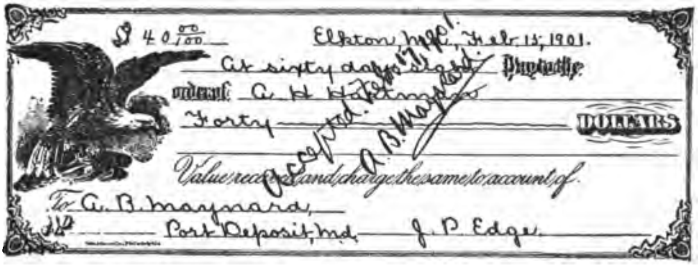
305. A **Sight Draft** is a draft that is payable whenever presented to the drawee for payment.

The purpose of the draft in Form 4 may be explained thus: Charles Harding, desiring to pay James Smith twenty dollars, orders M. S. Taylor, who owes him at least that amount, to pay the same to James Smith, or his order, at sight, and sends the draft to James Smith who may collect it from M. S. Taylor.

306. A **Time Draft** is a draft payable at a certain time after sight or after date.

FORM 5.

TIME DRAFT.



\$ 40 00/100 Elkton, Md., Feb. 15, 1901.

At sixty days sight Pay to the order of G. H. Hastings

Forty Dollars

Value received and charge the same to account of

To G. B. Maynard,

No. Port Deposit, Md. J. P. Edge.

Accepted Feb. 17, 1901. G. B. Maynard.

307. *IMPORTANT FACTS.*

1. All time drafts must be presented to the drawee for *acceptance.*

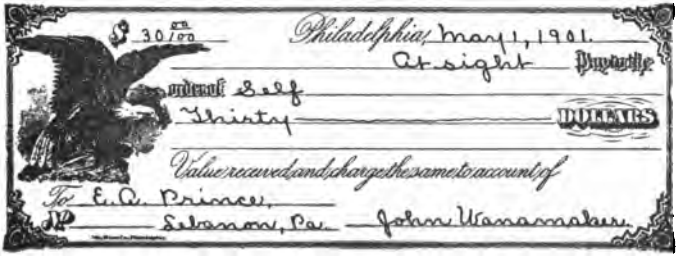
2. To **Accept** a draft, the drawee writes across the face the word "*accepted*" or "*seen,*" together with the date of acceptance and his name, unless the draft is payable "*after date,*" in which case the date of acceptance may be omitted.

3. A time draft made payable a certain number of days after sight becomes due that number of days after *acceptance;* but when made payable a certain time after date, it becomes payable at the expiration of that time after date.

Remark. In states where "days of grace" are allowed all time drafts become due three days later.

FORM 6.

308. TWO-PARTY DRAFT.



\$ 30 00/100 Philadelphia, May 1, 1901.

At sight Pay to the order of Self

Thirty Dollars

Value received and charge the same to account of

To E. A. Prince,

No. Lebanon, Pa. John Wanamaker.

The purpose of the draft in Form 6 may be explained thus: John Wanamaker wishes to collect a debt of $30 from E. A. Prince, of Lebanon, Pa. He makes himself the payee, then indorses the draft, and deposits it in a Philadelphia bank for collection, which bank forwards it to a bank in Lebanon, and this bank collects it from E. A. Prince.

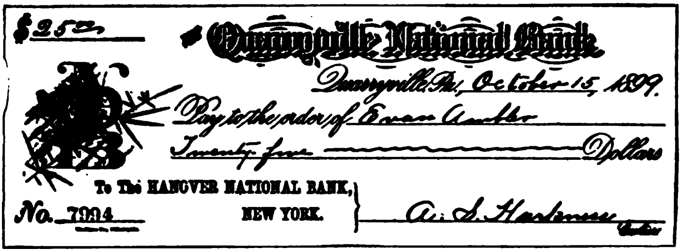
Note 1. John Wanamaker might also collect the above sum by making the bank in which he is a depositor the payee.

Note 2. Two-party drafts may also be time drafts.

309. A Bank Draft is a draft by one bank upon another in which it has deposits. It is used mainly to transmit money from one place to another.

FORM 7.

BANK DRAFT.



$25⁰⁰ Quarryville National Bank

Quarryville, Pa., October 15, 1899.

Pay to the order of Evan Ambler

Twenty-five Dollars

To The HANOVER NATIONAL BANK,
NEW YORK.

No. 7994

A. S. Harkness
Cashier

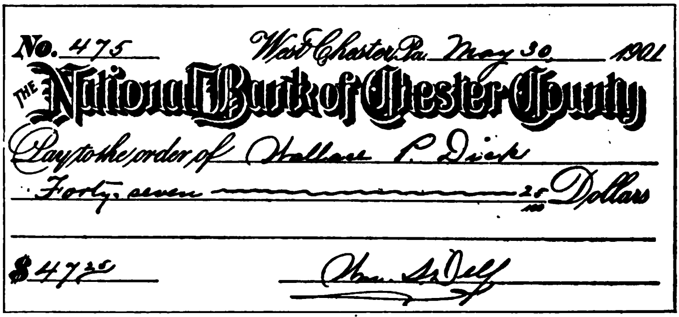
The purpose of the draft in Form 7 may be explained thus: Evan Ambler of Quarryville, Pa., desiring to send $25 to M. R. Walton of Erie, Pa., purchases the draft of the Quarryville National Bank of Quarryville, Pa., indorses it "Pay to the order of M. R. Walton, Evan Ambler," and sends it to M. R. Walton, who then indorses it and presents it for payment at a bank in Erie, which bank cashes it and collects it from the Hanover National Bank.

Note. All drafts made payable to the order of the payee are transferrable by indorsement, and hence negotiable.

310. A check is a sight draft on a bank, made by a depositor.

FORM 8.

A CHECK.



No. 475 West Chester Pa. May 30 1901

The National Bank of Chester County

Pay to the order of Wallace P. Dick

Forty-seven 25/100 Dollars

$47.25

Remark. The holder of a check should present it for payment on the day he receives it or on the first business day following.

EXERCISE 115.

1. In the sight draft, p. 242, who directs payment to be made? Who is ordered to pay? When is this draft due? Who is expected to pay it when due?

2. Suppose A owes you $50 and you owe B a like sum; write the sight draft that could be used both to collect from A and to pay B.

3. In the time draft, p. 243, name the drawer, the drawee, the payee. When is this draft due? Who pays it when due? Who receives the draft when paid?

4. Write a 30-day sight draft for $10, making yourself drawer, L. J. Harman drawee, and L. M. Winters payee. Date the draft Sept. 5, 1901, and have it accepted Sept. 5, 1901. When is it due? Who pays it when due? When is it due in states where "grace" is allowed?

5. Suppose you owe O. B. Manley $75, and S. B. Brooks owes you at least that sum; write the draft you could draw on S. B. Brooks in favor of O. B. Manley for the sum you owe the latter. Date the draft Dec. 15, 1901, and make it payable 60 days after date. Write the acceptance and find when the draft is due.

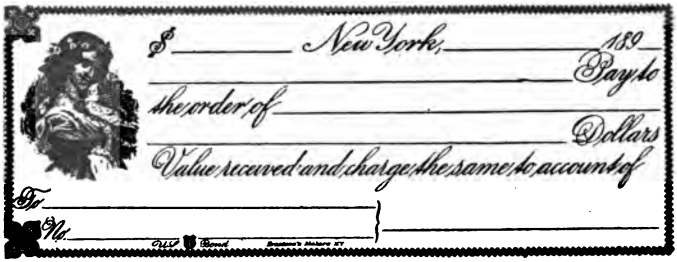
6. Name the drawer, the drawee, and the payee, in the draft in Form 6, p. 243. When is this draft due? Who pays it when due? Who receives the draft when paid?

7. Write the sight draft that you could draw on S. R. Roberts to collect a debt of $25. Explain the steps you would take after writing the draft to collect it.

8. Name the parties to the draft in Form 7, p. 244. Which bank mentioned in this draft is supposed to have deposits in the other? Write the indorsements as they would appear on the back of this draft when M. R. Walton has it cashed.

9. William Price of Lancaster, Pa., wishing to remit $50 to C. J. Allen of Altoona, Pa., purchases a draft for that amount from the Farmers' National Bank of Lancaster on Girard National Bank of Philadelphia. Write the draft and explain how it can be used in payment of the debt.

10. Fill out the following draft, making yourself drawer, A. R. Manning drawee, and S. J. Andrews payee. Make it payable 60 days after date, and find when it is due.



$________ New York, ____________ 189__

____________________________ Pay to

the order of ________________________

____________________________ Dollars

Value received and charge the same to account of

To ____________________

No. ____________________ ____________________

11. In the check on p. 245, name the drawer, the drawee, the payee. When is this check payable? Which of the parties named must be a depositor in the National Bank of Chester County? Which party named receives the sum mentioned in the face? Must he receive it direct from the bank named in the check? Which party will finally hold the check? Must it be indorsed before it is cashed? Who must first indorse it?

12. Suppose you have money deposited in the nearest bank; write the check which you would draw to pay Frank Hartman $40.

13. Suppose that James Manley draws a check in your favor for $20 on the Commonwealth National Bank of Harrisburg, Pa., and you have Wm. Brown cash the check for you. Write the check and the indorsement, and tell what Wm. Brown must do before he can have the check cashed.

14. You cashed for James Hammond a check drawn payable to his order, and Henry Eshelman cashed the check for you. Draw the back of the check and write thereon the proper indorsements.

311. The table on p. 248 gives for each state and territory of the United States the legal rate, the maximum contract rate, the statute of limitation, the time for payment of a note or draft falling due on Sunday or on a legal holiday, and whether or not grace is allowed.

The following notation is used in the table:

P represents previous business day; *S*, the succeeding business day; and *G*, three days of grace.

Note 1. By the statute of limitation is here meant the period of time from the date of the last acknowledgment of the debt, within which suit must be brought to collect a debt overdue.

Note 2. By maximum contract rate is meant the highest rate of interest that the law allows to be contracted for and collected.

STATES AND TERRITORIES.	INTEREST LAWS.				STATUTES OF LIMITATION.		
	Legal Rate.	Maximum Contract Rate.	Grace.	Payable.	Judgments.	Notes.	Open Accounts.
					Years.	*Years.*	*Years.*
Alabama	8	8	G	S	20	6	3
Arkansas	6	10	G	P	10	5	3
Arizona	6	Any		S	5	4	3
California	7	Any		S	5	4	2
Colorado	8	Any		S	20	6	6
Connecticut	6	6		S			6
Delaware	6	6		P*	10	6	3
Dist. of Columbia	6	10		S	12	3	3
Florida	8	10		S	20	5	2
Georgia	7	8		P	7	6	4
Idaho	7	12		S	6	5	4
Illinois	5	7		S	20	10	5
Indiana	6	8	G	P	20	10	6
Iowa	6	8		S	20	10	5
Kansas	6	10	G	P	5	5	3
Kentucky	6	6	G	P	15	15	2
Louisiana	5	8		S	10	5	3
Maine	6	Any		S	20	6	6
Maryland	6	6		P	12	3	3
Massachusetts	6	Any		S	20	6	6
Michigan	5	7	G	S	6	6	6
Minnesota	7	10		P	10	6	6
Mississippi	6	10	G	P	7	6	3
Missouri	6	8	G	S	10	10	5
Montana	8	Any		S	10	8	5
Nebraska	7	10	G	S	5	5	4
Nevada	7	Any	G	P	6	4	4
New Hampshire	6	6		S	20	6	6
New Jersey	6	6		S	20	6	6
New Mexico	6	12	G	S	7	6	4
New York	6	6		S	20	6	6
North Carolina	6	6	G	S	10	3	3
North Dakota	7	12		S	10	6	6
Ohio	6	8		S	15	15	6
Oklahoma	7	12	G	S	5	5	3
Oregon	6	10		S	10	6	6
Pennsylvania	6	6		S	5	6	6
Rhode Island	6	Any		S	20	6	6
South Carolina	7	8	G	S	20	6	6
South Dakota	7	12	G	S	10	6	6
Tennessee	6	6		S	10	6	6
Texas	6	10	G	P	10	4	2
Utah	8	Any		S	8	6	4
Vermont	6	6		S	8	6	6
Virginia	6	6		S	20	5	2
Washington	6	12		S	6	6	3
West Virginia	6	6		S	10	10	8
Wisconsin	6	10		S	20	6	6
Wyoming	8	12	G	P	5	5	8

* Unless due on Sunday or Monday holiday, in New Castle co.

Bank Discount.

312. **Bank Discount** is the amount deducted by a bank for paying a note or draft before it is due. It is the simple interest on the face of the note or draft from the day of discount to the day of maturity.

Note 1. Some banks occasionally discount interest-bearing notes. The discount is then reckoned on the amount of the note at maturity instead of on the face.

Note 2. In Pennsylvania, Delaware, Maryland, Missouri, and the District of Columbia, *the day of discount* as well as *the day of maturity* is counted in reckoning the time of discount.

313. The **Proceeds** of a note or draft is the sum discounted less the discount.

EXERCISE 116.

1. A note dated May 15, 1901, payable in 9 months, was presented at the First National Bank of Lancaster, Jan. 2, 1902, for discount. Find the discount and the proceeds, if the face of the note was $20. [See Form, § 299].

SOLUTION. 9 months after May 15, 1901, or Feb. 15, 1902 = the date of maturity.
Jan. 2, 1902, is the date of discount.
∴ 30 da. + 15 da., or 45 da. = the time of discount [§ 312, Note 2].
6%, the legal rate in Penn., is the rate of discount.
$0.20 = the discount on $20 for 60 da.
∴ ¾ of $0.20, or $0.15 = the discount on $20 for 45 da.
And, $20 − $0.15, or $19.85 = the proceeds.

2. Find the proceeds of a note dated June 1, 1901, payable in 60 days, if the face of the note was $500, and it was discounted June 2, 1901, at a Pennsylvania bank.

3. Find the proceeds of a note dated March 10, 1901, payable in 60 days, if the face of the note was $600, and it was discounted on the day of issue at a New Jersey bank.

4. Find the proceeds of a note of $450, dated July 5, 1901, and payable in 9 months, if discounted at a Pennsylvania bank, Aug. 10, 1901.

5. Find the proceeds of a note of $200, dated May 2, 1901, and payable in 30 days, if discounted at a New York bank on the date of the note.

6. Find the proceeds of a note of $240, dated Sept. 15, 1901, and payable in 90 days, if discounted by a Baltimore merchant, Sept. 30, 1901, at 4%.

7. Find the proceeds of a note dated July 2, 1901, and payable Jan. 2, 1902, if the face of the note was $200, and it was discounted at a Delaware bank, Sept. 1, 1901.

8. Find the proceeds of a note of $240, dated Sept. 19, 1899, and payable 6 months after date with interest at 5%, if the note was discounted at a Pennsylvania bank, Jan. 2, 1900. [See § 312, Note 1].

9. I owed L. B. Landis $500 and gave him my note payable in 90 days with interest at 4½%. If he had the note discounted the same day at a Pennsylvania bank, what did he receive?

10. Jan. 5, 1901, I bought from Mr. Kirk a horse for $150, and gave him in payment my note payable in 6 months with interest at 6%. If he had this note discounted at a Pennsylvania bank on the day he received it, find the proceeds.

11. A New Jersey farmer bought of Mr. Souder some cattle on Sept. 12, 1899, amounting to $348. He gave a note for the sum, payable in 90 days at a New Jersey bank, with interest at 6%. If Mr. Souder had the note discounted on the day the cattle were purchased, at the bank at which it was payable, what did he receive for it?

12. What should be the face of a note payable 60 days after date, to yield a proceeds of $250, if discounted at 6% on the day of issue, in a state in which 3 days of grace are allowed?

SOLUTION. 63 da. = the term of discount.
$0.0105 = the discount on $1 for 63 da.
∴ $1 − $0.0105, or $0.9895 = the proceeds of $1 of the note.
$250 = the entire proceeds.
∴ $250 : $0.9895, or $252.65 = the number of dollars in the face.
∴ $252.65 = the face of the note.

13. What should be the face of a note dated June 1, 1901, and payable in 60 days, to yield $100 proceeds, if discounted at a Pennsylvania bank on the date of the note?

14. What should be the face of a 90 day note to pay a debt of $340, if discounted at a New Jersey bank on the date of the note?

15. What should be the face of a 3 months note dated Jan. 12, 1901, to yield a proceeds of $60 when discounted at a Pennsylvania bank on the date of the note?

16. What should be the face of a note dated Mar. 1, 1901, and payable at a New York bank Aug. 12, 1901, to yield a proceeds of $400, if discounted June 13 at the bank at which it was payable?

17. What was the face of a 90 da. note dated May 7, 1902, that yielded a proceeds of $595 when discounted at a Baltimore bank July 7, 1902?

18. A note dated Dover, Del., May 1, 1901, was discounted June 1, 1901, and yielded a proceeds of $240. What was the face of this note, if it was payable 90 days after date and was discounted at a Delaware bank?

19. Write a promissory note dated Sept. 1, 1901, payable in 60 days, with yourself as payee and William P. Roberts as maker. Properly indorse it for discount and find the proceeds, if discounted at a bank in your state on the date of the note.

Domestic Exchange.

314. Exchange is the process of making payments by drafts. Exchange between two places in the same country is **Domestic Exchange.**

315. Exchange is at a *premium* when sight drafts sell above their face value, at a *discount* when they sell below their face value, and at *par* when they sell for their face value.

Premiums and discounts on drafts are reckoned as a certain per cent of the face or as a certain sum on each $1000 of the face.

Thus, if a draft of $500 is sold at a premium of $\frac{1}{8}$%, each dollar of the face is sold for 1.00\frac{1}{8}$, and the draft for 500 × 1.00\frac{1}{8}$.

When exchange is at a premium, a time draft is worth its face, minus the bank discount for the time the draft has yet to run, plus the exchange premium.

When exchange is at a discount, a time draft is worth its face, minus the bank discount for the time the draft has yet to run, minus the exchange discount.

Thus, let it be required to find the cost of a $500 draft due in 60 days, exchange $\frac{1}{4}$% discount.

SOLUTION. $0.01 = the bank discount on $1 for 60 da.
∴ $1.00 – $0.01, or $0.99 = the cost of $1 of the face, if exchange were at par.
$0.0025 = the exchange discount on $1 of the face.
∴ $0.99 – $0.0025, or $0.9875 = the cost of $1 of the face.
And, 500 × $0.9875, or $493.75 = the cost of the draft.

EXERCISE 117.

1. Find the cost of a draft of $125 at 1$\frac{1}{4}$% premium.

SUG. 1$\frac{1}{4}$% of $125, or $1.56 = the premium.

2. Find the cost of a draft for $500 at $\frac{1}{8}$% discount.

SUG. $\frac{1}{8}$% of $500, or $0.625, or $0.63 = the discount.

3. What is the cost of a sight draft of $250 when exchange is $\frac{1}{2}$% premium?

4. What is the cost of a sight draft for $500 when exchange is $\frac{1}{4}$% discount?

5. What is the cost in Philadelphia, on the day of date, of a draft for $140 payable 60 days after date, if exchange is $\frac{3}{4}$% discount?

6. Find the proceeds of a draft for $360 payable 90 da. after sight, if discounted at a Cleveland bank on the day of acceptance, exchange being $\frac{5}{8}$% premium.

7. What is the cost in Baltimore on Oct. 1 of a draft for $160, payable 30 days after sight and accepted Sept. 15, exchange being $\frac{7}{8}$% discount?

8. Mr. Jones of West Chester, Pa., wishes to send his friend Mr. Smith a sight draft for $100; what must he pay for it at a West Chester bank, exchange being $1\frac{1}{4}$% premium?

9. Roberts & Co. of Minneapolis sent 60 barrels of flour at $3.25 a barrel to James Brothers of Philadelphia, and drew on them for the amount, payable at sight, and had the draft cashed at a Minneapolis Bank; how much did they receive for it, exchange being 1% premium?

10. Peterson Harris of Pittsburg, Pa., drew on Mitchell Morris of Chicago for $500, payable at sight, and had the draft cashed by a Pittsburg bank; how much did he receive for it, exchange being $\frac{1}{2}$% discount?

11. I sold a horse to a New York drover for $200 and accepted in payment a draft on a New York bank for $200, payable in 60 da., which I had cashed in a Philadelphia bank on the day of issue; how much did I receive for it, exchange being at par?

12. What is the value in New Jersey of a draft for $500, payable 60 days after sight, if discounted 10 days after it is accepted, exchange being $\frac{3}{8}$% discount?

13. What should be the face of a sight draft to pay a debt of $600, when exchange is $\frac{1}{2}$% premium?

SUG. $1 + $\frac{1}{2}$% of $1, or $1.005 = the value of $1 of the face.
∴ $600 : $1.005 = the number of dollars in the face of the draft.

14. What should be the face of a 60 da. draft on a Philadelphia merchant to pay a debt of $250 in Chicago, if drafts on Philadelphia are selling in Chicago at $\frac{1}{8}$% discount?

SUG. $1 — bank discount of $1 for 60 da. at 5% — ¼% of $1 = the cost of $1 of the face.

15. Mr. Harrity owes a debt of $600 in San Francisco; what should be the face of a sight draft on New York to pay the debt, if exchange on New York is 1¼% premium in San Francisco?

16. Mr. Williamson sent to Jamison & Jones of Baltimore a 60 da. draft on New York to pay a debt of $600, when exchange on New York was ¾ premium in Baltimore; what was the face of the draft?

Supplement.

FOREIGN EXCHANGE.

316. **Foreign Exchange** is exchange between two places in different countries.

317. The face of a draft is usually expressed in the money of the country in which the foreign party lives.

Thus, the face of a draft on a party in England is written in *pounds, shillings*, and *pence;* on a party in France, in *francs;* on a party in Germany, in *marks;* and on a party in Holland, in *guilders*, or *florins*.

318. The following quotations show how 3 days' and 60 days' drafts on foreign countries were selling in New York at a certain date:

Sterling, 3 da., 4.88½	meaning	£1 = $4.88½.
Sterling, 60 da., 4.85	"	£1 = $4.85.
Francs, 3 da., 5.14⅜	"	5.14⅜ francs = $1.
Francs, 60 da., 5.16⅞	"	5.16⅞ francs = $1.
Reichsmarks, 3 da., 95⅜	"	4 marks = $0.95⅜.
Reichsmarks, 60 da., 94¾	"	4 marks = $0.94¾.
Guilders, 3 da., 40½	"	1 guilder = $0.40½.
Guilders, 60 da., 40¼	"	1 guilder = $0.40¼.

Note. Drafts on foreign parties are often called **Bills of Exchange.** Sometimes they are made out in sets of two or three, so worded that when one is paid the others are void. [See "New Complete Arithmetic."]

EXERCISE 118.

1. What is the value in Philadelphia of a draft on London for £60, exchange 4.85?

SOLUTION. \$4.85 = the value of £1.
∴ 60 × \$4.85, or \$291 = the value of the draft.

2. What is the value in New York of a 60 da. draft for £240 on London, exchange 4.85¼?

3. What is the value in Baltimore of a 3 da. draft for £25 6s. on a Liverpool merchant, exchange 4.84¼?

4. What is the value in Charleston of a 60 da. draft for £28 10s. 6d. drawn on a London banking house, exchange 4.84¾?

5. What is the value in Chicago of a 3 da. draft for 3250 francs on a Paris bank, exchange 5.14½?

SUG. 5.14½ francs = the value of \$1.
∴ 3250 francs : 5.14½ francs = the required number of dollars.

6. What is the value in Pittsburg of a 60 da. draft for 350 francs on a Paris merchant, exchange 5.16¼?

7. What is the value in Trenton, N. J., of a 3 da. draft for 5760 francs on a merchant in Calais, exchange 5.15?

8. What is the value in Buffalo of a 60 da. draft on a Paris bank for 500 francs, exchange 5.16¾?

9. What is the value in Philadelphia of a 30 da. draft for 2340 marks on a Berlin bank, exchange 95⅝?

SUG. \$0.95⅝ = the value of 4 marks.
∴ ¼ of 2340 × \$0.95⅝ = the value of the draft.

10. What is the value in Toledo of a 30 da. draft for 4800 marks on a Berlin merchant, exchange 94½?

11. What is the value in San Francisco of a 3 da. draft on a German Mercantile Co. for 4780 marks, exchange 95?

12. What is the value in St. Paul of a 3 da. draft of 280 guilders on a bank in Amsterdam, exchange 40¾?

Sug. $0.40¾ = the value of 1 guilder.
∴ 280 × $0.40¾ = the value of the draft.

13. What is the value in Charleston of a 60 da. draft on a merchant in Amsterdam for 2740 guilders, exchange 40?

14. What is the value in Washington of a 3 da. draft on a merchant in Amsterdam for 780 guilders, exchange 40⅜?

15. What is the face of a 60 da. draft on a London bank, that is worth in the U. S. $942.50, exchange 4.86¼?

Sug. $4.86¼ = the value of £1.
∴ $942.50 : $4.86¼ = the number of pounds in the draft.

16. What is the face of a 3 da. draft on a London merchant, bought for $731.25, exchange 4.87½?

17. What is the face of a 3 da. draft on a bank in Paris, bought for $400, exchange 5.14⅜?

Sug. 400 × 5.14⅜ fr. = the face of the draft. Why?

18. What is the face of a 60 da. draft on a merchant in Paris, bought for $250, exchange 5.13⅞?

19. What is the face of a draft on a bank in Berlin, bought for $76.30, exchange 95⅜?

Sug. $76.30 : ¼ of $0.95⅜ = the number of marks. Why?

20. What is the face of a 60 da. draft on an agent in Berlin, sold for $113.40, exchange 94½?

21. What is the face of a draft on an agent in Amsterdam, sold for $112.70, exchange 40¼?

Sug. $112.70 : $0.40¼ = the number of guilders. Why?

22. What is the face of a draft on a bank in Amsterdam, bought for $203.75, exchange 40¾?

23. I bought a bill of goods from a Paris merchant amounting to 5460 francs. I bought from a Philadelphia banker a draft on a bank in Paris for the amount, and forwarded it to the merchant; what did I have to pay for it, exchange 5.14½?

24. I have $1000 belonging to my brother, who lives in Germany, which I wish to remit to him by draft on Berlin; what must be the face of the draft, exchange $95\frac{3}{8}$?

PARTIAL PAYMENTS.

319. When a note is paid in part, such payment is called a **Partial Payment**, and it is receipted on the back of the note. This acknowledgment is called **Indorsement**.

320. The following are the rules most generally used for determining the balance due on a note with indorsements.

THE MERCHANTS' RULE.

The Merchants' Rule is used by business men in finding balances due on notes and accounts on which partial payments have been made.

RULE: *If the time does not exceed one year, find the amount of the principal from the time it began to draw interest to the day of settlement; find the amount of each payment from the day it was made to the day of settlement; subtract the sum of the amounts of the payments from the amount of the principal.*

If the time exceeds a year, find the balance due at the end of each year by the rule above stated, and treat it as a new principal.

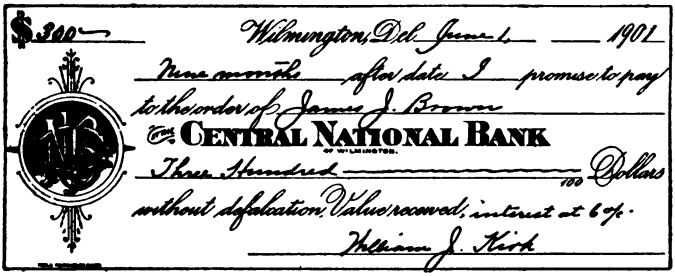
UNITED STATES RULE.

The rule adopted by the Supreme Court of the United States, known as the "United States Rule of Partial Payments," has been adopted as the legal rule by nearly all the states.

RULE: *Find the amount of the given principal to the time when the payment, or the sum of the payments, is* *equal to or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before.*

Note. New Hampshire, Vermont, and Connecticut have their own rules, which, being limited to these states, will not be considered here.

Find the balance due Dec. 15, 1902, on the following note:



$300— Wilmington, Del. June 1 1901

Nine months after date I promise to pay

to the order of James J. Brown

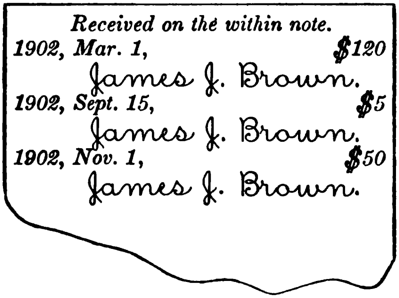
at the CENTRAL NATIONAL BANK OF WILMINGTON.

Three Hundred —— $\frac{}{100}$ Dollars

without defalcation. Value received, interest at 6%.

William J. Kirk

INDORSEMENTS.



Received on the within note.

1902, Mar. 1, *$120*

James J. Brown.

1902, Sept. 15, *$5*

James J. Brown.

1902, Nov. 1, *$50*

James J. Brown.

SOLUTION BY MERCHANTS' RULE.

Principal, June 1, 1901,	$300.00
Interest on principal for 1 year,	18.00
Amount, June 1, 1902,	$318.00

Amount brought forward		$318.00
First payment, Mar. 1, 1902,	$120.00	
Interest on first payment to June 1, 1902 (92 da.),	1.84	121.84
New principal, June 1, 1902,		$196.16
Interest on principal to Dec. 15, 1902 (197 da.),		6.44
Amount, Dec. 15, 1902,		$202.60
Second payment, Sept. 15, 1902,	$5.00	
Interest on second payment to Dec. 15, 1902 (91 da.),	.08	
Third payment, Nov. 1, 1902,	50.00	
Interest on third payment to Dec. 15, 1902 (44 da.),	.37	55.45
Balance due, Dec. 15, 1902,		$147.15

Solution by United States Rule.

Principal, June 1, 1901,	$300.00
Interest on prin. from June 1, 1901, to Mar. 1, 1902 (273 da.),	13.65
Amount, March 1, 1902,	$313.65
First payment,	120.00
New principal, Mar. 1, 1902,	$193.65
Interest on principal from Mar. 1, 1902, to Sept. 15, 1902 (198 da.), $6.39.	
(Interest due Sept. 15, 1902, exceeds payment then made and no settlement is made.)	
Int. on prin. from Mar. 1, 1902, to Nov. 1, 1902 (245 da.),	7.91
Amount, Nov. 1, 1902,	$201.56
Payments applied Nov. 1, 1902 ($5 and $50),	55.00
New principal, Nov. 1, 1902,	$146.56
Int. on new prin. from Nov. 1, 1902, to Dec. 15, 1902 (44 da.),	1.07
Amount due, Dec. 15, 1902,	$147.63

EXERCISE 119.

Find the balance due on the date of settlement by the Merchants' Rule:

1. On a note dated Sept. 1, 1899, for $300 with interest at 6%, containing the following indorsements:

June 1, 1900, $50. Sept. 1, 1900, $50.
Jan. 2, 1901, $100.
Settled June 1, 1901.

2. On a note dated June 1, 1900, for $500 with interest at 4½%, containing the following indorsements:

Apr. 1, 1901, $50. Aug. 1, 1901, $100.
Dec. 1, 1901, $150.
Settled, July 1, 1902.

3. On a note dated Albany, N. Y., July 1, 1900, for $500 with interest at the legal rate, containing the following indorsements:

May 1, 1901, $50. Jan. 2, 1902, $50.
July 1, 1901, $100. July 1, 1902, $5.
Settled, Dec. 31, 1902.

4. On a note dated Harrisburg, Pa., May 1, 1899, for $1000 with interest at 4½%, containing the following indorsements:

Mar. 20, 1900, $200. Sept. 1, 1901, $50.
June 15, 1900, $100. Dec. 31, 1901, $200.
Settled, Mar. 1, 1902.

5. On a note dated Dover, Del., Mar. 1, 1899, for $600 with interest at the legal rate, on which payments of $100 each were made at regular intervals of 6 mo. until Mar. 1, 1902, when the balance due was paid.

SUG. Regard 6 mo. as 6 calendar months, or half a year.

Find the balance due on the date of settlement by the U. S. Rule:

6. On a note dated Boston, Mass., Feb. 1, 1901, for $400 with interest at 4%, containing the following indorsements:

July 15, 1901, $200. Feb. 1, 1902, $100.
Sept. 15, 1902, $50.
Settled, Feb. 1, 1903.

7. On a note dated Bloomsburg, Pa., July 1, 1901, for $600 with interest at 4½%, containing the following indorsements:

Oct. 20, 1901, $200. Nov. 15, 1902, $200.
May 1, 1902, $10. Dec. 19, 1902, $100.
Settled, Feb. 1, 1903.

8. On the note mentioned in problem 2.

9. On the note mentioned in problem 3.

CHAPTER VII.

RATIO, PROPORTION, PARTNERSHIP, AND AVERAGE.

Ratio.

321. A **Simple Ratio** is an expression denoting that one number or amount called the **Antecedent** is to be measured by another number or amount called the **Consequent.**

Thus, the expression 15 yd. : 12 yd., read *15 yd. measured by 12 yd.*, is a simple ratio, in which 15 yd. is the antecedent and 12 yd. the consequent.

A simple ratio may also be expressed in the form of a fraction whose numerator is the antecedent and whose denominator is the consequent.

Thus, 15 : 12 may be written $\frac{15}{12}$.

322. The antecedent and the consequent are called the **Terms** of the ratio.

323. The **Value** of a ratio is the result obtained by performing the operation indicated.

324. A **Compound Ratio** is an expression denoting the product of two or more simple ratios.

Thus, $\left\{\begin{matrix}2:3\\4:5\\6:7\end{matrix}\right\}$ is equivalent to $\frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$, and may be read *2 measured by 3, times 4 measured by 5, times 6 measured by 7.*

325. The **Inverse** of a ratio is the ratio resulting from an interchange of terms.

Thus, the inverse of 4 : 5 is 5 : 4.

326. A ratio is in its *simplest form* when its terms are prime to each other.

EXERCISE 120.

Reduce to simplest form:

1. $3\frac{1}{2}$ bu. : 7 pk.

SOLUTION. $3\frac{1}{2}$ bu. : 7 pk. = 14 pk. : 7 pk. = 14 : 7 [§ 58] = 2 : 1 [§ 102]

2. $2\frac{1}{2} : 3\frac{1}{4}$. **5.** 12 men : 18 men. **8.** $62\frac{1}{2}$ ct. : $3.

3. $\frac{2}{3} : \frac{3}{4}$. **6.** $3\frac{1}{2}$ yd. : 15 yd. **9.** $4\frac{4}{5}$ lb. avoir : 15 oz.

4. \$$\frac{5}{6}$: \$$\frac{8}{9}$. **7.** $2\frac{3}{4}$ hr. : 45 min. **10.** \$3.12$\frac{1}{2}$: \$5.62$\frac{1}{2}$.

11. The ratio of two numbers is $12\frac{1}{2}$, and the antecedent is 8; what is the consequent?

12. The ratio of two amounts is $\frac{5}{6}$, and the consequent is $3\frac{3}{4}$ bushels; what is the antecedent?

13. If 3 times A's age equals 4 times B's, what is the ratio of A's age to B's?

14. $\frac{2}{3}$ of John's age equals $\frac{4}{5}$ of William's; what is the ratio of John's age to William's?

15. Two numbers are in the same ratio as 75 and 50; what is the ratio of the numbers? If the first is 12, what is the second?

16. Two fractions are in the same ratio as $\frac{2}{3}$ and $\frac{3}{4}$; what is their ratio? If the second is $\frac{8}{9}$, what is the first?

17. A cubic foot of olive oil weighs 57.19 lb. and a cubic foot of water 62.5 lb.; find to the nearest hundredth the ratio of the weight of olive oil to that of water.

18. Water weighs 62.5 lb. per cubic foot, and the ratio of the weight of silver to that of water is 10.47; find the weight of a cubic foot of silver.

19. Find the ratio of a pound troy to a pound avoirdupois.

20. Find the ratio of an ounce troy to an ounce avoirdupois.

21. The ratio of a cubic foot to a stricken bushel is about $\frac{4}{5}$; how many cubic feet are there in a bin that holds 240 stricken bushels?

Simple Proportion.

327. A **Proportion** is an expression of equality between equal ratios.

328. A **Simple Proportion** is a proportion of two simple ratios.

Thus, 6 : 9 = 8 : 12 is a simple proportion, and may be read *6 measured by 9 equals 8 measured by 12;* or, *the ratio of 6 to 9 equals the ratio of 8 to 12.*

329. A simple proportion has four terms; the first and fourth are the **Extremes**, and the second and third, the **Means**; the first and third, the **Antecedents**, and the second and fourth, the **Consequents**; the first and second, the **First Couplet**, and the third and fourth, the **Second Couplet.**

Note. The proportion 6 : 9 = 8 : 12 may also be written, $\frac{6}{9} = \frac{8}{12}$.

330. The ratio of two similar amounts may equal the ratio of two other similar amounts, and the two ratios together form a proportion.

Thus, 6 bu. : 9 bu. = $18 : $27, since each ratio is $\frac{2}{3}$.

331. Since the ratio of two similar amounts equals the ratio of their numerical values [§ 58], any ratio of similar amounts may be replaced by the ratio of their numerical values.

Thus, 6 bu. : 9 bu. = $18 : $27 may be written 6 : 9 = 18 : 27.

332. A Proportion whose terms are numbers is a **Numerical Proportion.**

Note. If a term of a proportion is unknown, it is generally represented by x.

333. A proportion is in its *simplest form* when its ratios are in the simplest form.

334. PRIN. *In a numerical proportion the product of the extremes equals the product of the means.*

Proof. Given $x : 5 = 7 : 9$, to prove that $9 \times x = 5 \times 7$.

$x : 5 = 7 : 9$ may be written $\frac{x}{5} = \frac{7}{9}$ [§ 329, Note].

Multiplying both ratios by 5×9, or 45, we have:

$$9 \times x = 5 \times 7.$$

COR. 1. *In a numerical proportion one extreme equals the product of the means divided by the other extreme.*

Given $x : 5 = 7 : 9$, to prove that $x = \frac{5 \times 7}{9}$.

$9\,x = 5 \times 7$ [§ 334].

$\therefore x = \frac{5 \times 7}{9}$.

COR. 2. *In a numerical proportion one mean equals the product of the extremes divided by the other mean.*

Given $4 : x = 5 : 10$, to prove that $x = \frac{4 \times 10}{5}$.

$5\,x = 4 \times 10$ [§ 334].

$\therefore x = \frac{4 \times 10}{5}$.

EXERCISE 121.

Reduce to simplest form:

1. x bu. : $2\frac{1}{2}$ bu. = \$$3\frac{1}{4}$: \$$4\frac{3}{4}$.

SOLUTION. Replacing the ratios by equal numerical ratios [§ 331], we have:

$$x : 2\tfrac{1}{2} = 3\tfrac{1}{4} : 4\tfrac{3}{4}.$$

Multiplying both terms of the first ratio by 2 and of the second by 4 [§ 100], we have:

$$2\,x : 5 = 13 : 19.$$

2. $x : 6 = 2\frac{1}{2} : 7\frac{1}{2}$.
3. \$ x : \$$5\frac{1}{2}$ = 12 bu. : 18 bu.
4. x yd. : $3\frac{1}{3}$ yd. = $\frac{2}{3} : \frac{3}{4}$.
5. $\frac{3}{4}\,x : 7\frac{1}{2} = 6\frac{1}{4} : 12\frac{1}{2}$.

Solve the following proportions; that is, find the value of the unknown term x:

6. $x : 6\frac{1}{4} = 7\frac{1}{2} : 15$.

Solution. Multiplying the terms of the first couplet by 4 and of the second by 2, to clear the proportion of fractional terms, we have:

$$4\,x : 25 = 15 : 30.$$

$$\text{Whence, } 4\,x = \frac{25 \times 15}{30} \text{ [§ 334, Cor. 1]} = \tfrac{25}{2}.$$

$$\text{And, } x = \tfrac{1}{4} \text{ of } \tfrac{25}{2}, \text{ or } \tfrac{25}{8}, \text{ or } 3\tfrac{1}{8}.$$

7. $x : 3\frac{1}{3} = 10\frac{1}{2} : 15.$

8. $x : 5\frac{5}{6} = 3.5 : 7.$

9. $x : 3.25 = 7.5 : 25.$

10. x rd. : 45 rd. = \$2.50 : \$7.50.

11. x men : 26 men = 15 da. : 30 da.

12. $\frac{2}{3}\,x : \frac{3}{4} = \frac{5}{6} : \frac{7}{8}.$

335. One quantity is directly proportional to another, if the ratio of any two amounts of the first equals the ratio of the two corresponding amounts of the second.

Thus, the cost of a quantity of sugar is directly proportional to the quantity bought; for, if 2 lb. cost 12 ct., 3 lb. cost 18 ct.

Here 2 lb. : 3 lb. = 12 ct. : 18 ct.

336. One quantity is inversely proportional to another, if the ratio of any two amounts of the first equals the inverse of the ratio of the two corresponding amounts of the second.

Thus, the time required to do a piece of work is inversely proportional to the force employed; for, if 4 men can do a piece of work in 6 da., 3 men can do it in 8 da.

Here 4 men : 3 men = the inverse of 6 da. : 8 da.

337. An easy method of testing whether one quantity is directly or inversely proportional to another is as follows:

1. *If when the second is doubled the first is also doubled, the first is directly proportional to the second.*

2. *If when the second is doubled the first is halved, the first is inversely proportional to the second.*

Thus, the quantity of sugar bought for a certain sum is directly proportional to the sum (the price per pound remaining the same), for if the sum is doubled the quantity bought is also doubled. But the quantity bought is inversely proportional to the price paid per pound (the quantity expended remaining the same), for if the price per pound is doubled the quantity bought is halved.

EXERCISE 122.

1. If 20 tons of hay cost \$300, how much hay can be bought for \$400?

Solution. Let $x =$ the number of tons that cost \$400.
Then, x tons cost \$400,
if 20 tons cost \$300.
The quantity of hay is directly proportional to the cost.
(Test: *If the cost is doubled the quantity of hay is doubled.*)
$\therefore x \text{ T.} : 20 \text{ T.} = \$400 : \$300$ [§ 335].
Or, $x : 20 = 400 : 300$ [§ 331].
Whence, $x = \frac{20 \times 400}{300}$, or $26\frac{2}{3}$.
$\therefore 26\frac{2}{3}$ tons $=$ the required amount.

Note. In the solution, the proportion of amounts may be omitted.

2. If 12 horses eat a quantity of hay in 10 weeks, how many horses would eat it in 30 weeks?

Solution. Let $x =$ the number of horses that would eat it in 30 weeks.
Then, x horses eat it in 30 weeks,
if 12 " " " " 10 " .
The quantity of stock fed is inversely proportional to the time.
(Test: *If the time is doubled the quantity of stock is halved.*)
$\therefore x : 12 =$ the inverse of $30 : 10$ [§ 336].
That is, $x : 12 = 10 : 30$.
Whence, $x = \frac{12 \times 10}{30}$, or 4.

3. In the following, tell whether the first quantity named is directly or inversely proportional to the second:

(*a*) Cost of wheat, the quantity bought.
(*b*) The distance a train runs in a certain time, the speed.
(*c*) The time required to do work, the force.
(*d*) The sum of money earned, the sum loaned.
(*e*) The time required to go a certain distance, the speed.
(*f*) The quantity bought for a certain sum, the cost per pound.

(*g*) The length of a pole, the length of its shadow.

(*h*) The quantity of work done in a certain time, the force.

4. If $4.50 was charged for the loan of $100 for 1 yr., what should be charged for the loan of $750 for the same time?

5. When flour is sold for $4.50 per barrel (196 lb.), what should be charged for 100 lb.?

6. If coal is retailed at $0.28 per 100 lb., what should be charged for a load containing 1440 lb.?

7. When hay is selling for $18 per ton, what should be charged for 1250 lb.?

8. If a 10-foot pole casts a 25-foot shadow, how high is a tree that casts a 72-foot shadow at the same time?

9. If 5 cu. ft. are equivalent to 4 bu., how many cubic feet are there in a bin that will hold 60 bu.?

10. If an arc of 90° measures 15 in., what is the length of an arc of 20° in the same circle?

11. 25 ct. will buy a certain quantity of sugar when it sells for 6¼ ct. per lb.; what would the same quantity cost when it sells for 8 ct. per lb.?

12. A train can run from A to B in 5½ hr., if it runs uniformly 30 mi. per hr.; what should be its speed to make the run in 3¼ hr.?

13. If 1⅛ bbl. of lime is used in laying 1000 bricks, how many barrels are used in laying 16225 bricks?

14. A grocer uses weights that are ¼ of an ounce too light to the pound; how many pounds does he charge for in selling a cheese whose true weight is 40 lb.?

15. When fruit puddine sells for $1 per dozen packages, what should be charged for 3 packages?

16. If 100 bu. of apples cost $75, what would 1 bbl. containing 2½ bu. cost at the same rate?

17. 60 geographic miles equal 69.16 statute miles; find the equatorial circumference of the earth in statute miles, the circumference being 21,600 geographic miles.

18. An Edison incandescent lamp burning 4 hr. each night will last about 200 nights; how many nights would it last, if it were to burn 3 hr. each night?

19. In a plot of a farm, 6.25 in. is made to represent a side 112.5 rd. long; what should represent the other sides whose lengths are 100 rd., 93.75 rd., and 78.125 rd.?

20. A cubic foot of water weighs 62½ lb. and a cubic foot of milk 64⅜ lb.; how many pounds of milk will a tank hold that holds 1000 pounds of water?

21. A square piece of timber 24 ft. long containing 576 board feet was sawed off 10 feet from one end; how many board feet were there in each piece?

22. A certain bin holds 2400 lb. of wheat; how many pounds of shelled corn would it hold?

Sug. The entire weight is directly proportional to the weight per bushel.

23. A certain mass of copper weighs 240 lb. Troy; how many pounds avoirdupois does it weigh?

Sug. The number of pounds is inversely proportional to the number of grains in a pound.

24. A passenger train runs 24¾ miles while a freight train runs 10⅝ miles; how far does the freight train run while the passenger train runs 99 miles?

25. If it takes 81 bricks to pave 2 square yards, how many bricks will it take to make a pavement 3 yards wide and 60 yards long?

26. A man used 600 shingles to make 100 sq. ft. of roof; how many more shingles must he use to complete a roof of 640 sq. ft.?

27. A farmer's crop of 240 bu. of oats weighs on an average 30 lb. to the bushel; if he sells it to a miller who takes 32 lb. to the bushel, for how many bushels should the farmer be paid?

28. A man bought sufficient fertilizer for 8 A. of land, sowing 240 lb. to the acre; he, however, sowed the amount of fertilizer bought on 10 A. How much did he sow to the acre?

Compound Proportion.

338. A **Compound Proportion** is a proportion one of whose ratios is compound.

339. PRIN. *In a proportion whose second ratio is compound, the first extreme equals the product of all the terms in the means divided by the product of all the terms in the last extreme.*

Let $x : 3 = \left\{\begin{matrix} 6 : 7 \\ 2 : 5 \\ 4 : 9 \end{matrix}\right\}$ be a proportion whose second ratio is compound.

Now, $\frac{x}{3} = \frac{6}{7} \times \frac{2}{5} \times \frac{4}{9}$.

Multiplying by $3 \times 7 \times 5 \times 9$,

we have $\cancel{3} \times 7 \times 5 \times 9 \times \frac{x}{\cancel{3}} = 3 \times \cancel{7} \times \cancel{5} \times \cancel{9} \times \frac{6}{\cancel{7}} \times \frac{2}{\cancel{5}} \times \frac{4}{\cancel{9}}$.

Or, $7 \times 5 \times 9 \times x = 3 \times 6 \times 2 \times 4$.

Whence, $x = \frac{3 \times 6 \times 2 \times 4}{7 \times 5 \times 9}$.

EXERCISE 123.

1. If a train running at the rate of 30 mi. per hour runs 240 mi. in 8 hr., in how many hours would it run 360 mi., running at the rate of 45 mi. per hour?

SOLUTION. Let x = the number of hours required; then we have the following:

STATEMENT.

Time.	Rate.	Distance.
x hr.	45 mi.	360 mi.
8 hr.	30 mi.	240 mi.

The time is inversely proportional to the rate and directly proportional to the distance.

$\therefore x : 8 = \left\{\begin{matrix} 30 : 45 \\ 360 : 240. \end{matrix}\right.$

Whence, $x = \frac{8 \times 30 \times 360}{45 \times 240} = 8$, or the number of hours required.

2. If 21 bu. of oats are required to feed 8 horses 7 da., how many bushels would be required to feed 16 horses 3 da.?

3. If 300 feet of boards are required to build a tight board fence 30 yd. long and 5 ft. high, how many feet would be required to build one 24 yd. long and 4½ ft. high?

4. If a lot 121 yd. long and 80 yd. wide contains 2 A., how wide must a lot 220 yd. long be to contain 3½ A.?

5. If $9 is charged for the use of $100 for 2 yr., what should be charged for the use of $250 for 2 yr. 6 mo.?

6. If a certain number of men can build a wall 60 yd. long, 2½ ft. wide, and 4 ft. high, in 10 da., in what time can they build a wall 48 ft. long, 3 ft. wide, and 3¾ ft. high?

7. If in example 6 the men were allowed 12 da. to build the second wall, how high could they build it, the other dimensions remaining the same? State and solve the problem.

8. If a wheel 6 ft. in circumference makes 880 revolutions in going a mile, how many revolutions would a wheel 4 ft. in circumference make in going 3520 ft?

9. In example 8, what would have been the circumference of the second wheel if it had made 660 revolutions in going 3520 ft? State the problem and solve it.

10. If 2 lamps burning 4 hr. per night for 6 nights consume 5 qt. of oil, how many gallons would be required per year for the 24 lamps of a village, each burning 7 hr. per night, provided the lamps are all of the same size?

11. If 1 man working 10 hr. per day can build a wall 24 ft. long, 10 ft. high, and 2 bricks thick, in 2 da., in how many days of 8 hr. each can 3 men build a wall 30 ft. long, 8 ft. high, and 3 bricks thick?

12. If a mow of hay 32 ft. long, 16 ft. wide, and 16 ft. high lasts 8 horses 20 weeks, how many weeks will a mow of hay 28 ft. long, 20 ft. wide, and 12 ft. high last 5 horses?

13. If 10 men in 20 da., working 8 hr. each, can pave a street 1 mi. long and 60 ft. wide, in how many days of 10 hr. each can 8 men pave a street 66 ft. long and 40 ft. wide?

Proportional Parts.

EXERCISE 124.

1. A and B together invest \$1500, and A's investment is to B's as 2 to 3; how much does each invest?

Solution (1) Divide \$1500 into 2 + 3, or 5, equal parts.
Then, 2 of these parts, or $\frac{2}{5}$ of \$1500, or \$600 = A's.
And, 3 " " " " $\frac{3}{5}$ of \$1500, or \$900 = B's.

Solution (2) A's : B's = 2 : 3.
Then, 3 A's = 2 B's.
And, A's = $\frac{2}{3}$ B's.
B's = $\frac{3}{3}$ B's.
∴ A's + B's = $\frac{5}{3}$ B's = \$1500.
$\frac{1}{3}$ B's = \$300.
B's = \$900.
A's = $\frac{2}{3}$ of \$900 = \$600.

2. The length of a field is to its width as 5 to 2, and their difference is 45 rd.; find the length and the width.

Solution. Let l represent the length and w the width.

$$\text{Then, } l : w = 5 : 2.$$
$$\text{Whence, } 2\,l = 5\,w.$$
$$\text{And, } l = \tfrac{5}{2}\,w.$$
$$w = \tfrac{2}{2}\,w.$$
$$\therefore\ l - w = \tfrac{3}{2}\,w = 45 \text{ rd.}$$
$$\text{Whence, } w = \tfrac{2}{3} \text{ of } 45 \text{ rd., or } 30 \text{ rd.}$$
$$\text{And, } l = \tfrac{5}{2} \text{ of } 30 \text{ rd., or } 75 \text{ rd.}$$

3. Divide \$100 into parts in the ratio of 3 to 2.

Sug. 1st part : 2d part = 3 : 2.

4. Divide \$250 into parts in the ratio of 1 to 4.

5. Divide 100 lb. into parts in the ratio of 5 to 4.

6. The sum of two numbers is 24 and their ratio is $\frac{3}{5}$; find the numbers.

7. The combined weight of a bushel of wheat and a bushel of oats is 92 lb., and the ratio of their weights equals the ratio of 15 to 8; find the weight of each.

8. The difference of two numbers is 15 and their ratio is $\frac{2}{3}$; find them.

9. A invested \$6 as often as B invested \$5; find their investments, if A's was \$75 more than B's.

10. A and B together invested \$275, of which A invested \$3 as often as B invested \$2; find their investments.

11. Divide 10 lb. into parts in the ratio of $\frac{1}{2}$ to $\frac{1}{3}$.

SUG. 1st part : 2d part $= \frac{1}{2} : \frac{1}{3} = 3 : 2$.

12. Divide $\frac{2}{3}$ into parts in the ratio of $\frac{2}{3}$ to $\frac{3}{5}$.

13. The sum of two numbers is 2.75 and their ratio equals the ratio of .5 to $.33\frac{1}{3}$; find them.

14. The difference of two fractions is $\frac{1}{12}$ and their ratio is $\frac{5}{7}$; find the fractions.

15. A grocer makes a mixture of 10 lb. of tea by using $\frac{1}{4}$ lb. of black tea to $\frac{3}{8}$ lb. of green tea; how many pounds of each kind are there in the mixture?

16. Solder for lead consists of 1 part tin to $1\frac{1}{2}$ parts lead; how much of each of these substances are necessary to make 2 lb. of solder?

17. Type-metal is made from antimony and lead mixed in the ratio of 1 part of the former to $4\frac{1}{2}$ parts of the latter; how much of each is required to make 100 lb. of type?

18. Two trains, A and B, start at the same time from two towns 34 miles apart and approach each other; how far must each train run before they meet, if A's rate is to B's as $\frac{2}{3}$ to $\frac{3}{4}$?

SUG. A's rate : B's rate $= \frac{2}{3} : \frac{3}{4} = 8 : 9$.
∴ A's distance : B's distance $= 8 : 9$.

19. Two men agreed to do a piece of work for \$36; if the first worked 2 da. as often as the second worked 3 da., how much pay should each receive?

20. A, B, and C invested jointly \$4400; if A invested \$2 as often as B invested \$3 and C \$6, how much did each invest?

SUG. Divide \$4400 into $2 + 3 + 6$, or 11 equal parts; then their shares are $\frac{2}{11}$, $\frac{3}{11}$, and $\frac{6}{11}$ of \$4400.

21. A miller ground 100 bu. of feed from oats, corn, and rye, mixed in the proportion of 2, 3, and 5; how many bushels of each did he use?

Partnership.

340. A **Partnership** is an association of individuals, called **Partners,** for the transaction of business.

Note. A partnership is sometimes called a **Firm.**

341. The **Capital Stock** is the amount which the partners invest in the business.

Note. The **Assets** of a firm are all the property belonging to the firm. The **Liabilities** are all the debts of the firm.

342. When the shares of the partners are invested for equal times, gains and losses are distributed among the partners in proportion to their respective shares.

343. When the shares of the partners are invested for unequal times, gains and losses are distributed among the partners in proportion to their respective shares and times, except when by special agreement some other plan is adopted.

EXERCISE 125.

1. A, B, and C form a partnership; A invests $4000, B $3000, and C $2000. If they gain $2700 the first year, what is each man's share of the gain?

SOLUTION. $4000 + $3000 + $2000, or $9000 = the capital stock.
$\frac{4000}{9000}$, or $\frac{4}{9}$ = the part of the capital stock owned by A.
$\frac{3000}{9000}$, or $\frac{1}{3}$ = " " " " " " " " B.
$\frac{2000}{9000}$, or $\frac{2}{9}$ = " " " " " " " " C.
$2700 = the total gain.
The gain is distributed in proportion to the shares.
∴ $\frac{4}{9}$ of $2700, or $1200 = A's gain.
$\frac{1}{3}$ of $2700, or $900 = B's gain.
And, $\frac{2}{9}$ of $2700, or $600 = C's gain.

2. Three brothers bought a farm for $4800, the oldest paying $2000, the second $1500, and the youngest $1300; if they gained in 1 year $720, what was each one's share of the gain?

3. A and B invested jointly $400 in a machine, of which sum A invested $250. During a certain season they cleared from it $240; how much of this should each receive?

4. M and N buy a pair of horses and a stage for $500. They agree that M shall receive $1.50 a day for driving the stage, and that the profits shall be shared in proportion to their investments. They run the stage 313 days during a certain year and receive $600 for carrying mail, $450 for carrying passengers, and $65.75 for carrying packages. If their running expenses for the year are $325, how much of the profit should each receive, if M contributes $300 and N $200?

5. Three men R, S, and T invested jointly $6000 in a laundry; at the end of the first year their gains were $2500, of which R received $1000, S $1200, and T the remainder. Find the sum invested by each.

6. M, N, and P in partnership gained during a certain year $3000, of which M received $1200, N $800, and P the remainder; if P's investment was $5000, find the investments of M and N.

7. A merchant failing in business has assets amounting to $2557.75. He owes A $4000, B $2500, and C $2000; how much should each receive?

8. A, B, and C started a coal yard, A furnishing $5000, and B $4000. C's gain for the first year was $198; find C's capital and A's and B's gain, if the total capital was $18,900.

9. A and B in partnership gain in 1 year $8560. A contributes $8000 and B $7000. A is to retain 25% of the profits for managing the business; how much of the balance should each receive?

10. A, B, and C bought a drove of horses for $7000 and paid freight $200. They sold the drove for $10,000; if A contributed $1800, B $2400, and C the remainder, how much of the profits should each receive?

11. A, B, and C formed a partnership. A invested $500 on Jan. 1, B $400 on March 1, and C $300 on July 1. At

the end of the year they had gained $590; what was each partner's share of the gain?

SOLUTION.

A's capital was in trade 12 mo., B's 10 mo., and C's 6 mo.
A's gain = the gain on $500 for 12 mo. = the gain on $6000 for 1 mo.
B's " = " " " $400 " 10 " = " " " $4000 " 1 "
C's " = " " " $300 " 6 " = " " " $1800 " 1 "
∴ the total gain = " " " $11,800 " 1 "
The total gain = $590.
∴ $590 = the gain on $11800 for 1 mo.
A's gain = $\frac{6000}{11800}$, or $\frac{30}{59}$, of $590, or $300.
B's " = $\frac{4000}{11800}$, or $\frac{20}{59}$, of $590, or $200.
C's " = $\frac{1800}{11800}$, or $\frac{9}{59}$, of $590, or $90.

12. Hall and Mills formed a partnership, Hall putting in $600 for 18 months, and Mills $800 for a year. They gained $510; what was each partner's share of the gain?

13. M, N, and P in partnership gained $2670. M's capital averaged $1200 for 10 months, N's $1500 for 9 months, and P's $100 for 12 months; what was each man's share of the gain?

14. M, N, and P invested jointly $3100. M's capital was invested 8 mo., and his gain was $360; N's capital was invested 9 mo., and his gain was $450; P's capital was invested 12 mo., and his gain was $720. Find the capital of each.

15. R and S entered into partnership Jan. 1, R investing $2000 and S $1500. July 1, R added $3000, and Aug. 1, S added $1500. Their gain at the end of the year was $1350; divide it properly between them.

16. A, B, and C contracted to do a piece of work. A sent 24 men for 45 da., B 40 men for 25 da., and C 35 men for 42 da. If the sum paid for doing the work was $355, find each partner's share.

17. R, S, and T entered into a partnership with a joint capital of $19,000. The first year R's capital was invested 8 mo. and his gain was $800; S's capital was invested 10 mo. and his gain was $400; T's capital was invested the whole year and his gain was $600. Find the capital of each.

Average.

EXERCISE 126.

1. A grocer mixes 50 lb. of Rio coffee costing 16 ct. per lb., 100 lb. of La Guayra costing 18 ct. per lb., and 250 lb. of Mocha costing 28 ct. per lb.; what was the average cost of the mixture per lb.?

SOLUTION. 50 × $0.16, or $8 = the cost of 50 lb. of Rio.
100 × $0.18, or $18 = " " " 100 " " La Guayra.
250 × $0.28, or $70 = " " " 250 " " Mocha.
∴ $96 = " " " 400 " " mixture.
And, $96 ÷ 400, or $0.24 = the cost of 1 lb. of mixture.

2. Find the amount of alcohol in each gallon of a mixture consisting of 20 gal. of wine, .18 of which is alcohol, and 10 gal. of wine, .22 of which is alcohol.

SOLUTION.

.18 × 20 gal., or 3.6 gal. = the amount of alcohol in the 20 gal.
.22 × 10 gal., or 2.2 gal. = " " " " " " 10 gal.
∴ 5.8 gal. = " " " " " " 30 gal.
And, 5.8 gal. ÷ 30, or .19⅓ gal. = the required amount.

3. Find the average cost per lb. of 10 lb. of Japan tea costing 40 ct. per lb., 40 lb. of Formosa costing 35 ct. per lb., 80 lb. of Imperial costing 45 ct. per lb., and 90 lb. of Ceylon costing 28 ct. per lb.

4. A firm employs 25 clerks at $1.25 per day, 4 drivers at $1.10 per day, 1 foreman at $4.50 per day; find the average daily wages paid.

5. Find the average cost per lb. of 84 lb. of "Keystone A" sugar costing 5.25 ct. per lb., 168 lb. "Centennial A" costing 5.10 ct. per lb., 252 lb. of "Franklin B" costing 5.20 ct. per lb., and 126 lb. "Keystone B" costing 5.05 ct. per lb.

6. Find the average cost per lb. of the following mixture: 49 lb. of almonds costing 25 ct. per lb., 5 lb. Brazils costing 12 ct. per lb., 10 lb. of filberts costing 13 ct. per lb., and 15 lb. of pecans costing 4 ct. per lb.

7. Find the average cost per lb. of the following mixture: 10 lb. of burnt almonds costing 16 ct. per lb., 80 lb. of caramels costing 8 ct. per lb., 100 lb. of chocolate drops costing 12 ct. per lb., and 40 lb. of gum drops costing 5 ct. per lb.

8. A town employs 1 principal at $150 per mo., 4 teachers at $80 per mo., 10 at $60 per mo., 12 at $45 per mo., and 6 at $40 per mo.; find the average salary paid per month.

9. A school was open 20 da. during a certain month; 6 pupils attended every day, 8 attended 19 da., 4 attended 18 da., 4 attended 17 da., 6 attended 16 da., and 1 attended 12 da. Find the average daily attendance for that month.

SUG. Divide the total number of days the pupils attended by the number of days the school was open.

10. Find the average time for the payment of $200 due in 6 mo. and $400 due in 8 mo.

SOLUTION. The use of $200 for 6 mo. = the use of $1 for 1200 mo.
The use of $400 for 8 mo. = the use of $1 for 3200 mo.
$\therefore$ The use of $600 for x mo. = the use of $1 for 4400 mo.
The use of $1 for 4400 mo. = the use of $600 for $\frac{1}{600}$ of 4400 mo., or $7\frac{1}{3}$ mo.
$\therefore$ x mo. = $7\frac{1}{3}$ mo. = the required time.

11. Find the average time for the payment of $300 due in 4 mo., $600 due in 6 mo., and $1000 due in 10 mo.

12. Jan. 4, 1901, J. P. Richards bought of Mench & Co. the following bill of goods: $300 on 15 days' credit; $250 on 30 days' credit; and $160 on 60 days' credit. If the entire bill was paid in one payment, on what date was it paid?

13. Jan. 1, Mr. Smith bought a bill of goods amounting to $600 on 6 mo. credit. Two months after the purchase he paid $300, and two months later $200 more. On what date was the balance due?

14. Jan. 1, Henry Cloud purchased a farm for $6000, agreeing to pay $\frac{1}{3}$ cash, $\frac{1}{3}$ in 6 mo., and the balance in 1 yr. If he paid the entire debt in one payment, on what date was it made?

CHAPTER VIII.

POWERS, ROOTS, AND MENSURATION.

Squaring and Cubing of Numbers.

344. The **Square** of a number is the result of using the number twice as a factor.

Thus, the square of 3 equals 3 × 3, or 9; the square of $\frac{2}{3}$ equals $\frac{2}{3} \times \frac{2}{3}$, or $\frac{4}{9}$; and the square of .05 equals .05 × .05, or .0025.

Note. The square of a number is also called the **Second Power** of the number.

345. The **Cube** of a number is the result of using the number three times as a factor.

Thus, the cube of 3 equals 3 × 3 × 3, or 27; the cube of $\frac{2}{3}$ equals $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$, or $\frac{8}{27}$; and the cube of .05 equals .05 × .05 × .05, or .000125.

Note. The cube of a number is also called the **Third Power** of the number.

346. The figure 2 written to the right above a number denotes that the number is to be squared, and the figure 3 similarly written denotes that the number is to be cubed.

Thus, 5^2, read *5 squared*, equals 5 × 5, or 25;
and 5^3, read *5 cubed*, equals 5 × 5 × 5, or 125.

EXERCISE 127.

Find the square and the cube of:

1. 1, 2, 3, 4, 5, 6, 7, 8, 9.

2. 10, 20, 30, 40, 50, 60, 70, 80, 90.

3. 100, 200, 300, 400, 500, 600, 700, 800, 900.

4. .1, .2, .3, .4, .5, .6, .7, .8, .9.

5. .01, .02, .03, .04, .05, .06, .07, .08, .09.

6. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$.

7. 18.

8. 35.

9. 49.

10. 125.

11. 404.

12. 842.

13. $\frac{2}{3}$.

14. $\frac{3}{4}$.

15. $\frac{4}{5}$.

16. $12\frac{1}{2}$.

17. $8\frac{1}{3}$.

18. $66\frac{2}{3}$.

19. .25.

20. .056.

21. 2.56.

347. A number that can be produced by squaring another number is called a **Perfect Square.**

Thus, 36 is a perfect square, being the square of 6.

348. A number that can be produced by cubing another number is called a **Perfect Cube.**

Thus, 8 is a perfect cube, being the cube of 2.

Square Root and Cube Root.

349. The **Square Root** of a given number is a number which squared will produce the given number.

Thus, the square root of 64 is 8, since $8^2 = 64$; the square root of $\frac{9}{16}$ is $\frac{3}{4}$, since $(\frac{3}{4})^2 = \frac{9}{16}$; and the square root of .0025 is .05, since $(.05)^2 = .0025$.

350. The **Cube Root** of a given number is a number which cubed will produce the given number.

Thus, the cube root of 125 is 5, since $5^3 = 125$; the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, since $(\frac{2}{3})^3 = \frac{8}{27}$; and the cube root of .125 is .5, since $(.5)^3 = .125$.

351. The symbol $\sqrt{}$, called the **Radical Sign**, written before a number indicates that the square root of the number is to be taken, and the symbol $\sqrt[3]{}$, written in a like position, indicates that the cube root is to be taken.

Thus, $\sqrt{4}$, read *the square root of 4*, equals 2; and $\sqrt[3]{27}$, read *the cube root of 27*, equals 3.

EXERCISE 128. ORAL.

Find the square root of:

1. 1; 100; 10,000; 1,000,000.

Sug. $\sqrt{1} = 1$, since $1^2 = 1$; $\sqrt{100} = 10$, since $10^2 = 100$, etc.

2. .01, .0001, .000001.

Sug. $\sqrt{.01} = .1$, since $(.1)^2 = .01$; $\sqrt{.0001} = .01$, since $(.01)^2 = .0001$.

3. 4, 9, 16, 25, 36.
4. 49, 64, 81, 100, 121, 144.
5. .04, .09, .16, .25, .36, .49, .64, .81.
6. $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$, $\frac{1}{81}$, $\frac{1}{100}$.
7. $\frac{4}{9}$, $\frac{9}{16}$, $\frac{4}{25}$, $\frac{9}{25}$, $\frac{16}{25}$, $\frac{25}{36}$, $\frac{4}{49}$, $\frac{9}{49}$, $\frac{16}{49}$, $\frac{25}{49}$, $\frac{36}{49}$.
8. $\frac{9}{49}$, $\frac{25}{64}$, $\frac{49}{64}$, $\frac{4}{81}$, $\frac{16}{81}$, $\frac{25}{81}$, $\frac{49}{81}$, $\frac{64}{81}$.
9. .0004, .0009, .0016, .0025, .0036, .0049, .0064, .0081.

352. It was shown in § 190 that the number of square units in a square equals the square of the number of units in the length; therefore,

Prin. *The number of units in the length of a given square equals the square root of the number of square units in the square.*

10. Find the length of squares that contain, respectively, 36 square inches, 49 square feet, 100 square yards, $\frac{4}{9}$ of a square rod, .0025 of a square mile.

Find the cube root of:

11. 1; 1,000; 1,000,000; .001; .000001.

12. 8, 27, 64, 125, 216, 343, 512, 729.

13. .008, .027, .064, .125, .216, .343, .512, .729.

14. $\frac{1}{8}$, $\frac{1}{27}$, $\frac{1}{64}$, $\frac{1}{125}$, $\frac{1}{216}$, $\frac{1}{343}$, $\frac{1}{512}$, $\frac{1}{729}$.

15. $\frac{8}{27}$, $\frac{27}{64}$, $\frac{8}{125}$, $\frac{27}{125}$, $\frac{64}{125}$, $\frac{125}{216}$, $\frac{8}{343}$, $\frac{27}{512}$, $\frac{64}{729}$.

353. It was shown in § 198 that the number of cubic units in a cube equals the cube of the number of units in its length; therefore,

PRIN. *The number of units in the length of a cube equals the cube root of the number of cubic units in the cube.*

16. Find the length of cubes that contain, respectively, 64 cubic inches, 125 cubic feet, $\frac{64}{125}$ of a cubic yard, .125 of a cubic foot.

GENERAL RULE FOR EXTRACTING SQUARE ROOT.[*]

354. The square root of numbers may be found by the following rule:

1. *Point off the number into periods of two figures each, beginning with the units.*
2. *Take the square root of the nearest square number below that expressed by the first period on the left, for the first figure of the root.*
3. *Subtract the square of the first figure of the root from the first period of the number and annex the next period to the remainder for a dividend.*
4. *Take twice the root found, as a trial divisor, and divide the dividend, exclusive of the right-hand figure, by it for the next term of the root.*
5. *Annex the last figure of the root now found to the trial divisor for the complete divisor.*

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[*] To Teacher. For explanation of this rule see "The New Complete Arithmetic" by authors, p. 309.

6. *Multiply the complete divisor by the last figure of the root now found and subtract the product from the remainder.*

7. *If the root is not fully found, repeat the above processes until all the periods of the number have been used.*

8. *Should a cipher occur in the root, annex a cipher to the trial divisor to form the next trial divisor, and bring down the next period.*

EXERCISE 129.

Extract the square root of:

1. 524176.

FORM.

$$\begin{array}{r|l|l}
 & 52'41'76 & 724 \\
 & 49 & \\
142 & 341 & \\
 & 284 & \\
1444 & 5776 & \\
 & 5776 &
\end{array}$$

EXPLANATION. Point off the number into periods of two figures each, beginning at units. The greatest number whose square is contained in 52 is 7, which is the first figure of the root. Square 7, subtract the result from 52, and annex 41 to the remainder, making the dividend 341. Double 7, the root found, for a trial divisor. Divide 341, exclusive of the right hand figure, by 14 for the next figure of the root, which is found to be 2. Annex 2 to 14, making the complete divisor 142. Multiply 142 by 2, the last figure of the root found, subtract the product 284 from 341, and annex 76 to the remainder, making the next dividend 5776. Double 72, the root already found, for a trial divisor. Divide 5776, exclusive of the right hand figure, by 144 for the next figure of the root, which is found to be 4. Annex 4 to 144, making the complete divisor 1444. Multiply 1444 by 4, the last figure of the root found, and subtract the product from 5776; nothing remains. Therefore, the square root of 524176 is 724.

2. 1024.
3. 2116.
4. 3481.
5. 4624.
6. 5776.
7. 7396.
8. 9025.
9. 14161.
10. 50625.
11. 106929.
12. 217156.
13. 258064.
14. 318096.
15. 363609.
16. 450241.
17. 498436.
18. 577600.
19. 643204.
20. 739600.
21. 824464.

22. Extract the square root, to hundredths, of 312.5.

FORM.

$$\begin{array}{r|l}
 & 3'12'.50'00 \;|\; \underline{17.677} \\
 & \underline{1} \\
27 & 212 \\
 & \underline{189} \\
346 & 2350 \\
 & \underline{2076} \\
3527 & 27400 \\
 & \underline{24689} \\
35347 & 271100
\end{array}$$

EXPLANATION. Point off the number into periods of two figures each, rightward and leftward from the decimal point. The square root is found to be nearer 17.68 than 17.67; hence, the result is taken to be 17.68.

Extract the square root of:

23. 53.29. **24.** 7.7284. **25.** 5852.25. **26.** 65.2864.

Extract the square root, to the nearest hundredth, of:

27. 32. **29.** 321. **31.** 3.42. **33.** .023.
28. 45. **30.** 508. **32.** 1.423. **34.** .0021.

355. The square of $\frac{4}{5} = \frac{4}{5} \times \frac{4}{5} = \frac{4 \times 4}{5 \times 5} = \frac{16}{25}$; hence, the square root of $\frac{16}{25} = \frac{4}{5}$, or $\frac{\sqrt{16}}{\sqrt{25}}$. Therefore,

PRIN. *The square root of a fraction equals the square root of the numerator divided by the square root of the denominator.*

Note. If a fraction in its lowest terms is a perfect square, its terms are perfect squares. When the terms of a fraction in its lowest terms are not perfect squares, it is best to reduce it to a decimal before extracting the square root.

Extract the square root of:

35. $\frac{121}{144}$. **37.** $\frac{256}{729}$. **39.** $7\frac{1}{9}$. **41.** $\frac{7}{8}$ to hundredths.
36. $\frac{400}{529}$. **38.** $\frac{1225}{64}$. **40.** $6\frac{115}{289}$. **42.** $\frac{16}{3}$ to hundredths.

GENERAL RULE FOR EXTRACTING CUBE ROOT.[*]

356. The cube root of numbers may be found by the following rule:

1. *Point off the number into periods of three figures each, beginning with the units.*

2. *Take the cube root of the nearest cube number below that expressed by the first period on the left, for the first figure of the root.*

3. *Subtract the cube of the first figure of the root from the first period of the number, and annex the next period to the remainder for a dividend.*

4. *Take three times the square of the root found for a trial divisor, and divide the dividend, exclusive of the two right-hand figures, by it for the next figure of the root.*

5. *Take three times the root previously found multiplied by the last figure found for the first correction, and the square of the last figure found for the second correction.*

6. *Place the right-hand figure of the first correction one order to the right of the trial divisor, and the right-hand figure of the second correction one order to the right of the first correction, then take the sum of the trial divisor and the two corrections for the complete divisor.*

7. *Multiply the complete divisor by the last figure of the root now found, and subtract the product from the dividend.*

8. *If the root is not fully found, annex the next period of the number to the remainder for a dividend and again find the trial divisor, the next figure of the root, the cor-*

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[*] **To Teacher. For explanation of this rule see "The New Complete Arithmetic," p. 314.**

rections, and the complete divisor as before; and so continue till all the periods of the number have been used.

9. *Should a cipher occur in the root, annex two ciphers to the trial divisor to form the next trial divisor, and annex another period to the dividend.*

EXERCISE 130.

Extract the cube root of:

1. 14526784.

Form.

	14'526'784	244
	8	
3 × 2 × 2 = 12	6526	
3 × 2 × 4 = 24		
4 × 4 = 16		
1456	5824	
3 × 24 × 24 = 1728	702784	
3 × 24 × 4 = 288		
4 × 4 = 16		
175696	702784	

Explanation. Point off the number into periods of three figures each, beginning at units. The greatest number whose cube is contained in 14 is 2, which is the first figure of the root. Cube 2, subtract the result from 14, and annex 526 to the remainder, making the dividend 6526. Take 3 times the square of 2, or 12, for a trial divisor. Divide 6526, exclusive of the two right-hand figures, by 12, making allowance for corrections to be added, for the next figure of the root, which is found to be 4. Take 3 times 2 times 4 for the first correction, and set it one place to the right of the trial divisor. Take the square of 4 for the second correction, and set it one figure to the right of the first correction. The sum of the trial divisor and the two corrections, 1456, is the complete divisor. Multiply 1456 by 4, the last figure of the root found, subtract the product 5824 from 6526, and annex 784 to the remainder, making the dividend 702784. Take 3 times the square of 24 for the trial divisor. Find as before the next figure of the root 4, the first correction 288, the second correction 16, the complete divisor 175696, the product 702784, and the remainder 0. The cube root is 244.

2. 3375. **5.** 195112. **8.** 1442897. **11.** 45118016.
3. 17576. **6.** 421875. **9.** 16777216. **12.** 131096512.
4. 50653. **7.** 681472. **10.** 28094464. **13.** 175616000.

14. Extract the cube root of 5, to the nearest hundredth.

FORM.

$$\begin{array}{r|l}
 & 5.000'000 \,\underline{|\,1.709} \\
 & 1 \\ \cline{2-2}
3 \times 1 \times 1 = 3 & 4000 \\
3 \times 1 \times 7 = 21 & \\
7 \times 7 = \underline{\ 49} & \\
559 & 3913 \\ \hline
\text{T. D.} = 3 \times 17 \ \times 17 \ = 867 & 87000000 \\ \hline
\text{T. D.} = 3 \times 170 \times 170 = 86700 & \\ \hline
\end{array}$$

EXPLANATION. Separate the number into periods of three figures each rightward and leftward from the decimal point. The cube root is found to be nearer 1.71 than 1.70. Therefore 1.71 is the cube root of 5, to the nearest hundredth.

Extract the cube root of:

15. 10.648. **17.** 16581.375. **19.** .039304. **21.** .000125.
16. 4.574296. **18.** .006859. **20.** .103823. **22.** 496793.088.

Extract the cube root, to the nearest hundredth, of:

23. 25. **25.** 4.5. **27.** 300. **29.** .8.
24. 126. **26.** 3.75. **28.** .005. **30.** .27.

357. The cube of $\frac{3}{4} = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64}$; hence, the cube root of $\frac{27}{64} = \frac{3}{4} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$. Therefore,

PRIN. *The cube root of a fraction equals the cube root of the numerator divided by the cube root of the denominator.*

Note. If a fraction in its lowest terms is a perfect cube, its terms are perfect cubes. If the terms of a fraction in its lowest terms are not perfect cubes, it is best to reduce the fraction to a decimal before extracting the cube root.

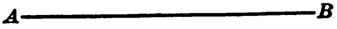
Extract the cube root of:

31. $\frac{64}{125}$. **33.** $\frac{125}{1728}$. **35.** $12\frac{19}{27}$. **37.** $\frac{3}{5}$ to hundredths.
32. $\frac{729}{1000}$. **34.** $\frac{9261}{10648}$. **36.** $5\frac{104}{125}$. **38.** $\frac{2}{7}$ to hundredths.

Mensuration of Surfaces.

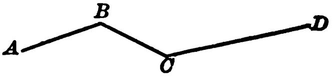
LINES.

358. A **Straight Line** is a line that has the same direction throughout; as *AB*.



359. A **Curved Line** is a line no part of which is straight; as *CD*.

360. A **Broken Line** is a line composed of two or more different straight lines; as *ABCD*.



361. The **Length** of a line is the number and kind of linear units it contains.

Thus, 4 ft. may be the length of a line.

SURFACES.

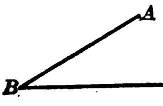
362. A **Plane Surface**, or a **Plane**, is a surface on which straight lines may be drawn in different directions.

363. A **Curved Surface** is a surface no part of which is plane; as the surface of a ball.

364. A **Plane Figure** is any portion of a plane surface bounded by a broken or a curved line; as a rectangle or a circle.

ANGLES.

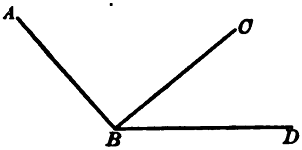
365. An **Angle** is the figure formed by two straight lines meeting at a point; as the angle *ABC*, or simply the angle *B*.



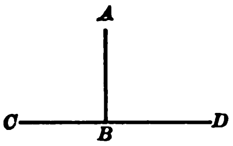
The lines AB and BC are the **Sides** of the angle, and the point B the **Vertex.**

Note. The symbol $\angle$ is read *angle.*

Thus, $\angle B$ is read *angle B.*

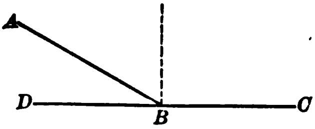


366. Adjacent Angles are angles that have the same vertex and a common side between them; as ABC and CBD.



367. When one straight line meets another straight line so as to make the adjacent angles equal, each angle is called a **Right Angle.**

Thus, the angles ABC and ABD are right angles.

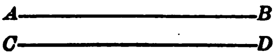


368. An **Obtuse Angle** is an angle greater than a right angle, and an **Acute Angle** is one less than a right angle.

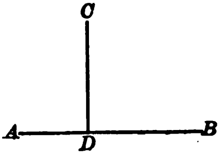
Thus, the angle ABC is an obtuse angle, and the angle ABD an acute angle.

PARALLEL AND PERPENDICULAR LINES.

369. Two lines are **Parallel** when they lie in the same plane and cannot meet however far they may be extended; as AB and CD.



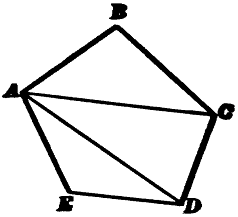
Parallel lines are everywhere the same distance apart.



370. Two lines are **Perpendicular** to each other when they form one or more right angles; as AB and CD.

PLANE FIGURES.

371. A **Polygon** is a plane figure bounded by straight lines called **Sides.**



The points where the sides meet are the **Vertices** of the polygon.

Thus, *AB*, *BC*, *CD*, *DE*, and *EA* are the sides, and the points *A*, *B*, *C*, *D*, and E, the vertices of the polygon *ABCDE*.

The **Perimeter** of a polygon is the sum of the sides.

A line joining two vertices of a polygon and not forming a side is a **Diagonal** of the polygon; as *AD* or *AC*.

372. A polygon of three sides is a **Triangle**; one of four sides, a **Quadrilateral**; of five sides, a **Pentagon**; of six sides, a **Hexagon**; of seven sides, a **Heptagon**; of eight sides, an **Octagon**; of nine sides, an **Enneagon**; of ten sides, a **Decagon.**

TRIANGLES.

373. A **Triangle** is a polygon of three sides.

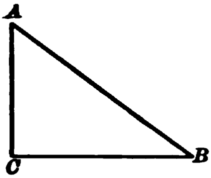


FIG. 1.

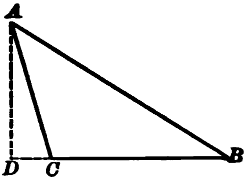


FIG. 2.

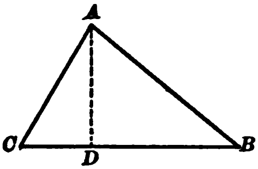


FIG. 3.

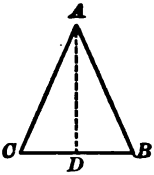


FIG. 4.

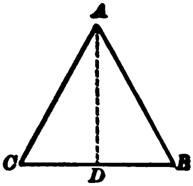


FIG. 5.

Thus, Figs. 1, 2, 3, 4, and 5 are triangles.

The **Base** of a triangle is the side upon which it is supposed to stand; as BC in the above triangles.

The **Altitude** of a triangle is the perpendicular drawn from the vertex of the angle opposite the base to the base or the base extended.

Thus, in Fig. 1, AC is the altitude, and in Figs. 2, 3, 4, and 5, AD is the altitude.

374. A **Right Triangle** [Fig. 1] is a triangle having a right angle. The side AB opposite the right angle C is called the **Hypotenuse**, and the sides AC and BC, the **Legs**.

375. An **Obtuse Triangle** [Fig. 2] is a triangle having an obtuse angle. In an obtuse triangle the altitude meets the base extended.

376. An **Acute Triangle** [Fig. 3, 4, or 5] is a triangle whose angles are all acute.

377. A **Scalene Triangle** [Fig. 3] is a triangle whose sides are unequal.

378. An **Isosceles Triangle** [Fig. 4] is a triangle two and only two of whose sides are equal. The equal sides are called the **Legs**.

379. An **Equilateral Triangle** [Fig. 5] is a triangle all of whose sides are equal.

380. *IMPORTANT TRUTHS.*

1. *The sum of the angles of a triangle equals 180°.*
2. *The angles opposite the legs of an isosceles triangle are equal.*
3. *Each angle of an equilateral triangle equals 60°.*
4. *The altitude of an isosceles triangle or of an equilateral triangle bisects the base and the angle opposite the base.*

5. *If a right triangle has an angle of 30°, the side opposite this angle equals one half the hypotenuse.*

6. *The square of the number of units in the hypotenuse of a right triangle equals the square of the number of units in one leg plus the square of the number of units in the other.*

7. *The square of the number of units in either leg of a right triangle equals the square of the number of units in the hypotenuse minus the square of the number of units in the other leg.*

AREA.

381. The **Area** of a plane figure is the amount of surface it contains expressed in square units.

382. *The number of square units in the area of a triangle equals one-half the number of units in the altitude times the number of units in the base.*

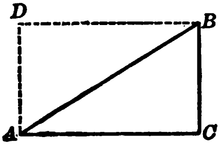


FIG. 1.

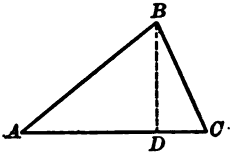


FIG. 2.

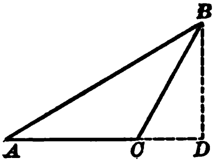


FIG. 3.

PROOF. Let the number of units in the base AC of the triangle ABC be 16 in Figs. 1 and 2, and 12 in Fig. 3; and let the number of units in the altitude of each be 9.

1. In Fig. 1, the right triangle $ABC = \frac{1}{2}$ the rectangle $DBCA$; for the triangle $ABD =$ the triangle ABC, which may be shown by placing one upon the other so as to make the sides about the right angles coincide.

Now, $9 \times 16 =$ the number of square units in the rectangle $DBCA$ [§ 194].

$\therefore \frac{1}{2}$ of $9 \times 16 =$ the number of square units in the right triangle ABC.

2. In Fig. 2, let 12 be the number of units in AD and 4 the number of units in DC.

Now, $\frac{1}{2}$ of $9 \times 12 =$ the number of square units in the right triangle ABD.

And, $\frac{1}{2}$ of $9 \times 4 =$ the number of square units in the right triangle DBC.

$\therefore$ $\frac{1}{2}$ of $9 \times (12 + 4)$, or $\frac{1}{2}$ of $9 \times 16 =$ the number of square units in the triangle ABC.

3. In Fig. 3, let 4 be the number of units in CD, then 16 will be the number of units in AD.

Now, $\frac{1}{2}$ of $9 \times 16 =$ the number of square units in the right triangle ABD.

And, $\frac{1}{2}$ of $9 \times 4 =$ the number of square units in the right triangle CBD.

$\therefore$ $\frac{1}{2}$ of $9 \times (16 - 4)$, or $\frac{1}{2}$ of $9 \times 12 =$ the number of square units in the triangle ABC.

383. To find the number of square units in the area of a triangle when the three sides are given, apply the following

Rule. *From half the sum of the number of units in the sides subtract the number of units in each side separately; multiply together the three remainders and the half sum, and extract the square root of the product.*

Illustration. Let it be required to find the area of a triangle whose sides are 5 ft., 7 ft., and 8 ft. long.

Solution. $\frac{1}{2}$ of $(5 + 7 + 8)$, or $10 =$ the number of units in $\frac{1}{2}$ the perimeter.

$\therefore$ $\sqrt{10 \times (10-5) \times (10-7) \times (10-8)}$, or $\sqrt{300}$, or $17.32 =$ the number of square feet in the area.

EXERCISE 131.

1. If two angles of a triangle are 36° and 59°, what is the size of the remaining angle?

2. If one of the equal angles of an isosceles triangle is 50°, what is the size of the angle opposite the base?

3. If the angle opposite the base of an isosceles triangle equals 50°, what is the size of each of the equal angles?

4. If one angle at the base of a right triangle equals 30° and the hypotenuse is 36 ft., what is the altitude?

5. One of the acute angles of a right triangle is 42° 15′; what is the size of the other acute angle?

6. One of the acute angles of a right triangle is three times the other; what is the size of each?

7. The angle *A* of the triangle *ABC* is *twice* the angle *B* or *three times* the angle *C*; what is the size of each angle?

8. The base of a right triangle is 36 ft. and the altitude 48 ft.; what is the length of the hypotenuse?

SUG. $h^2 = 36^2 + 48^2$ [§ 380, 6].

9. A ladder 35 ft. long is placed against a house so that the foot is 21 ft. from the base of the house and on the same level with it; how far does it reach up the house? [§ 380, 7.]

10. A pole 52 ft. high stands on level ground; how long a rope will it take to reach from the top to a stake 39 ft. from the foot of the pole?

11. The perpendicular drawn from the vertex of the right angle of a right triangle to the hypotenuse is 12 ft. long, and divides the hypotenuse into two parts 24 ft. and 6 ft. long, respectively; what is the length of each of the legs of the triangle?

12. Two ropes, one 42 ft. long and the other 50 ft. long, are fastened 30 feet above the ground to a pole and stretched to reach two stakes on opposite sides of the pole; how far are the two stakes apart, the surrounding ground being level?

13. Find the area of a triangle whose base is 20 ft. and whose altitude is 16 ft.

SOLUTION. $\frac{16}{2} \times 20$, or $160 =$ the number of square feet in the area [§ 382].

∴ 160 sq. ft. = the area [§ 381].

14. Find the number of acres in a triangular lot whose base is 10 ch. and whose altitude is 16 ch.

15. Find the area of a triangular lot whose sides are 13 rd., 14 rd., and 15 rd.

16. Find the area of an isosceles triangle one of whose legs is 40 ft. and one of whose equal angles is 30°.

SUG. Draw the altitude, then apply § 380, 5 and 7, to find the altitude and base.

17. Find the area of an isosceles triangle one of whose equal angles is 30° and whose altitude is 48 ft.

18. Find the area of an equilateral triangle one of whose sides is 5 rods.

19. Find the length of one side of a square that is equivalent to a triangle whose base is 25 ft. and whose altitude is 18 ft.

20. The area of a triangle whose base is 18 ft. is 436 sq. ft.; find its altitude.

21. The altitude of a triangle is 12 yd. and the area 60 sq. rd.; find the number of yards in the base.

22. What is the altitude of a triangle whose base is 16 ft., and whose area equals the area of a triangle whose base is 12 ft. and whose altitude is 9 ft?

23. The hypotenuse of a right triangle is 50 ch. and the altitude 30 ch.; find its area.

24. A square whose side is the hypotenuse of a right triangle contains 225 sq. yd., and another square whose side is the base of the same triangle contains 144 sq. yd.; what is the area of the triangle?

QUADRILATERALS.

384. A **Quadrilateral** is a polygon of four sides.

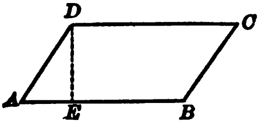


FIG. 1.

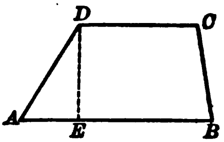


FIG. 2.

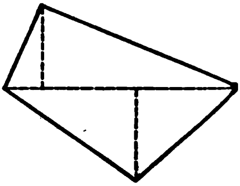


FIG. 3.

Thus, Figs. 1, 2, and 3 are quadrilaterals.

385. A **Parallelogram** [Fig. 1] is a quadrilateral whose opposite sides are parallel. The side *AB* on which it is supposed to rest is the **Base**, and the perpendicular *DE*, drawn from the side opposite the base to the base, is the **Altitude.**

386. A **Trapezoid** [Fig. 2] is a quadrilateral having only two sides parallel. The parallel sides *AB* and *DC* are the **Bases**, and the perpendicular *DE* joining the bases is the **Altitude.**

387. A **Trapezium** [Fig. 3] is a quadrilateral having no parallel sides.

PARALLELOGRAMS.

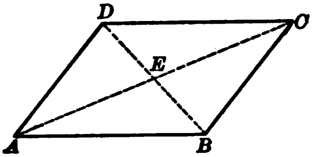


FIG. 4.

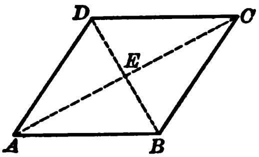


FIG. 5.

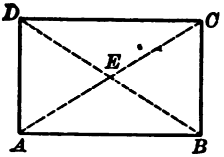


FIG. 6.

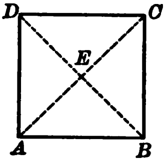


FIG. 7.

388. A **Rhomboid** [Fig. 4] is a parallelogram whose angles are not right angles and whose adjacent sides, as *AB* and *AD*, are unequal.

389. A **Rhombus** [Fig. 5] is a parallelogram whose angles are not right angles and whose sides are all equal.

390. A **Rectangle** [Fig. 6] is a parallelogram whose angles are all right angles.

391. A **Square** [Fig. 7] is a rectangle whose sides are all equal.

392. *IMPORTANT TRUTHS.*

1. *The opposite sides of a parallelogram are equal.*

2. *The opposite angles of a parallelogram are equal.*

3. *The sum of any two adjacent angles of a parallelogram is 180°.*

Thus, $\angle DAB + \angle ABC = 180°$, and $\angle ABC + \angle BCD = 180°$ [Figs. 4, 5, 6, and 7].

4. *The diagonals of a parallelogram bisect each other.*

Thus, $DE = EB$, and $AE = EC$ [Figs. 4, 5, 6, and 7].

5. *A diagonal of a parallelogram divides the parallelogram into two equal triangles.*

6. *The diagonals of a rectangle are equal.*

7. *The diagonals of a rhombus or of a square are perpendicular to each other.*

AREA.

393. *The number of square units in the area of a parallelogram equals the number of units in the altitude times the number of units in the base.*

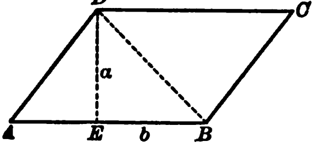


FIG 8.

PROOF. Let 8 represent the number of units in the base AB, and 5 the number of units in the altitude DE, of the parallelogram $ABCD$ [Fig. 8]. Draw the diagonal DB.

Now, $\frac{1}{2}$ of 5×8 = the number of square units in the triangle ABD.

But the parallelogram $ABCD$ = two times the triangle ABD [§ 392, 5].

$\therefore$ $2 \times \frac{1}{2}$ of 5×8, or 5×8 = the number of square units in $ABCD$.

394. *The number of square units in the area of a rhombus equals one-half the number of units in one diagonal times the number of units in the other diagonal.*

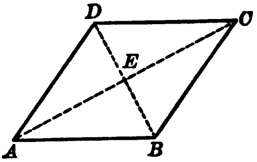


FIG. 9.

PROOF. Let 10 = the number of units in the diagonal AC, and 5 = the number of units in the diagonal BD, of the rhombus $ABCD$ [Fig. 9].

AC may be taken as the base and DE as the altitude of the triangle ADC [§ 392, 7].

$DE = \frac{1}{2}\ DB = \frac{1}{2}$ of 5 [§ 392, 4].

Now, $\frac{1}{2}$ of ($\frac{1}{2}$ of 5×10) = the number of square units in triangle ADC.

But rhombus $ABCD$ = 2 times triangle ADC [§ 392, 5].

$\therefore 2 \times \frac{1}{2}$ of ($\frac{1}{2}$ of 5×10), or $\frac{1}{2}$ of 5×10 = the number of square units in $ABCD$.

TRAPEZOID.

395. *The area of a trapezoid equals half the number of units in the altitude times the number of units in the sum of the bases.*

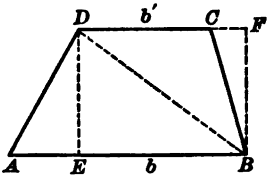


FIG. 10.

PROOF. Draw DB, the diagonal of the trapezoid $ABCD$. Draw DE, the altitude of the triangle ADB, and BF, the altitude of the triangle DBC [Fig. 10].

Now, $BF = DE$ [§ 369] = the altitude of the trapezoid.

Let 10 = the number of units in the base AB, 6 = the number of units in the base DC. and 5 = the number of units in the altitude DE or BF.

$\frac{1}{2}$ of 5×10 = the number of square units in triangle ADB.

$\frac{1}{2}$ of 5×6 = the number of square units in triangle BCD.

But, triangle ADB + triangle BCD = trapezoid $ABCD$.

$\therefore \frac{1}{2}$ of $5 \times (10 + 6)$ = the number of square units in $ABCD$.

TRAPEZIUM.

396. *The area of a trapezium equals the sum of the areas of the two triangles formed by drawing a diagonal.*

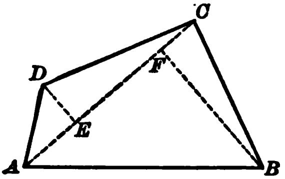


FIG. 11.

Thus, the area of the trapezium $ABCD$ equals the sum of the areas of the triangles ACD and ABC.

EXERCISE 132.

1. The angle A of the parallelogram $ABCD$ [Fig. 8] is 48° ; what is the size of each of the other angles of the parallelogram?

2. The base of a rectangle is 48 ft. and the altitude 36 ft.; what is the diagonal?

3. The diagonal of a rectangle is 65 ft. and the base 52 ft.; what is the altitude and what the area?

4. One diagonal of a rhombus is 24 yd. and the other 32 yd.; what is the length of one side and what the area?

5. Find the length of the diagonal of a square whose side is 23 ft.

6. Find the length and the area of a square whose diagonal is 36 ft.

7. Find the area of a parallelogram whose base is 36 ch. and whose altitude is 28 ch.

8. The area of a parallelogram is 180 sq. rd. and the base is 60 rd.; what is the altitude?

9. The area of a field in the form of a rhomboid is 9 A. 96 P. and the altitude is 32 rd.; what is the base?

10. The perimeter of a rectangle is 112 ft. and the width is 24 ft.; what is the area and what the diagonal?

11. One side of a rhombus is 48 ft. and one angle 30°; find the area.

12. The area of a rhombus is 90 perches and one diagonal is 10 rd.; what is the other diagonal?

13. A square lot contains an acre; what is its length to the nearest hundredth of a yard?

14. Find the area of a trapezoid $ABCD$, if the base AB is 24 ft., the base DC 18 ft., and the altitude DE 9 ft.

15. Find the area of the trapezium $ABCD$ [Fig. 11], if the diagonal AC is 10 ft., the perpendicular DE 2 ft., and the perpendicular BF 6 ft.

16. The four sides of a tract of land are $AB = 14$ ch., $BC = 18$ ch., $CD = 22$ ch., and $DA = 20$ ch., and the diagonal $AC = 30$ ch.; find the area.

17. How many board feet are there in a board 16 ft. long, 18 in. wide at one end and 14 in. wide at the other end?

18. How many square feet are there in the face of a wall 40 ft. long, 8 ft. high at one end, and 6 ft. high at the other end?

19. A piece of carpet 18 ft. long is cut bias at an angle of 30°, and one end is $4\frac{1}{2}$ ft.; how many square yards are there in the piece?

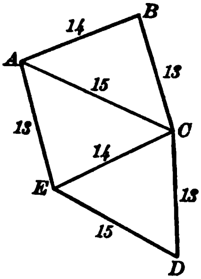
20. A rectangular field contains 5 acres, and the length is to the width as 3 to 2; how many panels of fence 8 ft. long will be required to fence it?

21. Find the area of a rhombus whose base and shorter diagonal are each one foot.

22. What is the altitude of a parallelogram 36 ft. long, that contains as much surface as a parallelogram 32 ft. long and 24 ft. wide?

23. Find the side of a square that contains as much surface as a trapezoid whose bases are 16 ft. and 9 ft., and whose altitude is 8 ft.

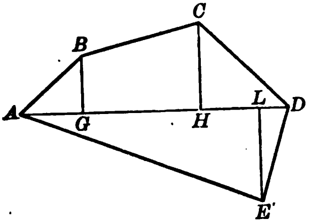
24. A roof is 50 ft. long and 14 ft. wide on each side; find the cost of slating it at \$7 per square (100 sq. ft.).



25. Find the area of the polygon $ABCDE$ in which $AB = 14$ rd., $BC = 13$ rd., $CD = 13$ rd., $DE = 15$ rd., $EA = 13$ rd., $AC = 15$ rd., and $EC = 14$ rd.

Note. The area of any polygon may be found by dividing it into triangles and taking the sum of their areas.

26. One side of a quadrilateral field measures 24 ch. 80 li.; another side opposite and parallel to it measures 18 ch., 40 li; if the distance between these sides is 12 ch. 60 li., find the number of acres in the field.



27. Find the area of the polygon $ABCDE$ in which $AG = 2$ ch., $GH = 4$ ch., $HL = 2$ ch., $LD = 1$ ch., and the perpendiculars $BG = 2$ ch., $CH = 3$ ch., and $EL = 3$ ch.

Note. The area of a polygon may be found by dividing it into right triangles and trapezoids, and taking the sum of their areas.

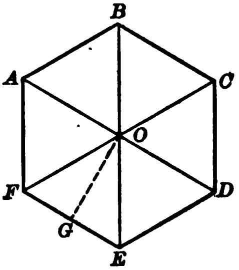
28. Draw three trapezoids of different shapes, having the sum of the parallel sides in each 4 in. and the altitude 2 in. Draw the first trapezoid having two right angles, the second two obtuse angles adjacent to the same base, and the third two obtuse angles not adjacent to the same base. Find the area of each.

29. The diagonals of a rhombus are 8 ft. and 6 ft., respectively; find the perimeter.

30. One diagonal of a field in the form of a trapezium is 24^m, and the perpendiculars from the opposite vertices to this diagonal are 42^m and 86^m, respectively; find the area in ares.

REGULAR POLYGONS AND CIRCLES.

397. A **Regular Polygon** is a polygon whose sides and whose angles are equal.



A REGULAR POLYGON.

The **Center** of a regular polygon is a point within, equally distant from the vertices; as the point O.

A **Radius** of a regular polygon is a line drawn from the center to a vertex; as OA.

An **Apothem** of a regular polygon is a perpendicular drawn from the center to a side; as OG.

398. *IMPORTANT TRUTHS.*

1. *The radii of a regular polygon are equal.*
2. *The apothems of a regular polygon are equal.*
3. *The radii of a regular polygon divide it into equal isosceles triangles.*
4. *The side of a regular hexagon equals the radius.*
5. *The apothem of a regular hexagon equals* $\frac{1}{2}\sqrt{3}$ *times the radius.*

AREA.

399. *The number of square units in the area of a regular polygon equals one-half the number of units in the apothem times the number of units in the perimeter.*

Let it be required to find the area of the regular polygon $ABCDEF$ [§ 397], when $EF = 6$ in. and $OG = 3\sqrt{3}$ in.

The perimeter $= 6 \times 6$ in., or 36 in.

$\frac{3}{2}\sqrt{3} \times 6 =$ the number of square units in triangle OEF.

$6 \times$ triangle $OEF =$ the regular polygon.

$\therefore 6 \times \frac{3}{2}\sqrt{3} \times 6 =$ the number of square units in $ABCDEF$.

That is, $\frac{3}{2}\sqrt{3} \times 36$ sq. in. = the area of the regular polygon.

EXERCISE 133.

1. Find the area of a regular hexagon, one of whose sides is 5 ft.

2. Find the area of a regular hexagon whose radius is 6 yd.

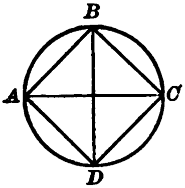
3. Find the area of a regular hexagon whose apothem is 8 rd.

4. Find the area of a regular pentagon, one of whose sides is 11.76 ft., and whose apothem is 8.09 ft.

5. Find the area of a regular octagon, one of whose sides is 3.83 yd., and whose apothem is 4.62 yd.

6. Find the radius of a circle whose area is $113\frac{1}{7}$ sq. ft.

Note. For treatment of the circle, see p. 178.

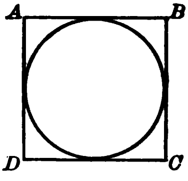


An Inscribed Square.

7. Find the side of the square *ABCD* inscribed in a circle whose diameter *AC* is 16 in.

8. Find the difference between the circumference of the circle and the perimeter of the inscribed square, when the diameter of the circle is 16 in.

9. Find the difference between the area of the circle and the area of the inscribed square, when the diameter of the circle is 16 in.



A Circumscribed Square.

10. Find the perimeter and area of a square circumscribed about a circle whose diameter is 8 ft.

11. Find the difference between the circumference of a circle and the perimeter of a circumscribed square whose side is 12 ft.

12. Find the difference between the area of a circle and the area of a circumscribed square whose length is 10 ft.

13. Find the area of the largest square that can be cut from a circular board whose diameter is 20 in.

Sug. The diameter of the circle = the diagonal of the square.

14. Find the area of the largest circle that can be cut from a square board 15 in. long.

15. How much farther is it around the edge of a square pond that contains 1386 sq. rd. than around a circular pond of the same size?

16. A cow is fastened by a rope 20 yd. long to a stake in the center of a square lot containing 1600 sq. yd.; over how much land can she graze?

17. What is the radius of a circle whose circumference equals the sum of the circumferences of two circles whose radii are 6 ft. and 3 ft.?

18. What is the diameter of a circle whose area equals the sum of the areas of two other circles whose diameters are 8 ft. and 6 ft.?

19. Find the number of feet in the diameter of a circle whose circumference is 1 mile.

20. If the length of a quadrant is $1\frac{4}{7}$ ft., what is the area of the circle?

21. The apothem of a regular octagon is 1.2071 times one side; find the area of a regular octagon whose perimeter is 32 ft.

Mensuration of Solids.

400. A **Solid** is any bounded portion of space; as a cube or a cylinder.

401. The **Volume** of a solid is the amount of space it contains expressed in cubic units.

402. If a solid is bounded by plane figures, these figures are the **Faces** of the solid.

403. The lines in which the faces meet are the **Edges** of the solid, and the points in which the edges meet are the **Vertices**.

RIGHT PRISMS.

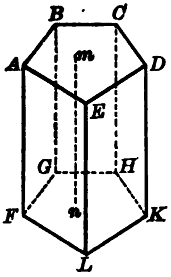


FIG. 1.

404. A **Right Prism** is a solid bounded by two equal and parallel polygons, called **Bases**, and rectangles, called the **Lateral Faces.** The lines in which the lateral faces meet are the **Lateral Edges.**

Thus, *ABCDE* – *FGHKL* is a right prism whose bases are the equal and parallel polygons *ABCDE* and *FGHKL*, whose lateral faces are the rectangles *AELF*, *EDKL*, *CDKH*, *BCHG*, and *ABGF*, and whose lateral edges are *AF*, *EL*, *DK*, *CH*, and *BG*.

The **Altitude** of a right prism is the perpendicular line joining the bases, as *mn*.

The **Lateral Area** of a right prism is the sum of the areas of the lateral faces.

Note. When the base of a right prism is a regular polygon, the figure is called a **Regular Prism.**

405. A **Rectangular Solid** is a right prism all of whose faces are rectangles [Fig. 2].

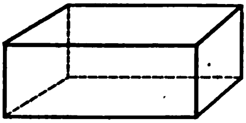


FIG. 2.

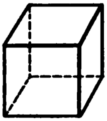


FIG. 3.

406. A **Cube** is a rectangular solid all of whose faces are equal squares [Fig. 3].

Note. For treatment of the cylinder, see p. 179.

407. *IMPORTANT TRUTHS.*

1. *The number of square units in the lateral area of a right prism equals the number of units in the altitude times the number of units in the perimeter of the base.*

2. *The number of cubic units in the volume of a right prism equals the number of units in the altitude times the number of square units in the base.*

EXERCISE 134.

1. Find the lateral area of a right prism the perimeter of whose base is 24 ft. and whose altitude is 3 ft.

SOLUTION. 3×24, or $72 =$ the number of sq. ft. in the lateral area [§ 407, 1]. $\therefore$ 72 sq. ft. = the lateral area.

2. Find the volume of a right prism the area of whose base is 14 sq. ft. and whose altitude is 7 ft.

SOLUTION. 7×14, or $98 =$ the number of cu. ft. in the volume [§ 407, 2]. $\therefore$ 98 cu. ft. = the volume.

3. Find the lateral area of a right prism whose base is a square 4 ft. long and whose altitude is 3 ft.

4. Find the volume of a right prism the area of whose base is 12 sq. ft. and whose altitude is 18 in.

5. Find the area of the entire surface of a regular square prism one side of whose base is 5 ft. and whose altitude is 4 ft.

6. Find the lateral area of a regular hexagonal prism one side of whose base is 8 ft. and whose altitude is 10 ft.

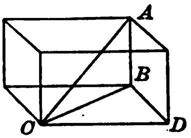
7. What is the altitude of a prism whose volume is 288 cu. ft. and the area of whose base is 48 sq. ft.?

8. What is one side of the base of a regular square prism whose volume is 150 cu. ft. and whose altitude is 6 ft.?

9. Find the weight of a pillar of uniform size whose base is a circle with a radius of 12 in. and whose height is 14 ft., if 1 cu. ft. of granite weighs $165\frac{3}{4}$ lb.

10. What must be the depth of a bin 10 ft. long and 6 ft. wide to hold 10 tons of wheat, regarding 1 bu. as equal to $\frac{4}{5}$ of a cu. ft.?

11. What is the distance from the upper corner of a room to the opposite lower corner, if the length of the room is 16 ft., the width 14 ft., and the height 12 ft.?



SUG. $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$ and $\overline{BC}^2 = \overline{BD}^2 + \overline{CD}^2$; whence, $\overline{AC}^2 = \overline{AB}^2 + \overline{BD}^2 + \overline{CD}^2$.

Note. The line AC is the diagonal of the room.

12. Find the diagonal of a cube 6 ft. long.

13. Find the number of square feet in the surface of a cube whose diagonal is 8 ft.

14. How many cartloads (cubic yards) must be removed in digging a well 60 ft. deep, if the diameter is 5 ft.?

PYRAMIDS AND CONES.

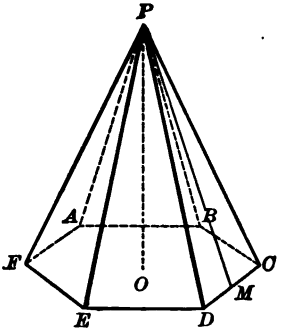


FIG. 1.

408. A **Regular Pyramid** is a solid bounded by a regular polygon, called the **Base**, and a number of equal isosceles triangles, called the **Lateral Faces**, which meet at a point called the **Vertex** or **Apex.**

Thus, $P - ABCDEF$ [Fig. 1] is a regular pyramid. $ABCDEF$ is the base, P the vertex, and the equal isosceles triangles PAB, PBC, PCD, PDE, PEF, and PFA are the lateral faces.

The sum of the areas of the lateral faces is the **Lateral Area.**

The **Altitude** is the perpendicular drawn from the vertex to the center of the base; as PO.

The **Slant Height** is a line drawn from the vertex to the middle point of a side of the base; as PM.

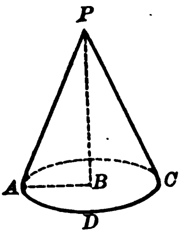


FIG. 2.

409. A **Right Cone** is a solid formed by revolving a right triangle about one of its legs as an axis.

Thus, $P - ADC$ [Fig. 2], formed by revolving the right triangle PAB about PB as an axis, is a right cone.

The curved surface formed by the hypotenuse PA is the Lateral Surface and its area the Lateral Area.

The circle formed by the base *AB* is the Base of the cone.

The point *P* is the Vertex, or Apex, of the cone.

A straight line, *PC*, drawn from the vertex to the circumference of the base is the Slant Height of the cone.

410. *IMPORTANT TRUTHS.*

1. *The number of square units in the lateral area of a regular pyramid equals one-half the number of units in the slant height times the number of units in the perimeter of the base.*

2. *The number of square units in the lateral area of a right cone equals one-half the number of units in the slant height times the number of units in the circumference of the base.*

3. *The number of cubic units in the volume of a regular pyramid or of a right cone equals one-third the number of units in the altitude times the number of square units in the base.*

EXERCISE 135.

1. Find the lateral area of a regular pentangular pyramid one side of whose base is 6 ft., and whose slant height is 8 ft.

SOLUTION.

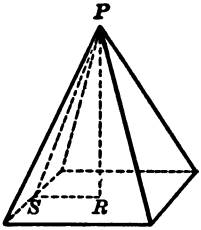
5×6, or 30 = the number of units in the perimeter of the base.
$\frac{1}{2}$ of 8, or 4 = " " " " " half the slant height.
∴ 4×30, or 120 = " " " sq. units in the lateral area [§410, 1].
∴ 120 sq. ft. = the lateral area.

2. Find the volume of a right cone, the diameter of whose base is 28 in. and whose altitude is 48 in.

SOLUTION. $\frac{1}{2}$ of 28, or 14 = the number of inches in the radius.
$\frac{22}{7} \times 14^2$, or 616 = " " " square inches in the base.
$\frac{1}{3}$ of 48, or 16 = " " " inches in $\frac{1}{3}$ of the altitude.
∴ 16×616 = " " " cu.in. in the volume [§410, 3].
Or, $\frac{16 \times 616}{1728}$, or $5\frac{19}{27}$ = " " " cu. ft. " " "
∴ $5\frac{19}{27}$ cu. ft. = the volume.

3. Find the lateral area of a right cone, the circumference of whose base is 26 in. and whose slant height is 15 in.

4. Find the volume of a regular square pyramid, the area of whose base is 42 sq. in., and whose altitude is 18 in.

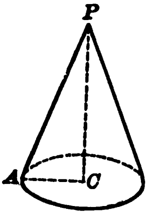


5. A side of a regular square pyramid is 6 ft., the altitude *PR* is 8 ft.; find the length of the slant height *PS*.

Sug. *PRS* is a right triangle.

6. Find the lateral area of a regular square pyramid, one side of whose base is 18 in. and whose altitude is 12 in.

7. Find the lateral surface of a regular hexagonal pyramid, one side of whose base is 6 in. and whose slant height is 20 in.



8. The slant height of a right cone, the radius of whose base is 6 in., is 10 in.; find the altitude.

9. Find the cubical contents (volume) of a right cone, the radius of whose base is 12 in. and whose slant height is 15 in.

10. Find the area of the entire surface of a right cone, the slant height of which is 25 in. and the altitude 20 in.

11. Find the slant height of a regular pyramid whose lateral area is 192 sq. in., and the perimeter of whose base is 24 in.

12. Find the altitude of a right cone whose volume is 352 cu. in., and whose diameter is 8 in.

13. Find a side of the base of a regular square pyramid whose volume is 300 cu. in., and whose altitude is 9 in.

14. Find the radius of the base of a right cone whose lateral surface is 47⅐ sq. ft., and whose slant height is 5 ft.

15. A conical wine glass 3 in. deep and 3 in. wide at the top is ⅔ full of water; how many cubic inches of water in it?

16. How many square yards of canvas are required to make a conical tent 18 ft. in diameter at the base and 12 ft. high?

17. The slant height of a conical piece of lead is 5^{cm} and the diameter of its base is 6^{cm}; find its volume in cubic centimeters.

18. Find the volume of a regular pyramid whose base is a square containing 36^{qcm} and whose slant height is 5^{cm}.

FRUSTUMS.

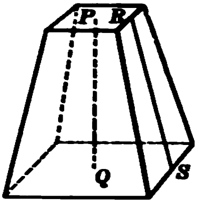


FIG. 1.

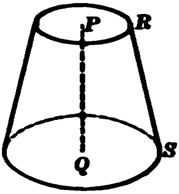


FIG. 2.

411. The **Frustum** of a regular pyramid or of a right cone is the part between the base and a plane parallel to the base.

Thus, Fig. 1 is the frustum of a regular square pyramid and Fig. 2 the frustum of a right cone.

The base of the pyramid or of the cone and the face parallel to the base are the **Bases** of the frustum.

Note. The frustum of a regular pyramid or of a right cone is a **Regular Frustum.**

The **Altitude** of a regular frustum is the line joining the centers of the bases of the frustum; as *PQ*.

The **Slant Height** of a regular frustum is that part of the slant height of the regular pyramid or of the right cone included beween the bases of the frustum; as *RS*.

412. The **Lateral Surface** of a regular frustum is the entire surface less the bases, and the area of the lateral surface is the **Lateral Area.**

413. To find the number of square units in the lateral area of a regular frustum:

RULE: — *Multiply the number of units in the sum of the perimeters or circumferences of the bases by half the number of units in the slant height.*

414. To find the number of cubic units in the volume of a frustum:

RULE: — *Multiply the number of square units in the lower base plus the number of square units in the upper base plus the square root of the product of these numbers by one-third the number of units in the altitude.*

EXERCISE 136.

1. Find the lateral area of the frustum of a right cone, the diameters of whose bases are 5 ft. and 3 ft., and whose slant height is 8 ft.

SOLUTION. $\frac{22}{7} \times 5 + \frac{22}{7} \times 3$, or $\frac{22}{7} \times 8$, or $\frac{176}{7}$ = the number of feet in the circumferences of the bases.

$\frac{1}{2}$ of 8, or 4 = the number of feet in half the slant height.

$\therefore 4 \times \frac{176}{7}$, or $\frac{704}{7}$, or $100\frac{4}{7}$ = the number of sq. feet in the lateral area.

$\therefore 100\frac{4}{7}$ sq. ft. = the lateral area.

2. Find the volume of the frustum of a regular square pyramid, the areas of whose bases are 9 sq. in. and 4 sq. in., and whose altitude is 6 in.

SOLUTION. $\frac{1}{3}$ of $6 \times (9 + 4 + \sqrt{9 \times 4})$, or 2×19, or 38 = the number of cubic inches in the volume.

3. Find the lateral area of the frustum of a regular hexagonal pyramid, the sides of whose bases are 10 in. and 5 in., respectively, and whose slant height is 8 in.

4. Find the volume of the frustum of a right cone, the radii of whose bases are 16 in. and 10 in., and whose altitude is 5 in.

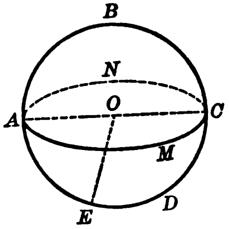
5. How many square feet of zinc will be required to line the sides and bottom of an ash hopper 5 ft. square at the top and 3 ft. square at the bottom, if the slant height is 6 feet?

6. How many cubic inches of water will a vessel hold that is 6 in. in diameter at the top, 4 in. in diameter at the bottom, and 9 in. deep?

7. How many quarts of milk, to the nearest hundredth, will a tin bucket hold whose top diameter is 14 in., bottom diameter 10 in., and depth 8 in.?

8. Find, correct to the nearest gallon, the capacity of a vat in the form of a frustum of a cone, if the circumferences of its bases are $37\frac{5}{7}$ ft. and $18\frac{6}{7}$ ft., respectively, and its slant height 5 ft.

SPHERES.



SPHERE.

415. A **Sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the **Center.**

A **Diameter** of a sphere is a straight line drawn from surface to surface through the center; as *AC*.

A **Radius** of a sphere is a line drawn from the center to the surface. It is half a diameter; as *OC*.

A **Circumference** of a sphere is the circumference of a circle whose radius is the radius of the sphere; as *AMCN*.

416. *IMPORTANT TRUTHS.*

1. *The number of square units in the area of a sphere is* $\frac{88}{7}$ *times the square of the number of units in the radius.*

2. *The number of cubic units in the volume of a sphere equals* $\frac{88}{21}$ *times the cube of the number of units in the radius.*

EXERCISE 137.

1. Find the area of a sphere whose radius is 6 in.

Sug. $\frac{88}{7} \times 6^2 =$ the number of sq. in. in the area of the sphere.

2. Find the volume of a sphere whose diameter is 8 in.

Sug. $\frac{88}{21} \times 4^3 =$ the number of cu. in. in the volume of the sphere.

3. Find the area of a sphere whose diameter is $2\frac{1}{3}$ ft.

4. Find the volume of a sphere whose diameter is 7 ft.

5. Find the radius of a sphere whose area is 154 sq. ft.

Sug. $\frac{88}{7} \times r^2 = 154$.

6. Find the diameter of a sphere whose volume is $1437\frac{1}{3}$ cu. in.

7. The radius of a sphere is 14 in.; find its circumference.

8. The circumference of a sphere is 66 in.; find its diameter.

9. The circumference of a sphere is 132 in.; find its area and its volume.

10. The area of a sphere is 9856 sq. in.; find its circumference.

11. The volume of a sphere is 113⅐ cu. ft.; find its circumference.

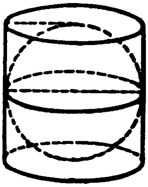
12. The area of a sphere is 7546 sq. in.; find its volume.

Sug. First find the radius.

13. The volume of a sphere is 179⅔ cu. ft.; find its area.

14. What is the diameter of a ball that weighs as much as 8 balls of the same material, each 1 in. in diameter?

15. Find the diameter of a ball whose area equals the sum of the areas of 16 balls, each 4 in. in diameter.



An Inscribed Sphere.

16. Find the area of a sphere inscribed in a cylinder, the diameter of whose base is 6 in. and whose altitude is 6 in.

17. Find the volume of a cylinder circumscribed about a sphere whose surface is 176 sq. ft.

18. A ball 3 in. in diameter was dropped in a cylindrical mug, 3 in. in diameter and 3 in. high, full of water; how many cubic inches of water remained in the mug?

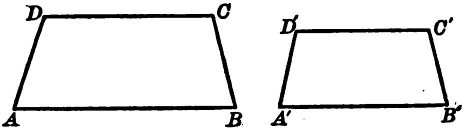
SIMILAR FIGURES.

417. Similar Figures are such as have the same shape. In such figures lines and angles that are like placed are said to be **Corresponding** or **Homologous.**

418. *In similar polygons the homologous angles are equal and the homologous lines are proportional.*

Thus, in the similar trapezoids $ABCD$ and $A'B'C'D'$.

$$\angle A = \angle A'$$
$$\angle B = \angle B'$$
$$\angle C = \angle C'$$
$$\angle D = \angle D'.$$



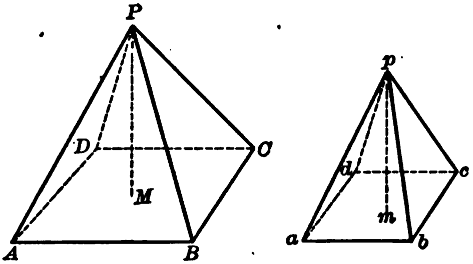
Also, $AB : A'B' = BC : B'C' = CD : C'D' = DA : D'A'$.

419. *All regular polygons of the same number of sides are similar. All circles are similar.*

Thus, all equilateral triangles are similar, all squares are similar, all regular pentagons are similar, etc.

420. *In similar solids all homologous faces are similar.*

Thus, in the similar pyramids $P-ABCD$ and $p-abcd$, base $ABCD$ is similar to base $abcd$, face PAB is similar to face pab, etc.



421. *In similar solids homologous lines are proportional.*

Thus, $PA : pa = AB : ab = PM : pm$, etc.

422. *All cubes are similar and all spheres are similar.*

423. *IMPORTANT TRUTHS.*

1. *The perimeters or circumferences of two similar figures are to each other as any homologous lines.*

2. *The areas of two similar surfaces are to each other as the squares of any two homologous lines.*

3. *The lateral areas or the entire areas of two similar solids are to each other as the squares of any two homologous lines.*

4. *The volumes of two similar solids are to each other as the cubes of any two homologous lines.*

5. *The amounts of water or other liquid that flow through two similar orifices are to each other as the squares of the homologous dimensions (lines) of the orifices.*

EXERCISE 138.

1. Two homologous sides of two similar triangles are 4 ft. and 6 ft., and the altitude of the first is 3 ft; what is the homologous altitude of the second?

SUG. Let x = the number of feet in the altitude of the second; then, since the homologous altitudes are to each other as the homologous sides [§ 418],

$$x \text{ ft.} : 3 \text{ ft.} = 6 \text{ ft.} : 4 \text{ ft., or } x : 3 = 6 : 4.$$

2. The diameters of two circles are 7 ft. and 14 ft., and the area of the first is 38.5 sq. ft.; what is the area of the second?

SUG. Let x = the number of sq. ft. in the area of the second. Since the areas are to each other as the squares of the diameters [§ 423, 2],

$$x : 38.5 = 14^2 : 7^2.$$

3. The altitudes of two similar pyramids are 3 ft. and 5 ft., and the lateral area of the second is 169 sq. ft.; what is the lateral area of the first?

SUG. Let x = the number of sq. ft. in the lateral area of the first. Since the lateral areas are to each other as the squares of the altitudes [§ 423, 3],

$$x : 169 = 3^2 : 5^2.$$

4. The diameters of two spheres are 8 ft. and 1 ft., and the volume of the second is $\frac{11}{21}$ cu. ft.; what is the volume of the first?

SUG. Let x = the number of cubic feet in the volume of the first. Since the volumes are to each other as the cubes of the diameters [423, 4],

$$x : \tfrac{11}{21} = 8^3 : 1^3.$$

5. The circumferences of two circles are 22 ft. and 66 ft., and the diameter of the first is 7 ft.; what is the diameter of the second?

SUG. Let x = the number of feet in the diameter of the second. Since the circumferences are to each other as the diameters,

$$22 : 66 = 7 : x.$$

6. The perimeters of two regular hexagons are 30 ft. and 36 ft., and the apothem of the second is $3\sqrt{3}$ ft.; what is the apothem of the first?

SUG. Let x = the number of feet in the apothem of the first. Since the perimeters are to each other as the apothems,

$$30 : 36 = x : 3\sqrt{3}.$$

7. The entire surfaces of two cubes are 96 sq. in. and 216 sq. in., and a side of the second is 6 in.; what is a side of the first?

8. The circumferences of two circles are $3\frac{1}{2}$ ft. and $1\frac{1}{2}$ ft., and the diameter of the first is $\frac{49}{44}$ ft.; what is the diameter of the second?

9. The volumes of two similar cones are 77 cu. ft. and 308 cu. ft., and the radius of the base of the first is 6 ft.; what is the radius of the base of the second?

10. The altitudes of two similar prisms are 3 in. and 5 in., and the entire surface of the first is 256 sq. in.; what is the entire surface of the second?

11. A farmer has two fields of the same shape, their lengths being 30 rd. and 20 rd.; if the width of the second is 8 rods, what is the width of the first?

12. The diameters of two cylindrical vessels of the same shape are 15 in. and 20 in.; if the second holds 81 gal., how much will the first hold?

13. How many times as much water will flow through an orifice 3 in. in diameter in a given time, as through one $1\frac{1}{2}$ in. in diameter?

14. If it takes 112 panels of fence to enclose a field 18 rd. long, how many panels will be required to enclose a similarly shaped field 24 rd. long?

15. If it takes a pipe 2 in. in diameter 18 hr. to empty a cistern, how long would it take a pipe $1\frac{1}{2}$ in. in diameter to empty it?

SUG. The time is inversely proportional to the quantity emptied per hr., and the quantity emptied per hr. is directly proportional to the squares of the diameters.

∴ the time is inversely proportional to the squares of the diameters.

16. If a cylindrical vessel whose inner diameter is $18\frac{1}{2}$ in. and depth 8 in., holds one bushel, what are the dimensions of a similarly shaped vessel that holds 27 bushels?

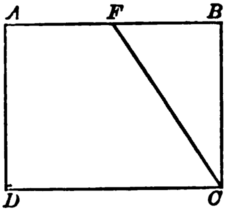
17. If a ball whose diameter is 6 inches has a surface of $452\frac{4}{7}$ square inches, what is the diameter of a ball whose surface is 16 times as great?

18. If the surface of a cube whose diagonal is 8 ft. is 128 sq. ft., what is the diagonal of a cube whose surface is 1152 sq. ft.?

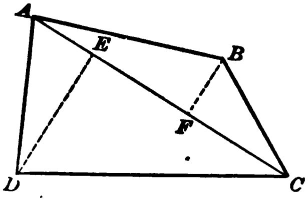
EXERCISE 139.

Miscellaneous Problems.

1. How many rods of fence are necessary to enclose a square tract of land containing 40 A. and divide it into 16 equal square lots?



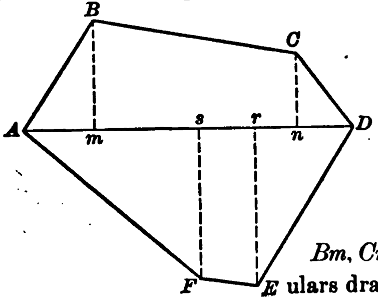
2. In the rectangular field *ABCD*, *AB* is 50 rd. and *BC* 40 rd. If *F* is the middle point of *AB* and a fence is built from *C* to *F*, how many acres will the triangle *FBC* contain? Find the length of the fence.



3. *ABCD* is a field whose diagonal *AC* is 18 ch., and the perpendiculars *DE* and *BF* are 12 ch. and 6 ch.; find the number of acres in the field.

4. Find the number of acres in the field *ABCDEF*, having given the following measurements:

$AB = 15$ rd.
$DC = 10$ rd.
$DE = 20$ rd.
$AF = 25$ rd.
$Am = 9$ rd.
$As = 20$ rd.
$Ar = 26$ rd.
$An = 32$ rd.
$AD = 38$ rd.



Bm, *Cn*, *Fs*, and *Er* are perpendiculars drawn to the diagonal.

5. How many square feet of boards are required for the sides and ends of a barn 50 ft. long, 36 ft. wide, and 24 ft. high to the gable, the ridge pole being 16 ft. higher than the gable?

6. Find the length of the hand rail required for a flight of stairs containing 15 steps, each 12 in. high and 9 in. wide.

7. A rug 24 ft. long and 18 ft. wide is placed in the center of a floor 28 ft. long and 22 ft. wide; how many square feet of the floor remain uncovered?

8. Find the length that must be cut from a board 18 in. wide in order that the piece cut off may contain 1 sq. ft.

9. A has a rectangular lot 60 ft. front and 120 ft. deep, on which he builds a house 48 ft. long and 42 ft. wide. The remainder of the lot is made into a yard; find its area.

10. A rectangular glass 36 in. long and 20 in. wide is surrounded by a frame 6 in. wide; find the number of square inches in the frame.

11. Find the number of leaves 6 in. long and 4 in. wide that can be cut from a sheet of paper 34 in. long and 26 in. wide, if the sheets are cut so as to make the least possible waste.

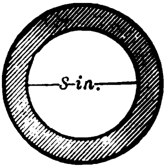
12. How many circular tops 4 in. in diameter can be cut for tin cans from a sheet of tin 30 in. long and 24 in. wide?

13. How many revolutions per minute are made by a locomotive wheel 16 ft. in circumference when the locomotive is running 45 mi. per hour?

14. The length of the minute hand of the clock on City Hall, Philadelphia, is 10 ft. 8 in.; how many feet does the point of this hand move in an hour?

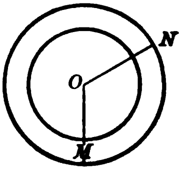
15. The diameter of the dial of the clock mentioned in example 14 is 22 ft.; find the area of the dial.

16. A line 50 ft. long extends from the top of a center pole 40 ft. high to the edge of a circus ring; find the area of the ring.



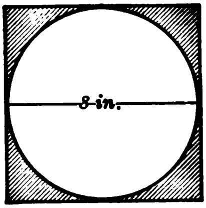
17. How many square inches are there in the surface of a frame 1 in. wide around a clock dial 8 in. in diameter?

18. How many ares are there in a piece of land in the form of a square one hectometer long?



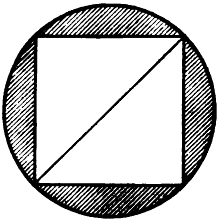
19. Find the number of square feet in the walk surrounding the circle whose radius *OM*, is 6 yd., if the distance *ON*, from the center of the circle to the outer edge of the walk, is 8 yd.

20. The carpet on a floor 16 ft. square is surrounded by a border which covers $\frac{15}{64}$ of the area of the floor; how many feet wide is the border?



21. From a square board whose side was 8 in., a circular board as large as possible was cut; how many square inches of surface were wasted?

22. Trees are planted 40 ft. apart along both sides of a street for a distance of half a mile; find the number of trees.



23. A square court as large as possible is made in a circle whose diameter is 66 ft., and the remainder of the circle sodded; how many square feet are sodded?

24. Find the number of square feet in an equilateral triangle whose altitude is 5 ft.

25. How many cubic yards of earth must be removed in digging a well 8 ft. in diameter and 40 ft. deep?

26. How many board feet of inch boards will it take to make a closed box 4 ft. long, 3 ft. high, and $3\frac{1}{2}$ ft. wide, outside measurement?

27. How many bushels of wheat would the box mentioned in example 26 hold? [§ 205, Note 2.]

28. How many bushels of corn in the ear will a corn-crib hold that is 36 ft. long, $3\frac{1}{2}$ ft. wide at the bottom, $4\frac{1}{2}$ ft. wide at the top, and 9 ft. high? [§ 205, Notes 3, 4.]

29. Find the weight of a granite block 4 ft. long, 3 ft. wide, and 18 in. high, if granite weighs 2.78 times as much as an equal bulk of water, and water weighs 62.5 lb. to the cubic foot.

30. Find the weight in kilograms of a column of water in a pipe 14^{m} high, whose inner diameter is 5^{dm}.

31. The diameters of the earth and moon are as 1 to $\frac{3}{11}$; how many times as large as the moon is the earth?

32. A cube whose edge was 20 in. was plated uniformly with 1261 cu. in. of copper; find the thickness of the plating.

33. How many bushels of ashes, heaped measure, will a hopper hold whose upper base is 4 ft. square, lower base 9 ft. square, and depth 6 ft., regarding 1 cu. ft. as equal to $\frac{16}{25}$ of a heaped bushel?

34. Find the depth of a cubical tank that holds 847 hhd.

SUG. 1 gal. = 231 cu. in.

35. For one inch of rainfall what weight of water falls on a square yard of ground, if 1 cu. ft. of water weighs 62.5 lb.?

36. What will it cost to cover with gold leaf worth 10 ct. per square inch a cube whose volume is 1728 cu. in.?

37. A cubic foot of iron was formed into a bar 4 in. square; find its length.

38. Find the cost, at 15 cents per square meter, of lining the sides and bottom of a cubical tank that will hold just 8^{MT} of water. [§ 219.]

39. Find the weight, to the nearest pound, of a hollow cylindrical roller made of cast iron, weighing 550 lb. per cubic foot, if it is 8 ft. long, its outer diameter 4 ft., and the thickness of the iron 2 in.

40. How many square yards of rubber cloth would be required to make 100 footballs 8 in. in diameter?

41. A globe 6 in. in diameter is covered with copper to the thickness of $\frac{1}{2}$ inch; find the value of the copper at eight cents per cubic inch.

Supplement.

TIMBER MEASURE.

424. Squared timber is usually sold by the cubic foot. The number of cubic feet in a stick is determined by the following:

RULE. *Multiply together the number of inches in the thickness, the number of inches in the width, and the number of feet in the length, and divide the product by 144.*

Thus, the number of cubic feet in a stick of timber 8 in. thick, 10 in. wide, and 16 ft. long, is

$$\frac{8 \times 10 \times 16}{144}, \text{ or } 8\tfrac{8}{9}.$$

425. One half of the sum of the end diameters of a log, excluding the thickness of the bark, is called its **Mean Diameter.**

426. When a log to be squared is sold by the cubic foot, the number of cubic feet — squared timber — in it is found by the following:

RULE. *Multiply four-ninths of the square of the number of inches in the mean diameter by the number of feet in the length and divide the product by 144.*

Thus, the number of cubic feet of squared timber in a log whose mean diameter is 16 in. and whose length is 18 ft., is

$$\frac{\frac{4}{9} \times (16)^2 \times 18}{144}, \text{ or } 14\tfrac{2}{9}.$$

Note 1. In some localities the number of cubic feet — squared timber — in a log is found by the following:

RULE. *Multiply half the square of the number of inches in the mean diameter by the number of feet in the length and divide the product by 144.*

Thus, the number of cubic feet of squared timber in a log whose mean diameter is 18 in. and whose length is 20 ft., is

$$\frac{\frac{1}{2} \times (18)^2 \times 20}{144}, \text{ or } 22\tfrac{1}{2}.$$

Note 2. If a log tapers suddenly at the butt, the diameter at the middle of the log is sometimes used instead of the mean diameter.

Note 3. In a log of inferior quality, the diameter of the smaller end is sometimes used instead of the mean diameter.

427. To determine the number of board feet of square-edged inch boards that can be sawed from a log, the following rule, called **Doyle's Rule**, is generally used:

RULE. *Subtract 4 from the number of inches in the diameter, square the result, divide by 16, and multiply the quotient by the number of feet in the length.*

Thus, the number of board feet of square-edged inch boards that can be sawed from a log 30 ft. long and 24 in. in diameter is

$$\frac{(24-4)^2}{16} \times 30, \text{ or } 750.$$

Note. By *the diameter* in the above rule is meant the mean diameter, the diameter at the middle of the log, or the diameter at the smaller end, depending on the agreement between the buyer and the seller.

EXERCISE 140.

1. Find the number of cubic feet in a stick of timber 6 in. thick, 9 in. wide, and 24 ft. long.

2. Find the number of cubic feet in a stick of timber 22 in. thick, 24 in. wide, and 35 ft. long.

3. Find the number of cubic feet — squared timber — in a log 36 ft. long, if the diameters of the ends are 34 in. and 20 in.

4. How many cubic feet — squared timber — would be estimated in the log mentioned in problem 3, if the rule stated in note 1, Art. 426, were applied?

5. Find the amount of square-edged inch boards that can be sawed from a log 16 in. in diameter and 18 ft. long.

6. Find the amount of square-edged inch boards that can be sawed from a log 12 in. in diameter and 20 ft. long.

7. Find the number of cubic feet — square timber — in a log 40 ft. long, if the diameters of the ends are 60 in. and 45 in.

8. Find the amount of square-edged inch boards that can be sawed from a log 25 in. in diameter and 35 ft. long. (Answer to the nearest foot.)

Find the amount of square-edged inch boards that can be sawed from the following:

9. A log 20 in. in diameter and 20 ft. long.
10. A log 24 in. in diameter and 36 ft. long.
11. A log 36 in. in diameter and 30 ft. long.
12. A log 40 in. in diameter and 16 ft. long.
13. A log 44 in. in diameter and 18 ft. long.
14. A log 60 in. in diameter and 32 ft. long.
15. A log 32 in. in diameter and 28 ft. long.
16. A log 22 in. in diameter and 12 ft. long.
17. A log 12 in. in diameter and 24 ft. long.
18. A log 24 in. in diameter and 24 ft. long.
19. A log 12 in. in diameter and 12 ft. long.
20. A log 24 in. in diameter and 12 ft. long.

MISCELLANEOUS PROBLEMS.

1. Write in figures and in words a number consisting of four units of the fifth order, two of the third, five of the second, and four of the first.

2. Write in words and in figures a number consisting of six units of the fourth period, twenty-four of the third, six hundred of the second, and one of the first.

3. Write in figures a number whose highest order is ten millions, point it off in periods of three figures each, and write the number in words.

4. Name the letters employed in the Roman system, and the number for which each stands.

5. Write the number of the present year by the Roman system.

6. Write in figures:

Twenty-seven million fifty thousand forty-six; seventy-five billion sixty million four hundred ninety; MCCIX; MDCCCXLIX; CDXCIV.

7. In 1900 the popular vote for McKinley was 7,208,244, for Bryan 6,358,789, for Woolley 208,835, for Debs 86,686, for Maloney 39,759, for Barker 50,373, for Ellis 5,698, and for Leonard 1,057; by how many did McKinley's vote exceed the combined vote for the others?

8. Which is nearer to a million dollars and by how much, $1,100,000 or $900,009?

9. If the cost of sending a telegram is 25 cents for ten words and two cents for each additional word, find the cost of sending a telegram of 36 words.

10. Multiply 7845 by 2478, and write in words the numbers represented by the four partial products.

11. A farmer bought 8 steers for $32 a head, and after feeding them for 6 months at a cost of $35 per month, sold them for $498; find the average gain per head.

12. In an election in which A and B were the only candidates for a given office, the total number of votes cast for both was 2028; if A was elected by a majority of 106, how many votes did each receive?

13. A bought 24,000 ft. of lumber at $32 per M, and gave in payment 1400 rails at $6 per C and the remainder in cash; how much cash did he pay?

14. The total amount paid to 993,529 pensioners in the United States for the year ending June 30, 1900, was $139,381,522; find to the nearest dollar the average amount paid each pensioner for that year.

15. In 1899 there were in the United States about 11,000,000 cows, which produced about 1,430,000,000 lb. of butter during that year; find to the nearest pound the average amount of butter per cow.

16. There were 20,281,334 bu. of wheat produced from 1,502,321 acres in Pennsylvania in 1900; find to the nearest bushel the average amount produced per acre.

17. After selling 6 loads of wheat, each containing 75 bu., a farmer had left 42 bu.; how much wheat had he at first?

18. A bankrupt who can pay $40 on $100 of his debts, owes B $2400; how much will B receive?

19. The quotient resulting from solving a problem in division is 24; what would the quotient be if both dividend and divisor were multiplied by 4? Why?

20. In shipping 100 bushels of wheat, which rate is cheaper, $0.05 per bushel (60 lb.) or 0.08\frac{3}{4}$ per 100 lb.? By how much is it cheaper?

21. The sum of two numbers is 984, and the larger is 62 more than the smaller; find the numbers.

22. The sum of two numbers is 1002, and the larger is twice the smaller; find the numbers.

23. A drover paid \$396 for an equal number of cows and calves, paying \$40 for each cow and \$4 for each calf; how many of each did he buy?

24. Simplify

$$\left\{(87 + 13) \times 5 + 10\right\} \times \left\{275 \div \overline{15 - 10} \times 4\right\}.$$

25. Give all the sets of factors of 24. How many of these sets are prime factors?

26. Tell whether or not each of the following numbers is (1) prime, (2) composite, (3) even, (4) odd; and give the reason for your answer:

18, 15, 23, 2, 1, 49, 81, 121, 93, 69.

27. Tell at sight which of the numbers 2, 3, 4, 5, and 9 will divide:

109, 111, 124, 819, 478, 215, 2340.

28. Find all the common divisors of 96 and 144.

29. Find the greatest common divisor of 1032 and 1505.

30. Find the sum of the prime numbers to 100.

31. The factors of the dividend are 7, 11, 2, 3, 2, and 5; and those of the divisor are 7, 11, and 5; find the quotient.

32. One boat leaves a certain wharf every Wednesday, and another leaves the same wharf every 10 days; if both left the wharf Jan. 18, on what date did they again leave on the same day?

33. What is the least sum of money that can be paid in either \$20 bills or \$50 bills?

34. Find the least number of dollars that a man can divide equally among his 3 sons, or among his 4 daughters, or among all his children.

35. A contractor has workmen enough to divide into groups of 12 men or 14 men, and in either case have one man left for overseer; how many groups of 12 men each can he form?

36. Supply the missing number:

$$\tfrac{2}{3} + 22\tfrac{1}{4} + ? + 148\tfrac{1}{12} + 6\tfrac{1}{6} = 200.$$

37. Simplify:

$$\frac{\frac{3}{4} \text{ of } \frac{4}{5} \div 1\frac{2}{3} + 1 - 16\frac{2}{3} \div 33\frac{1}{3} \times 2\frac{1}{2}}{100 \div 14\frac{2}{7} \times \frac{6}{7} - 4\frac{2}{3} + 1\frac{1}{4} \div 2}.$$

38. Find the diameter of a wheel that makes 168 revolutions in a half mile.

39. A can do a piece of work in 4 da. and B and A together can do it in 1 da.; in what time can B alone do it?

40. A can do a piece of work in 2 da., and B can do it in 5 da.; after they have worked together 1 da., in what time can A finish the work?

41. If A can cut a cord of wood in $\frac{3}{4}$ of a day, and B can cut a cord in $\frac{2}{3}$ of a day, in what time can they together cut a cord?

42. Divide .02002 by .002, and write the result in words.

43. Simplify $\frac{.02 \div .002}{.002 \div .02} \div \frac{.02 \div 200}{20 \div .02} \times \frac{.16\frac{2}{3} \div 8\frac{1}{3}}{100 \div 66\frac{2}{3}}$.

44. Ice is $\frac{10}{11}$ as heavy as water, and a cubic foot of water weighs $62\frac{1}{2}$ lb.; how many cubic feet of ice weigh 100 lb?

45. A dealer bought 10,000 cigars at $3.50 per C, and retailed them at the rate of 4 for 25 ct.; find his profit.

46. A passenger on a moving train observes that it takes him just 45 seconds to pass a freight train 1320 ft. long, standing on a side track; at what rate in miles per hour is his train running?

47. One boy is 125 yd. behind another and walks 450 yd. to overtake him; how many times as fast does the first boy walk as the second?

48. The minute hand of a clock moves how many times as fast as the hour hand?

49. The minute hand of a clock points to 3 and the time is between 7 and 8 o'clock; how many minute spaces must the minute hand gain on the hour hand to overtake it?

50. What part of a minute space does the minute hand of a clock gain on the hour hand in one minute? In 15 min.?

51. How long will it take the minute hand of a clock to gain 1 minute space on the hour hand? 5 minute spaces?

52. When it is eight o'clock, how many minute spaces is the minute hand of a clock behind the hour hand? How long will it take the minute hand to overtake the hour hand?

53. At what time after five o'clock are the hour and the minute hands of a clock together?

54. Two boys are 1 mile apart and are approaching each other. The first walks 3 rods every time the second walks 2 rods; how far must each walk before they meet?

55. One man is half a mile behind another, and takes 5 steps while the other takes 3 steps; how many steps must the first take to overtake the second, if their steps are $2\frac{1}{2}$ ft. long?

56. How many gallons of water must be mixed with 20 gal. of wine worth $3 per gallon, in order to reduce the value of the mixture to $2.50 per gallon?

57. How much alloy must be mixed with 3 pwt. 8 gr. of pure gold to make gold 18 carats fine?

SUG. Gold 18 carats fine is $\frac{18}{24}$ pure.

58. The distance from New York to Liverpool is 3050 nautical miles, or knots; find the distance in statute miles.

59. The weight of a common brick is about 4.5 lb. A man who wishes to haul a ton, as nearly as possible, should load how many bricks?

60. The whitewash adopted by the U. S. Government is composed of 10 parts of freshly slaked lime to 1 part of hydraulic potash, and the whole is mixed with salt water. How much potash must be mixed with 2 bu. 2 pk. of lump lime, if lump lime in slaking doubles its volume?

61. A cubic foot of dry oak wood weighs 59.375 lb.; find the weight of an oak plank 12 ft. × 1 ft. × 2 in.

62. The U. S. silver dollar contains 371.25 gr. of pure silver; how many dollars could be made from each pound (troy) of pure silver used, and how many grains of silver would remain?

63. Sound travels 1090 ft. per second; express this rate in miles per hour.

64. A druggist bought 3 lb. 4 oz. of a certain drug at $2 per pound (avoir.) and sold it in packages of 1 ʒ 2 ℈ each for 5 ct. per package; find his profit.

65. When a locomotive is running at the rate of 40 mi. per hour, how many revolutions does the driving-wheel make per minute, if its circumference is 20 ft.?

66. A farmer sold 20 bu. of corn and 30 bu. of wheat for $38; if he received 10 cents more per bushel for the wheat than for the corn, how much did he receive per bushel for each?

67. Find the cost of 2 T. 240 lb. of hay at $14.50 per ton.

68. How long will it take a column of men 880 yd. long to march through a street 1 mi. long, if it marches at the rate of 2½ mi. per hour?

69. Find the cost of 8 pieces of scantling, each 16 ft. × 4 in. × 2 in., at $24 per M board feet.

70. A farmer sold three loads of wheat weighing 2142 lb., 2163 lb., and 2205 lb., at $0.80 per bushel (60 lb.); how much did he receive for them?

71. Coal is sold in Pennsylvania by the long ton; a dealer in that state who sells coal at $0.28 per 100 lb., should charge how much for a ton?

72. An arc of 22° 30′ is 6 in. long; find the length of the circumference of the circle.

73. A train 176 yd. long running at the rate of 20 mi. per hour crossed a bridge in 2 min. 15 sec.; how many yards long was the bridge?

74. A cubic foot of dry air weighs 566.9 gr.; find in pounds (avoir.) the weight of the air in a room 20 ft. × 14 ft. × 10 ft. (Cancel.)

75. The five-cent piece weighs 77.16 gr. and is three fourths copper and one fourth nickel; how many ounces (troy) of copper are used in making $5 worth of five-cent pieces?

76. At what price per yard must goods that cost $0.40 per yard be marked, in order that the dealer may sell them at a discount of 10% and still make 12½%?

77. A merchant sold a couch for $15, which was at a discount of 25% from the marked price. If his gain was 20%, find what per cent the marked price was above cost.

78. A dealer makes a profit of 33⅓% when he sells at retail, and he sells at wholesale at a discount of 12% of his retail price; find his gain per cent at wholesale.

79. If envelopes are bought at the rate of 10 ct. for a package of 25, and sold at the rate of 2 for a cent, find the gain per cent.

80. A coat was marked to gain 20%, but it was sold out of season for $6, which was at a loss of 20%; what was the marked price?

81. A merchant's retail price is 20% above his wholesale price; what per cent does he gain at retail, if he gains 10% at wholesale?

82. When ⅔ of the cost equals ⅘ of the selling price, what is the gain or loss per cent?

83. If a bankrupt owes $2500 and his assets are $850, how many cents can he pay on the dollar?

84. B spent $260 for apples at the rate of $1.30 per bushel. Keeping a part of them for his own use, he retailed the others at 40% profit and cleared $13 on the cost of all; how many bushels did he keep?

85. A dealer bought lemons at 20 ct. a dozen and twice as many at 30 ct. a dozen; if he mixed them and retailed them all at 40 ct. a dozen, find his gain per cent.

86. The publisher's price of a book is $0.90, but he sells to retailers at a discount of 16⅔%; if the retailer sells the book at the publisher's price, what per cent does the retailer gain?

87. What should be the selling price per pound of a mixed spice formed of 8 lb. costing 50 ct. per pound, 8 lb. costing 40 ct. per pound, and 4 lb. costing 60 ct. per pound, in order that the profit may be 25%?

88. A merchant bought woollen blankets at $4 a pair and marked them at an advance of 25%; if he afterward sold them at a discount of 10%, find his gain per cent.

89. A dealer bought 50 bu. of white lime at 20 ct. a bushel, and retailed it by the peck at such a price as allowed him a profit of 60% after paying $4 for hauling; find the retail price.

90. A dealer bought a bunch of 72 bananas for $0.90. He retailed half of them at the rate of 2 for 5 ct., a dozen of them at the rate of 3 for 5 ct., and the remainder at 1 ct. each; find his gain per cent.

91. What per cent is gained on tablets bought at $1 a dozen and retailed at 10 ct. each?

92. A dealer sold 2 cows for $40 each, gaining 33⅓% on one of them and losing 11⅑% on the other; find the cost of each and the dealer's gain per cent on the transaction.

93. Goods are marked to gain 20%, but are sold at a discount of 20% from the marked price; find the gain or loss per cent.

94. A merchant sold clothing on a commission of 20%; if his commission for a certain year was $2421, find the net proceeds of the sales.

95. What sum must be sent to an agent so that after taking out his commission of 2½%, he may have sufficient money to buy 60 tons of fertilizer at $18 per ton?

96. My agent purchased for me 1000 bu. of wheat at $0.80 per bushel. I sent him a check for $856, which paid the cost of the wheat, freight charges amounting to $40, and his commission; what rate of commission did he charge me?

97. A commission merchant sent his principal $225 as the net proceeds of a sale amounting to $250. If the agent retained the balance as commission, what rate did he charge?

98. A real estate dealer sold for me a lot of land, and after retaining his commission of 2½%, sent me $292.50; what commission did he charge?

Sug. 97½% of the selling price = $292.50.

99. After deducting a commission of 4% and paying other charges amounting to $6.50, an agent remitted to his principal $435.10 as the net proceeds of a shipment of baled hay; find the commission charged.

SUG. $435.10 + $6.50 = the sum remaining after the agent deducted 4% of the sales.

100. During March an agent sold $2000 worth of goods and was paid a commission of $250. During April he sold $1684 worth; what should be his commission for April, the rate being the same?

101. What rate of dividend does stock pay, if an investor realizes 4½% on his investment, the stock costing him $120 per share?

102. A sent his broker $1895 to pay for 40 shares of P. R. R. stock ($50), brokerage ¼%; find the market value of each share.

103. Find the profit on 20 shares of stock ($50), bought at $58¼ per share and sold at $60¼, if two semi-annual dividends of 2½% each were received between the transactions, brokerage ¼% on the sale and the purchase.

104. What sum must be invested in stock at 199%, paying quarterly dividends of 2%, in order that the annual income may be $200, brokerage ⅛%?

105. How many shares of stock at 90½% can be purchased with the proceeds of 60 bonds ($100) at 110¼%, brokerage in each transaction ⅛%; and what will be the surplus?

106. A sold 60 shares of Lehigh Valley R. R. stock ($50) at $34⅞, brokerage ¼%, and bought with the proceeds U. S. 4's of 1925 at 137½%, brokerage ⅛%; how many $100 bonds did he buy and what was the surplus from the sale?

107. Wishing to buy 60 shares of Phila. Traction stock ($50) at $99, brokerage ¼%, I instruct my broker to sell sufficient Delaware and Hudson stock at 138½%, brokerage ⅛%, to make the purchase; how many shares must he sell and what will be the surplus?

108. What must be paid for 6% stock ($50) in order that it may pay an investor 8% interest on his investment, making no allowance for brokerage?

109. What must be the market price of 6% stock in order that it may yield 5% on the investment, brokerage ⅛%?

110. What must be the market price of 6% stock ($50) that it may yield 5% on the investment, brokerage ¼% ?

111. July 1st, A borrowed $200 at 4½%, and when the principal had amounted to $203.75, he paid the debt; on what date did he make payment?

112. Find the difference between the common and the accurate interest on $200 from May 1st to Dec. 1st at 4½%.

113. A bought flour at $4.10 a barrel, payable in 6 months. If he wishes to pay cash, what should he pay per barrel, money being worth 5% ?

114. A bought goods to the amount of $202 on 6 mo. credit, but he paid the bill in 3 mo.; how much did he pay, money being worth 4%?

115. A laborer deposited $100 in a savings bank at the beginning of the year, and withdrew principal and interest at the end of the year; how much was he entitled to, if the savings bank paid 3% interest compounded quarterly?

116. Write a promissory note for $100, dated to-day and payable in 60 da. Make yourself payee and John Martin maker. Properly indorse the note for discount and find its value on the day of issue.

117. Write the check that you would use to draw $10 from the nearest bank, if you had money deposited in it.

118. A man divided $3600 among his six children, 3 boys and 3 girls, giving to each boy twice as much as to each girl; how much did each receive?

119. Two men are to receive $48 for painting a barn. If they work together 6 days and then one of them finishes the work in 4 days more, how much should each receive?

120. A and B in partnership gained \$40; A owned $\frac{4}{5}$ of the stock, lacking \$12; and B's share of the gain was \$10. What was the whole stock and the share of each man?

121. Three men invested jointly \$3000, and after a certain time, when the gains were to be divided, it was found that A was entitled to \$3 as often as B to \$4 and C to \$5; find the investment of each.

122. If it costs \$1800 to pave a street 1320 ft. long and 60 ft. wide, what length of street 40 ft. wide should be paved for \$1000?

123. A pole 10 ft. long casts a shadow 12 ft. long at the same time that a tree casts a shadow 75 ft. long; how high is the tree?

124. A, B, and C jointly invest \$2000, \$2500, and \$3000 respectively, and after being in business six years, dissolve partnership. If \$12,000 is realized from the sale of their joint property, how much of this should each receive?

125. Knowing that a geographic mile equals 1′ of the earth's equatorial circumference, find the number of statute miles in an arc of 20° of this circumference.

126. Find $\sqrt{11\frac{1}{9}}$.

127. Find $\sqrt{11025}$ by first separating the number into its prime factors.

128. How many sq. ft. of paper are required to cover a box 1 ft. × 8 in. × 6 in.?

129. The entire surface of a cubical box is 54 sq. in.; find its diagonal.

130. If a ball 1 in. in diameter weighs 1 lb., what is the weight of a ball $1\frac{1}{2}$ in. in diameter, made of the same material?

131. A piece of land in the form of a trapezoid contains 1 A.; if the length of one of the parallel sides is 30 rd. and of the other 10 rd., how far are they apart?

132. How long a board must be used to brace a gate 8 ft. long and 6 ft. high, if the brace reaches from the upper right-hand corner to the lower left-hand corner?

133. The standard U. S. bushel is a cylinder 18½ in. in diameter and 8 in. deep; show that it contains about 2150.42 cu. in. (Use $\pi = 3.1416$.)

134. Two vessels are of the same shape and one is twice as high as the other; if the first holds 1.5 gal., how many gallons will the second hold?

135. A rug 18 ft. long and 12 ft. wide left a margin of 2½ ft. wide uncovered; find the cost of waxing this margin at 5 ct. a square foot.

136. A stream at a certain point is 40 yd. wide and on an average 11 ft. deep; if the water flows past this point at the rate of 2100 yd. per hour, how many gallons pass this point per minute? (Cancel.)

SUG. 1 cu. ft. = 1728 cu. in.; 1 gal. = 231 cu. in.

137. A surveyor's chain just reaches around a rectangular strip of land 22 ft. long; how many square feet does the strip contain?

138. A standard township is 6 mi. square; how many 40-acre farms can be formed from it, and how many miles of fence will enclose them, each farm being a square?

139. Find the difference between the inner and the outer surface of a closed box 2 ft. × 1 ft. × 6 in., outside measurement, the box being made of boards ½ inch thick.

140. How many liters of water will a tank hold whose base is a square 3^m long and whose depth is 6^m?

141. Find the value of the potatoes that just fill a bin 20 ft. × 12 ft. × 6 ft. at $0.80 a bushel. [§ 205, Note 3.]

142. How much will it cost to plaster the walls and ceiling of a room 36 ft. long, 24 ft. wide, and 8 ft. high, at $0.20 a square yard, allowing 420 sq. ft. for doors and windows?

143. Find the cost of digging and walling the cellar for a house 40 ft. long, 36 ft. wide, if the cellar is dug to a depth of 9 ft. and the walls are 10 ft. high and 18 in. thick, the cost of excavating being 20 ct. a cubic yard and of building the wall $1 a perch.

144. How many tons of ice can be cut from a pond 110 ft. long and 60 ft. wide, if the ice averages 9 in. in thickness, and ice is $\frac{10}{11}$ as heavy as water? [§ 199, Note 2.]

145. A slate 8 ft. long and 2 ft. wide is surrounded by a frame 4 in. wide; how much farther is it around this frame on the outside than on the inside? How many square inches in the frame?

146. A cylindrical tank 40 ft. high and 15 ft. in diameter is represented in a drawing as 15 in. high; what is the length of the diameter as represented in the drawing?

147. How many cubic feet are there in a conical pit 6 ft. across the top and 5 ft. from the edge to the deepest point?

148. A lawn 60 yd. long and 50 yd. wide is surrounded by a walk 6 ft. wide; find the cost of graveling this walk at $0.12 per square yard.

149. How many rods of hedge fence will enclose the walk described in problem 148 and separate it from the lawn?

150. How many cubic feet of granite are there in a pillar 4 ft. square and 8 ft. high? How much does it weigh, if 1 cu. ft. of granite weighs 173¾ lb.?

151. How many square ft. of zinc are required to line the sides and the bottom of a box 6 ft. long, 4 ft. wide, and 3 ft. deep, inside measurement?

152. What is the diameter of a wheel that makes 480 revolutions in going a mile?

153. A lady found that if she paid $1.10 per yard for silk, she would lack $1.20 of having enough to pay for the quantity desired, but that if she paid $.85 per yard, she would have $1.80 remaining; how much money had she?

154. In a Centigrade thermometer the freezing point is Zero and the boiling point is 100°; in a Fahrenheit the freezing point is 32° and the boiling point is 212°. What degree Centigrade corresponds to 68° Fahrenheit?

155. Alcohol is .792 times as heavy as water; find the weight of a liter of alcohol.

156. A rectangular piece of land containing 1¼ A. is 121 yd. long; find its width.

157. How many perches of stone laid dry will build a wall 132 ft. long, 1½ ft. wide, and 4 ft. high?

158. At $5 a rod, what will it take to fence a rectangular lot containing 2 A., if it is twice as long as wide?

159. A silver dollar weighs 412.5 gr. and is 10% copper; if 1 lb. (troy) of copper is worth 24 ct., what is the value of the copper in a silver dollar?

160. Two trolley cars leave a station together and run in opposite directions, the first at the rate of 20 mi. per hour and the second at the rate of 25 mi. per hour; in how many minutes will they be 1 mi. apart?

161. A mixed tea containing 40 lb. is 25% black and the remainder green. How many pounds of black tea added to the mixture will make the new mixture half black and half green?

162. A man was willed $8000. If he earns $400 a year and spends $650 a year, in what time will all the money be gone?

163. A boy agreed to work for a merchant for 20 wk. on condition that he should receive $20 and a suit of clothes; at the end of 12 wk., when the boy stopped work, it was found that he was entitled to $8 and the suit. Find the value of the suit.

164. Two corn-shellers can together shell 250 bu. of corn in 10 hr.; but after running 6 hr. one of them is stopped and the other completes the work of shelling the 250 bu. in 10 hr. more. In what time would each alone shell this amount?

165. One pipe can fill a tank in 20 hr. and another can fill it in 25 hr. The tank can be emptied by a spigot in 15 hr. If the tank is empty and the pipes and spigot set to flow at the same time, in how many hours will the tank be filled?

166. A barn valued at $1800 was insured for ¾ of its value; find the annual premium, if the rate charged was $2.40 per $1000 of insured value.

167. Find the number of tons in the weight of a consignment of 225 bbl. of pork, each barrel weighing 200 lb. (Cancel.)

168. A barrel of sugar weighing 240 lb. was bought at $4.80 per hundredweight and sold at $5\frac{1}{2}$ ct. per pound; if the freight charges were 40 ct., find the gain.

169. A has his life insured for $8000; what sum does he pay in premiums in 20 yr. if he pays an annual premium of $37.50 per $1000?

170. A and B bought a keg of nails for $2.80, A paying $1.60 of this price and B the remainder. B paid the freight amounting to $0.30, but kept the keg, worth $0.10; which owes the other, and how much does he owe, if they shared the nails equally?

171. A painter received $3 per day for his labor and paid $5 per week for his board, that being his only expense; at the expiration of 10 wk. he had saved $100. How many days was he idle?

172. A class of 150 pupils was examined in grammar, 20% of them being boys and the rest girls. If $16\frac{2}{3}$% of the boys and 20% of the girls failed, what per cent of the class passed?

173. Find $\sqrt[3]{216}$ by first reducing the number to its prime factors.

174. An agent received $2000 to invest in wheat at $0.80 per bushel after taking out his commission of 5%; find the number of bushels purchased and the surplus.

175. The area of a trapezoid is 32 sq. ft. If its altitude is 4 ft. and the difference of its parallel sides is 4 ft., find the parallel sides.

176. If 8 men do $\frac{2}{3}$ of a piece of work in 4 da., in what time should 6 men finish it?

177. A rectangular lot 100 ft. × 48 ft. is surrounded by a ditch 1 ft. wide; find the depth of this ditch, if the earth removed from it and spread over the lot raised its surface 2 in.

178. A liter of mercury weighs $13^{kg}\ 598^{g}$; how many times as heavy as water is mercury?

179. The four sides of a tract of land measure 108⅓ rd., 116⅔ rd., 75 rd., and 100 rd.; the last two sides form a right angle. Find the number of acres in the tract.

180. The perimeter of an isosceles triangle is 45 ft. and the base is one half of one of the equal sides; find the base.

181. How many posts set 12 ft. apart are necessary to fence a rectangular tract of land 8 rd. wide, containing 2 A.?

182. Find the duty on a painting valued at $40,000, if the rate of *ad valorem* duty is 20%.

Note. A tax levied by a government on the invoice value of goods imported from a foreign country is called an **Ad Valorem Duty**.

183. A jeweler imported $10,500 worth of cut diamonds; if the rate of *ad valorem* duty is 10%, find the duty.

184. Find the duty on 1000 bu. of potatoes, if the duty is $0.25 per bushel.

Note. A tax levied by a government on the amount of goods imported from a foreign country, irrespective of their value, is called a **Specific Duty**.

185. Find the duty on 25 boxes of macaroni each containing 25 lb., if the duty is 1½ ct. per pound.

186. Find the duty of 10 rolls of Wilton carpet, each roll containing 90 sq. yd. and invoiced at $2.50 per square yard, if the duty is 60 ct. per square yard and 40% *ad valorem.*

187. Find the lateral area of the frustum of a right cone the radii of whose bases are 10 ft. and 4 ft., respectively, and whose altitude is 8 ft.

188. Two men entered into partnership. The first invested $250 dollars for 40 days and lost $80. The second invested $350 and lost $70; how many days was the capital of the second invested?

189. A and B entered into partnership. A invested $450 for 8 months and gained $180. B invested his capital for 7 months and gained $175; what was B's capital?

The Sensenig-Anderson Series of Arithmetics.

ANSWERS

TO ESSENTIALS OF ARITHMETIC.

Exercise 11, page 16.

2. 1239
3. 1882
4. 2288
5. 31329
6. 222754
7. 354423
8. 308425
9. 115087
10. 278
11. 1899
12. 2882
13. 12733
14. 25083
15. 38921
16. 16555
17. 18613
18. 14369

Exercise 12, page 17.

3. 1776
4. 1359
5. $15923
6. 9482 lb.
7. $25783
9. 10777
11. 20655
12. 51179194
8. B 10150; C 10906
10. $2465; $3029; $3377; $3722; $12593
13. Males, 36486; Females, 35294; Total, 71780

Exercise 16, page 24.

2. 642
3. 190
4. 403
5. 195
6. 287
7. 1212
8. 3306
9. 1458
10. 1722
11. 1396
12. 1831
13. 2285
14. 2079
15. 4942
16. 7626
17. 194
18. 157
19. 700
20. 1377
21. 629
22. 2988
23. 1416
24. 1380
25. 882

Exercise 17, page 25.

3. 310 mi.
4. 139
5. 544
6. $535
7. $755
8. 4422
9. $2467
10. 42 yr.
11. 8954
12. 1008 loads
13. $4045
14. 713
15. 119 A.
16. $136
17. $1050
18. $73931
19. 106; 107; 108
21. 13981 R.; 4629 D.; 5002 R.
20. Exports: $30202038 in 1899; $11401725 in 1897
Imports: $37939907 in 1899; $22528888 in 1897
Increase in Exports: $18800313
Increase in Imports: $15411019
Increase in Exports above increase in Imports: $3389294

Exercise 20, page 32.

2. 4758
3. 5922
4. 7880
5. 4770
6. 3040
7. 8704
8. 19308
9. 8228
10. 30040
13. 3480
14. $19250
15. 3540
16. 3800 mi.
17. 8100
18. $2850
19. $1550; $316
20. 2640 lb.

Exercise 21, page 33.

5. 320; 3200; 32000
6. 5500; 55000; 550000
7. 13500; 135000; 1350000
8. 31560; 315600; 3156000
9. 198500
10. 519000
11. 553000
12. 784000
13. 810000
14. 400000
15. 20000
16. $35000
17. 2000
18. $125000
19. $7000
20. $12000

Exercise 22, page 34.

7. 1058
8. 4704
9. 4977
10. 1764
11. 4214
12. 9072
13. 16555
14. 58032
15. 33761
16. 65142
17. 34970
18. 142196
19. 182790
20. 348348
21. 383614
22. 142272
23. 111616
24. 211932
25. 230202
26. 154736
27. 477006
28. 39552
29. 204408
30. 586704
31. 480198
32. 1170984
33. 2367408
34. 1913849
35. 3208842
36. 2996001
37. 8000
38. 7200
39. 14000
40. 112000
41. 72000
42. 62400
43. 476000
44. 255000
45. 1320000
46. 891000
47. 646400
48. 866400
49. 272160
50. 41262
51. 35712
52. 144400
53. 2100000
54. 6480000
55. 1960200
56. 98280000

Exercise 23, page 37.

3. 19200
4. 384 mi.
5. 1530
6. 147840
7. 21600
8. 107520
9. 65400 ft.
10. 26420 lb.
11. 384 da.
12. 34000
13. 1620
14. $784
15. 624 gal.
16. $2288
18. 6540 ft.

Exercise 26, page 42.

2. 166
3. 149
4. 125
5. 62
6. 125
7. 108
8. 83
9. 718
10. 1157
11. 1642
12. 997
13. 894
14. 1016
15. 727
16. 1782^{1}
17. 5172^{3}
18. 7130^{1}
19. 1924^{6}
20. 20687^{1}
21. 17583^{1}
22. 4395^{8}
23. 3392
24. 29921^{2}
25. 18724
26. 1750
27. 1646^{3}
28. 982
29. 1251^{2}
30. 1122^{3}
31. 2469^{1}
32. 337^{1}
33. 2991
34. 1855^{1}
35. 2021
36. 100168^{2}
37. 24287^{1}
38. 1475
39. 11111^{2}
40. 22446^{6}

Exercise 27, page 43.

3. $35
4. 60
5. $691
6. 1090 ft.
7. $3
8. 24 ft.
9. 131 lb.
10. 901 bu.
11. 1446 lb.
12. $147
13. $7
14. 28 bu.

Exercise 29, page 45.

2. 5^{2}
3. $3^{\underline{11}}$
4. 5^{5}
5. $5^{\underline{76}}$
6. 6
7. 9^{7}
8. $5^{\underline{24}}$
9. 9^{3}
10. $9^{\underline{45}}$
11. 10^{7}
12. $9^{\underline{17}}$
13. $7^{\underline{86}}$
14. $9^{\underline{27}}$
15. 9^{3}
16. $7^{\underline{48}}$
18. 25
19. 24
20. 23
21. 23
22. 14
23. 24
24. 68
25. 97
26. 89
27. 57
28. 15
29. 21
30. 68
31. 99
32. 87
34. 106
35. 108
36. 208
37. 309
38. 406
39. 404
40. 708
41. 809
42. 806
43. 760
44. 750
45. 990
46. 709
47. 908
48. 605
49. 609
50. 601
51. 909
53. 16
54. 32
55. 45
56. 54
57. 44
58. 56
59. 44
60. 56
61. 28
62. 48
63. 52
64. $107^{\underline{108}}$
65. 236
66. 61
67. 80
68. 65
69. $722^{\underline{324}}$
70. $1665^{\underline{211}}$
71. $885^{\underline{175}}$
72. $112^{\underline{302}}$

Exercise 30, page 47.

3. 32
4. 23
5. 18
6. 16
7. 13
8. 12
9. 12
10. 11
11. $29^{\underline{5}}$
12. $15^{\underline{38}}$
13. $15^{\underline{13}}$
14. $30^{\underline{66}}$
15. $35^{\underline{28}}$
16. $85^{\underline{84}}$
17. 34
18. 28
19. 28
20. 21
21. 19
22. $13^{\underline{50}}$
23. $12^{\underline{60}}$
24. $8^{\underline{542}}$
25. $10^{\underline{846}}$
26. 104
27. 60
28. $68^{\underline{120}}$
29. $101^{\underline{142}}$
30. $101^{\underline{288}}$
31. 18
32. 23
33. 6
34. 28
35. 24
36. 14
37. 15
38. 18
39. $3^{\underline{500}}$
40. $4^{\underline{4800}}$
41. $5^{\underline{7420}}$
42. $31^{\underline{800}}$
43. $16^{\underline{2640}}$
44. $7^{\underline{7400}}$
45. $2^{\underline{19548}}$
46. $2^{\underline{1876}}$
47. $1^{\underline{19701}}$

Exercise 31, page 48.

2. 360 lb.
3. $56
4. 25
5. 30 yr.
6. $120
7. $526
8. $275
9. 107
10. 6 da.
11. $4
12. 9 mi.
13. N. Y., 97 A.; Pa., 87 A.; N. J., 86 A.; Del., 113 A.; Md., 121 A.

Exercise 34, page 53.

4. 218
5. 327
6. 406
7. 638
8. 89
9. 706
10. 1200
11. 2046
12. 7
13. 385
14. 720
15. 144
16. 3115
17. 206
18. $403^{\$\underline{140}}$
19. 202
20. 108
21. $228^{\underline{265}}$ rd.
22. $20^{\underline{2886}}$ sec.

Exercise 35, page 54.

2. 238
3. 200
4. 1760
5. 50
6. 7
7. 20
8. 60
9. 28
10. 47
11. 215
12. 10
13. 60
14. 4
15. 1

Exercise 36, page 56.

2. 24
3. $54
4. 86
5. 44
6. 760
7. 2961
8. 700
9. 184 yd.
10. $1
11. 1312
12. 100

Exercise 38, page 60.

2. 2, 3, 5
3. 2, 3, 3, 3
4. 2, 2, 2, 3, 3
5. 2, 2, 3, 5
6. 3, 5, 5
7. 2, 3, 3, 5
8. 2, 5, 11
9. 2, 2, 3, 13
10. 2, 2, 2, 5, 7
11. 2, 7, 13
12. 3, 5, 7, 7
13. 3, 3, 5, 17
14. 2, 3, 3, 5, 7
15. 7, 11, 13
16. 2, 3, 3, 5, 5, 7

Exercise 40, page 63.

3. 12
4. 8
5. 14
6. 7
7. 18
8. 25
9. 56
10. 30
11. 7
12. 8
13. 10
14. 26
15. 35
16. 48
17. 108
18. 192
19. 625
20. 138
21. 165
22. 112
23. 78
24. 56
25. 6 lb.
26. 8 ft.; 74
27. $6
28. 14 ft.
29. 4 ft.
30. 35 ft.
31. 77; 18
32. 45
33. $121

Exercise 42, page 67.

3. 84
4. 672
5. 252
6. 198
7. 840
8. 375
9. 260
10. 945
11. 1176
12. 1440
13. 450
14. 312
15 324
16. 450
17. 840
18. 6050
19. 11858
20. 5005
21. 8190
22. 4860
23. 500 lb.
24. 168 ct.; 7, 6
25. 60 in.
26. 24
27. 120 in.
28. 60; 3
29. 120
30. 360 A.
31. 122 ft.
32. $550
33. 1460 lb.
34. 1720

Exercise 43, page 69.

6. 156
7. 182
8. 210
9. 255
10. 270
11. 272
12. 306
13. 342
14. 360
15. 375
16. 800
17. 266
18. 2400
19. 8000
20. 1920
21. 900
22. 2
23. 4
24. 2
25. 4
26. 5
27. 7
28. 9
29. 11
30. 6
31. 14
32. 4
33. 3
34. 3
35. 2
36. 6
37. 8
38. 35
39. 100
40. 108 yd.
41. 30
42. 2548 lb.
43. 15
44. 8 da.
45. 9
46. 36
47. 32

Exercise 44, page 73.

2. 17
3. 19
4. 23
5. 29
6. 31
7. 37
8. 41
9. 43
10. 47
11. 49
12. 51
13. 71
14. 73
15. 79
16. $127; 3, 7, 9
17. $67; 24, 35, 42
18. $87

Exercise 45, page 75.

3. 1287
4. 714
5. 665
6. 920
7. 3770
8. 6324
9. 20790
10. 6630
11. 2220
12. 13104
13. 1014
14. 5046
15. $3400
16. 32800 lb.
17. 4900 lb.

Exercise 48, page 82.

5. $\frac{3}{4}$
6. $\frac{5}{6}$
7. $\frac{8}{9}$
8. $\frac{7}{12}$
9. $\frac{12}{13}$
10. $\frac{9}{14}$
11. $\frac{15}{16}$
12. $\frac{24}{25}$
13. $\frac{27}{28}$
14. $\frac{5}{7}$
15. $\frac{31}{32}$
16. $\frac{29}{31}$
17. $\frac{41}{47}$
18. $\frac{9}{10}$
19. $\frac{1}{8}$
20. $\frac{127}{8}$
21. $\frac{239}{6}$
22. $\frac{175}{2}$
23. $\frac{987}{8}$
24 $\frac{1412}{9}$
25. $\frac{4682}{15}$
26. $\frac{480}{13}$
27. $\frac{732}{15}$
28. $\frac{197}{2}$
29. $\frac{5632}{25}$
30. $\frac{4865}{19}$
31. $\frac{3745}{17}$
32 $\frac{10123}{20}$
33 $\frac{7502}{25}$
34. $\frac{100101}{100}$
35. $12\frac{5}{7}$
36. $11\frac{1}{2}$
37. 16
38. $48\frac{2}{5}$
39 78
40. $193\frac{2}{3}$
41. $34\frac{4}{5}$
42. $50\frac{5}{11}$
43. $40\frac{1}{2}$
44. 16
45. $29\frac{9}{31}$
46. $336\frac{1}{2}$
47. 91
48. $42\frac{1}{7}$
49. $28\frac{6}{7}$
50. $\frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{12}{60}$
51. $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}, \frac{7}{12}$
52. $\frac{16}{36}, \frac{21}{36}, \frac{10}{36}, \frac{7}{36}$
53. $\frac{20}{30}, \frac{24}{30}, \frac{9}{30}, \frac{4}{30}$.
54. $\frac{15}{18}, \frac{14}{18}, \frac{5}{18}$,
55. $\frac{12}{28}, \frac{10}{28}, \frac{3}{28}$
56. $\frac{72}{78}, \frac{15}{78}, \frac{8}{78}$
57. $\frac{3}{5}, \frac{2}{5}, \frac{3}{5}$
58. $\frac{85}{180}, \frac{57}{180}$
59. $\frac{40}{315}, \frac{6}{315}$
60. $\frac{168}{875}, \frac{240}{875}$
61. $\frac{407}{1155}, \frac{156}{1155}$

Exercise 50, page 85.

4. $2\frac{13}{60}$
5. $2\frac{1}{24}$
6. $2\frac{1}{36}$
7. $\frac{25}{32}$
8. $\frac{1}{6}$
9. $\frac{25}{156}$
10. $\frac{51}{650}$
11. $\frac{86}{323}$
12. $5\frac{1}{16}$
13. $13\frac{1}{12}$
14. $19\frac{1}{24}$
15. $17\frac{25}{32}$
16. $\frac{32}{63}$
17. $\frac{17}{24}$
18. $\frac{22}{35}$
19. $\frac{125}{156}$
20. $41\frac{1}{3}$
21. $47\frac{1}{12}$
22. $117\frac{31}{42}$
23. $54\frac{7}{12}$
24. $13\frac{13}{24}$ ft.
25. $\frac{181}{300}$
26. $53\frac{1}{4}$ mi.
27. $\frac{5}{72}$
28. $\frac{9}{35}$

Exercise 52, page 88.

5. $\frac{11}{24}$
6. $\frac{11}{36}$
7. $\frac{47}{75}$
8. $\frac{53}{72}$
9. $\frac{3}{5}$
10. $\frac{1}{24}$
11. $\frac{1}{90}$
12. $\frac{1}{60}$
13. $\frac{1}{60}$
14. $\frac{1}{210}$
15. $\frac{6}{35}$
16. $\frac{3}{110}$
17. $\frac{5}{48}$
18. $\frac{7}{60}$
19. $\frac{9}{100}$
20. $13\frac{3}{20}$
21. $11\frac{3}{4}$
22. $88\frac{1}{2}$
23. $19\frac{5}{42}$
24. $62\frac{53}{72}$
25. $\frac{17}{24}$
26. $\frac{5}{18}$
27. $\frac{19}{30}$
28. $2\frac{1}{24}$
29. $22\frac{11}{12}$
30. $2\frac{19}{80}$
31. $\frac{1}{2}$
32. $3\frac{5}{12}$
33. $\frac{25}{48}$
34. $\frac{2}{21}$
35. $4\frac{7}{80}$
36. $34\frac{9}{26}$
37. $\frac{11}{14}$
38. $\$98\frac{17}{20}$
39. $\frac{1}{4}$
40. $23\frac{1}{3}$ mi.
41. $58\frac{3}{8}$

Exercise 56, page 93.

1. $2\frac{9}{13}$
2. $3\frac{1}{2}$
3. $2\frac{4}{5}$
4. 9
5. 25
6. 217
7. $7\frac{14}{15}$
8. $90\frac{10}{21}$
9. $33\frac{9}{47}$
10. $7\frac{1}{5}$
11. $28\frac{1}{3}$
12. $80\frac{1}{2}$
13. 208
14. $144\frac{2}{3}$
15. $156\frac{1}{3}$
16. $\frac{15}{56}$
17. $\frac{3}{10}$
18. $\frac{1}{132}$
19. $\frac{24}{65}$
20. $\frac{45}{68}$
21. $\frac{49}{72}$
22. $\frac{21}{50}$
23. $\frac{10}{39}$
24. $\frac{1}{5}$
25. $\frac{1}{60}$
26. $\frac{1}{2}$
27. 300
28. 114
29. $141\frac{3}{5}$
30. $7\frac{1}{13}$
31. 225
32. 750
33. 900
34. 4092
35. $7380\frac{5}{8}$
36. $19\frac{1}{4}$
37. $84\frac{3}{8}$
38. $14\frac{7}{12}$ in
39. $115
40. 1375 lb.
41. $585\frac{3}{4}$
42. $23\frac{5}{8}$
43. $330
44. 3978
45. $\$31\frac{7}{20}$
46. $\$8\frac{1}{48}$
47. $8\frac{409}{1152}$ lb.

Exercise 60, page 99.

1. $\frac{2}{45}$
2. $\frac{15}{28}$
4. $\frac{2}{17}$
5. $\frac{2}{45}$
6. $\frac{6}{37}$
7. $\frac{6}{11}$
8. $\frac{18}{65}$
9. $\frac{9}{238}$
10. $\frac{19}{192}$
11. $\frac{145}{169}$
12. $\frac{3}{20}$
13. $\frac{8}{165}$
14. $\frac{9}{35}$
15. $\frac{1}{148}$
16. $\frac{4}{121}$
17. $\frac{33}{148}$
18. $\frac{7}{85}$
19. $\frac{9}{41}$
20. 18
21. $1\frac{1}{5}$
22. 81
23. 300
24. $166\frac{2}{13}$
25. $1\frac{19}{21}$
26. $1\frac{1}{30}$
27. $1\frac{24}{31}$
28. $1\frac{2}{43}$
29. $228\frac{3}{7}$
30. $3\frac{9}{13}$
31. 20
32. $9\frac{1}{3}$
33. 7
34. $1\frac{1}{2}$
35. $\frac{77}{138}$
36. $3\frac{1}{3}$
37. $2\frac{7}{9}$
38. $4\frac{5}{72}$
39. $\frac{12}{25}$
40. 6
41. $3\frac{3}{19}$
42. $\frac{5}{19}$
43. $\frac{1}{14}$
44. 10
45. $9\frac{33}{50}$
46. $14\frac{29}{35}$
47. $\frac{1}{17}$
48. $1\frac{1}{14}$
49. $2\frac{5}{7}$
50. $3\frac{2}{3}$
51. $\frac{5}{8}$
52. 6
53. 25
55. $\frac{9}{8}$
56. $43\frac{5}{9}$
57. $\frac{10}{11}$
58. $360; $480
59. 1000 ft.
60. $\frac{3}{8}$ mi.
61. $100; $18
62. 60 lb; 32 lb.
63. $\frac{189}{50}$

Exercise 62, page 103.

1. $\frac{2}{3}$
2. $\frac{3}{16}$
3. $8\frac{3}{4}$
4. $\frac{9}{10}$
6. $\frac{5}{54}$
7. $\frac{2}{25}$
8. $\frac{2}{27}$
9. $\frac{3}{80}$
10. $\frac{1}{66}$
11. $\frac{1}{250}$
12. $\frac{1}{33}$
13. $\frac{4}{93}$
15. 21
16. 22
17. $1\frac{1}{15}$
18. $4\frac{1}{3}$
19. $3\frac{1}{2}$
20. $41\frac{3}{5}$
21. 700
22. 6250
24. $\frac{3}{7}$
25. $1\frac{2}{5}$
26. 1
27. $1\frac{13}{27}$
28. $\frac{8}{15}$
29. $1\frac{95}{143}$
31. $\frac{2}{3}$
32. $1\frac{2}{5}$
33. $1\frac{1}{2}$
34. $1\frac{1}{5}$
35. $\frac{3}{100}$
36. 100
37. $\frac{5}{6}$
38. $\frac{1}{7}$
39. $\frac{2}{3}$
40. $351\frac{9}{16}$
41. 1
42. $69\frac{333}{343}$
43. 5
44. $1\frac{7}{20}$
46. 8
47. 704
48. 32
49. 7
50. $6\frac{2}{3}$
51. $11\frac{5}{11}$
52. $\frac{81}{365}$
53. $\frac{1}{20}$
54. $\frac{2}{25}$
55. $333\frac{1}{3}$

Exercise 63, page 106.

5. $\frac{2}{3}$
6. $\frac{5}{12}$
7. $1\frac{1}{2}$
8. $11\frac{1}{9}$
9. $2\frac{1}{12}$
10. $\frac{1}{3}$
11. 6
12. $1\frac{1}{2}$
13. $\frac{32}{35}$
14. $33\frac{1}{3}$
15. $\frac{7}{500}$
16. 2
17. $\frac{1}{5}$
18. $\frac{3}{7}$
19. $\frac{1}{5}$
20. 16
21. 2
22. $1\frac{35}{38}$
23. 1
24. $1\frac{21}{40}$
25. $1\frac{1}{3}$
26. $3\frac{7}{10}$
27. $\frac{3}{160}$
28. $1\frac{5}{12}$

Exercise 65, page 110.

1. $10\frac{5}{12}$
2. 2240
3. $195
4. $35\frac{1}{7}$
5. $43\frac{24}{25}$
6. $4500
7. 20 bu.
8. $2\frac{3}{4}$
9. 525 lb.
10. $\$63\frac{1}{3}$
11. $25
12. $\$93\frac{3}{4}$; $\$31\frac{1}{4}$
13. 160
14. 60
15. $\frac{16}{27}$
16. $9\frac{3}{8}$ in.
18. $7\frac{1}{2}$; $12\frac{1}{2}$
19. 270
20. $2000 ; $3000
21. 96 qt.
22. $\frac{7}{8}$
23. $\frac{1}{5}$
24. 24
25. $41\frac{1}{4}$
26. $48\frac{8}{15}$ ft.
27. 144
28. 14 ; $\frac{1}{4}$ yd.
29. $56\frac{1}{4}$ min. ; $18\frac{3}{4}$ min.
30. 18 days
31. $\frac{1}{5}$
32. $\frac{6}{7}$
33. $\$20\frac{1}{4}$
34. 21
35. 150
36. $71\frac{1}{2}$ yr.
37. 16
38. $\$7\frac{1}{2}$
39. $\frac{8}{5}$
40. $150
41. $7\frac{1}{2}$ da.
42. 56 lb.
43. 39
44. $1\frac{11}{24}$ hr.
45. 16 ct.
46. $48\frac{3}{4}$ ct.
47. $35\frac{1}{5}$
48. 32
49. $\frac{4}{17}$
50. 6
51. 45 ; 75
52. $40
55. $95\frac{5}{24}$; $65\frac{11}{24}$
56. 1
57. $\$1\frac{3}{4}$
17. Three hundred four six hundred twenty-seconds

Exercise 67, page 117.

1. .4
2. .05
3. .67
4. .008
5. .012
6. .132
7. .0006
8. .0408
9. .4756
10. .00040
11. .00302
12. .000084
13. 1.5
14. 2.12
15. $.22\frac{2}{9}$
16. 600.006
17. 4.04
18. 400.04

Exercise 68, page 118.

2. $\frac{4}{5}$
3. $\frac{3}{50}$
4. $\frac{3}{25}$
5. $\frac{1}{200}$
6. $\frac{81}{125}$
7. $\frac{61}{200}$
8. $\frac{9}{20}$
9. $4\frac{1}{4}$
10. $36\frac{1}{2}$
11. $1\frac{1}{20}$
12. $\frac{1}{1250}$
13. $\frac{801}{2000}$
14. $\frac{1}{6}$
15. $\frac{5}{6}$
16. $\frac{5}{8}$
17. $\frac{3}{16}$
18. $\frac{1}{7}$
19. $\frac{3}{8}$
20. $\frac{2}{3}$
21. $\frac{5}{8}$
22. $\frac{7}{8}$
23. $\frac{5}{6}$
24. $\frac{2}{9}$
25. $4\frac{4}{9}$
26. $6\frac{1}{3}$

Exercise 69, page 118.

2. .75
3. .375
4. .625
5. .1875
6. .1
7. .225
8. .24
9. .08
11. $.83\frac{1}{3}$
12. $.88\frac{8}{9}$
13. $.28\frac{4}{7}$
14. $.41\frac{2}{3}$
15. $.714\frac{2}{7}$
16. $.555\frac{5}{9}$
17. $1.166\frac{2}{3}$
18. $.583\frac{1}{3}$

Exercise 70, page 119.

2. $.41\frac{2}{3}$
3 $.83\frac{1}{3}$
4. $.166\frac{2}{3}$
5. $.142\frac{6}{7}$
6. $.583\frac{1}{3}$
7. $.2857\frac{1}{7}$
8. $.6\frac{2}{3}$
9. $.12\frac{1}{2}$
10. $.63\frac{3}{4}$
11. $.5\frac{5}{8}$
12. $.33\frac{1}{3}$
13. $.1\frac{2}{3}$
14. $.00\frac{5}{9}$

Exercise 71, page 120.

2. 30.0735
3. 418.54
4. 57.30476
5. 57.19
6. 46.3184
7 3.1075
8 20.9271
9. 1.317
10. 16.194
12. $2.2\frac{3}{8}$
13. $6.1\frac{5}{84}$
14. $1.94\frac{13}{20}$
15. $1.04\frac{61}{240}$
17. 3.706
18. 5.125
19. 18.1075
20. .4125
21. 16.6125
22. 40.7102
23. 1.99625
24. .05
25. 3.551
27. $.0\frac{9}{20}$
28. $.0\frac{1}{8}$
29. $2.5\frac{5}{6}$
30. $07\frac{1}{2}$
31. $2.12\frac{1}{2}$
32. $.13\frac{1}{3}$
33. $36\frac{9}{40}$
34. .000099
35. 39.37079
36. .09999
37. $.074
38. $11.75
39. $127.50
40. $158.15
41. 6671.577
42. 8.4 ft.

Exercise 72, page 123.

2. .375; 375; 3.75; 37.5
3. 125; 12500; 125000; 1250
5. 11.25; 112.5; 1125; 11250
6. .0136; .136; 1.36; 13.6
8. 2.5; .036; .018; .25; .07
9. 28.2
10. .6
11. 172.725
12. .15
13. 270
14. 33660
15. 16350
16. 21.875
17. .0001584
18. .44308
19. .0000025
20. 25000250
22. 27.33$\frac{1}{3}$
23. .20$\frac{5}{8}$
24. .64$\frac{8}{15}$
25. .27
26. 6.25 ct.
27. 125 ct.
28. 103.125
29. 792
30. 14.45 mi.
31. 460 cwt.
32. 2, 3, 6, 5
33. $40.625
34. $785
35. 449.28 lb.
36. 146.25 lb.
37. $240
38. 733.04 ft.

Exercise 73, page 125.

2. .8567; .08567; .008567
3. .075; .0075; 000075
4. .003075; .03075; .3075
5. 200; 2; .2
10. .01; .005
11. .013
12. .041
13. .00006
14. 3.25
15. .0327
16 .0456
17. 3.6
18. .000003
19. .0024
20. 10; 1000
21. 10; .01
22. 3.6
23. 1.26
24. 12
25. 30250
26. 250
27. 8
28. 3.24
29. .0024
31. $\frac{2}{45}$
32. $\frac{1}{240}$
33. 3
34. .02$\frac{3}{8}$
35. .81$\frac{1}{4}$
36. .00$\frac{1}{4}$
37. .24$\frac{8}{33}$
38. 1.76$\frac{8}{17}$
39. .13$\frac{1}{3}$
40. 6 ct.
41. 5 ct.
42. 4.25 mi.
43. 58$\frac{1}{3}$ lb.
44. $8
45. 600 bu.
46. 94461
47. 1.36 lb.
48. $7
49. $.06
50. 6$\frac{1}{24}$ in.
51. $5.65
52. $0.50
53. 156.25 lb.

Exercise 74, page 128.

2. 47.8; 4.78; .478
3. .000275; .0275; .00275
4. 2.5; .025; .25
6. .7; .1; .35
7. .025; .0125; .01$\frac{2}{3}$
8. .0035; .0002$\frac{1}{3}$; .0028
9. .0004; .00002$\frac{2}{3}$; .0002
10. 50; 5; 500
11. .003; .015; .1; 150
13. $\frac{17}{2800}$
14. 321$\frac{3}{7}$
15. $\frac{3}{2000}$
16. $\frac{15}{28}$
17. $\frac{2}{25}$
18. $\frac{1}{10}$
19. .66$\frac{2}{3}$
20. 4.50
21. .56$\frac{1}{4}$
22. .68$\frac{3}{4}$
23. 20
24. 6 hr.
25. 6$\frac{13}{22}$
26. 80
27. 55$\frac{5}{9}$
28. 80
29. 2.78
30. 1$\frac{4}{11}$
31. $\frac{23}{32}$; $\frac{9}{32}$
32. 9.5
33. 1$\frac{1}{3}$; $\frac{3}{4}$
34. 200
35. 203
36. 3

Exercise 75, page 130.

9. 375.022
10. 3.205557
11. .003
12. $185.40
13. $89.95
14. 8.75
15. 26.46
16. 9.31
20. 25.2; 10.8; 5.4
21. 4$\frac{4}{5}$
22. 3.38
23. $217.05
24. 1.03
25. $164.45
26. $89.98
27. $48.84
28. .405; .088
29. 2.0828 lb.
30. 434507; 509025
31. $6.27
32. 13.764
33. 1$\frac{7}{17}$
17. 66 lb.; 26.7 lb.; 2.8 lb.; 1.8 lb.; 2.7 lb.

Exercise 76, page 134.

9. .05; .25; .7
10. .12$\frac{1}{2}$; .16$\frac{2}{3}$; .33$\frac{1}{3}$
11. .005; .018; .05
12. .185; .208
13. $3.25
14. $16.08
15. $0.35
16. 0.18\frac{3}{4}$
17. $4.164
18. $12.062
19. $20.004
20. $0.253
21. $23.75
22. $112.50
23. $150
24. $661.54
25. $16.25
26. $4.12
27. $9.60
28. 84; 80; 3456
29. 48; 18
30. 5; 10
31. 70 yr.
32. $1.80
33. $36
34. $2.16

Exercise 77, page 137.

1. \$20.74
2. \$160.50
3. \$30
4. \$12.98
5. \$65.75
6. \$0.14

Exercise 78, page 140.

2. 250 lb.
3. 16 cwt.
4. 1920 gr.
5. 4800 lb.
6. 35000 gr.
7. 4480 lb.
9. $4\frac{1}{6}$ lb.
10. $\frac{25}{48}$ pwt.
11. $4\frac{1}{2}$ lb.
12. $6\frac{1}{4}$ T.
13. $3\frac{3}{4}$ T.
14. $\frac{1}{4}$ cwt.
15. $\frac{1}{32}$ lb.
16. $\frac{1}{25}$ T.
17. 36000 gr.
18. $1\frac{3}{25}$ T.
19. \$32.03
20. \$0.27
21. $\frac{5}{16}$
22. 80
23. $166\frac{2}{3}$

Exercise 79, page 142.

2. 144 in.; 63 in.; 24 in.; 27 in.
3. 8000 li.; 7500 li.
4. 1760 yd.; $24\frac{3}{4}$ yd.; $4\frac{1}{8}$ yd.
6. $21\frac{9}{11}$ rd.
7. $\frac{1}{2}$ mi.
8. 8.4 ch.; .105 mi.
9. $\frac{3}{35}$
10. $\frac{1}{4}$
11. $\frac{3}{32}$
12. $\frac{33}{50}$
15. $5\frac{1}{6}$
16. 32 rd.
17. 120
18. 660
19. $207\frac{21}{44}$
20. $\frac{3}{128}$

Exercise 80, page 145.

1. 432; 1296
2. 45; $272\frac{1}{4}$
4. 4840 sq. yd.; 43560 sq. ft.
5. 400 sq. rd.; 25 sq. ch.
3. $151\frac{1}{4}$ sq. yd ; $1361\frac{1}{4}$ sq. ft.; 196020 sq. in.
6. $2\frac{2}{9}$
7. $\frac{11}{40}$; $1\frac{1}{4}$
8. 1.545 A.
9. $2\frac{1}{4}$ A.
10. $\frac{4}{1089}$
11. $\frac{3}{5}$; $\frac{5}{16}$
12. 36
13. 7.225
14. $2\frac{3}{4}$
15. 140
16. $46\frac{2}{9}$
20. 12

Exercise 81, page 149.

1. 108 cu ft.; 186624 cu. in.
2. $20\frac{1}{4}$ cu. ft.; 34992 cu. in.
3. $2\frac{1}{2}$ cu. ft.; $\frac{5}{54}$ cu. yd.
4. $\frac{1}{6}$; $\frac{1}{54}$
5. 512
6. 1536
8. $3\frac{3}{4}$
9. 256
10. 384

Exercise 82, page 151.

1. 5; 14; 48
2. 18; 40; 6
3. $\frac{1}{2}$; $\frac{3}{8}$; $\frac{1}{16}$; $\frac{5}{8}$; $\frac{3}{16}$
4. $\frac{3}{4}$; $\frac{5}{8}$; $\frac{5}{16}$; $\frac{1}{2}$;
5. 616; 1075.21
6. 7.48; 9.31
9. $76\frac{4}{5}$
10. $17\frac{7}{25}$
11. 300
12. 75
13. $32\frac{8}{11}$
14. 384

Exercise 83, page 154.

1. 1440
2. 3600
3. 146 da
4. 152 da
5. $\frac{6}{73}$; $\frac{13}{73}$; $\frac{18}{73}$
6. 183
7. 82
8 167
9. 121
10. 3 yr. 137 da.; 3 yr. 63 d.
11. $\frac{78}{365}$
12. 24
13. 1600 and 2000
15. Apr. 30
16. Nov. 30
17. Dec. 1
18. Mar. 30
19. Apr. 30
20. Nov. 2
21. Nov. 28
22. 40; 42; 4.8
23. $13\frac{1}{3}$; $11\frac{1}{4}$; 45
24. $33\frac{3}{4}$′; 2025″
25. $1\frac{1}{2}$
26. $\frac{1}{16}$
27. 58yr.
28. 40yr.

Exercise 84, page 156.

2. 96 d.
3. 3360 d.
4. $13\frac{1}{3}$ s.
5. 168 d.
7. £$\frac{2}{5}$
8. £$3\frac{3}{4}$
9. .05 s.
10. £$11\frac{59}{120}$
11. \$87.60
12. 24.3325 ct.
13. 121.6625 ct.
14. 48.665 ct.
15. \$24.125
16. \$17.9352
17. £20 55
18. 4.11 s.
19. 414.4 francs
20. 400 marks

Exercise 86, page 159.

2. 8820 lb.
3. 15912″
4. 1068 d.
5. $771\frac{1}{2}$ sq. ft.
6. 3178 gr.
8. .04 of a rt.∠
9. £$4\frac{49}{120}$
10. $1\frac{6}{73}$ yr.
11. $5\frac{17}{32}$ bu.
12. .12 T.
13. $\frac{35}{33}$ rd.
14. 2.64 ch.; .033 mi.
16. $15\frac{1}{6}$ yd.
17. $776\frac{1}{2}$ gal.
18. $34\frac{61}{240}$ hr.
19. $97\frac{5}{6}$ sq. ft.
20. $274\frac{1}{2}$ cu. ft.
22. 1 da. 2 hr.
23. 1 lb. 10 pwt.
24. 5 yd. 9 in.
25. 2 rd. 4 yd. 2 ft.
26. 3 sq. rd. 4 sq. ft. 36 sq. in.
27. 1 mi. 7 rd. 1 yd. 1 ft. 6 in.
29. 4 yd. $4\frac{1}{2}$ in.
30. 14 hr. 24 min.
31. 5 s. 8 d. $2\frac{2}{7}$ far.
32. 22 sq. yd. 6 sq. ft. 27 sq. in.
33. 4 A. $106\frac{2}{3}$ sq. rd.
34. 2 mi. 29 rd. 1 ft. 6 in.
36. 1 A. 64 sq. rd.
37. 18 sq. yd. 1 sq. ft. 50.4 sq. in.
38. 2° 4′ 12″
39. 4 yd. $4\frac{1}{2}$ in.
40. 16 da. 10 hr. 12 min.
41. 7 oz. 16 pwt.

Exercise 87, page 164.

4. 4.5^{Kg}
5. 450^{mg}; 700^{mg}; 5^{mg}
6. 4756.845^{Kg}; 4.756845^{MT}
7. 8.7968^{Km}; 8796800^{mm}
8. 45.675^{dl}; 4567.5^{ml}; 456.75^{cl}
9. 40^{cl}; 4^{cl}; $.5^{\mathrm{cl}}$; 400^{ml}; 40^{ml}; 5^{ml}
10. 60^{cm}; 75^{cm}; 7.5^{cm}; 600^{mm}; 750^{mm}; 75^{mm}
11. 8000^{ml}; 200^{ml}; 61^{ml}
12. 200^{mg}; 40^{mg}; 7500^{mg}
13. $.4^{\mathrm{g}}$; $.07^{\mathrm{g}}$; $.254^{\mathrm{g}}$
14. $.0225^{\mathrm{g}}$; $.21875^{\mathrm{g}}$; 7.456^{g}
16. 6057.6^{m}
17. 2247.084^{g}
18. 4007975^{g}
19. 2^{Kg} 756^{g} 456^{mg}
20. 1^{Km} 425^{m} 620^{mm}
21. 65^{Kl} 327^{l} 400^{ml}
22. 4^{MT} 278^{Kg} 564^{g}

Exercise 88, page 165.

3. 1.4575^{Ha}; 14575^{ca}
4. 75^{qdm}; 7500^{qcm}
5. 175^{qdm}; 1750000^{qmm}; 1.75^{ca}
6. $.004^{\mathrm{qm}}$; 40^{qcm}; 4000^{qmm}
8. 450270^{qm}
10. $.0406^{\mathrm{qm}}$; 4.06^{qdm}
11. 40075^{qm}; 400.75^{a}
12. $.00015^{\mathrm{qm}}$; 150^{qmm}
13. 14.07^{a}; $.1407^{\mathrm{Ha}}$
14. 45^{qDm} 67^{qm} 65^{qdm}
15. 7^{Ha} 58^{a} 32.4^{ca}
16. 45^{Ha} 67^{a} 40^{ca}
17. 4^{Ha} 56^{a} 78^{ca}
19. 5400^{cdm}; 5400000^{ccm}; $5400000000^{\mathrm{cmm}}$
21. $.016^{\mathrm{cbm}}$; 16000^{ccm}; 16000000^{cmm}
22. $.00025^{\mathrm{cbm}}$; $.25^{\mathrm{cdm}}$; 250000^{cmm}
23. $.0000075^{\mathrm{cbm}}$; 7500^{cmm}
24. 146^{cbm} 750^{cdm}
25. 4^{cbm} 462^{cdm} 150^{ccm}

Exercise 89, page 167.

1. 2.54
2. 1.99
3. 6.45
4. 11.96
5. 61.775 A.; 40 47^{a}.
6. 16.30
7. 1.31
8. 55 115 lb.; 373.25^{g}
9. 56.75 bu.; 528.34 gal.
10. 4.41^{l}; 3.79^{l}
12. 1.03^{Kg}
13. 20.94^{g}
14. 9.225
15. 50
16. 1^{g}
17. 175
18. 2.5; 2.5
19. 26^{Hl} 4^{Dl} 6^{l}
20. 23.4375
21. 7920^{mg}
22. 9.999^{MT}
23. 40
24. \$3708
25. 31.24^{Kg}
26. 43.2; 43.2^{MT}
27. 21.95
28. 1492^{g}

Exercise 90, page 170.

1. 5
2. 48
3. $57.31
4. $52\frac{1}{2}$
5. $1.87
6. $66.08
7 27; $22\frac{3}{4}$

Exercise 91, page 171.

2. $8.82
3. $8.88
4. $6.23

Exercise 92, page 173.

1. 20
2. $16\frac{16}{21}$; $33\frac{11}{21}$
3. $17.85
4. 15.84
5. 1458 laths; $16\frac{1}{35}$ lb.; 8.16 bu.
16.32 bu.; 1.02 bu.

Exercise 93, page 174.

1. $13\frac{1}{3}$ bd. ft.
2. 16
3. $3.60
4. 120
5. 750
6. $221.18
7. $6.72
8. $2.35
9. $23.31
10. 132
11. 600
12. 250
13. $31\frac{1}{4}$
15. 7889

Exercise 94, page 177.

2. 16
3. $8\frac{184}{297}$
4. $69.09
5. $1\frac{1}{11}$
6. $302.40
7. $216.80

Exercise 95, page 178.

1. 88 ft.
2. $3\frac{1}{7}$
3. 154
4. $7\frac{6}{7}$ ft.
5. $\frac{99}{224}$
6. 28 ft.
7. 616
8. 420
9. $56\frac{4}{7}$ in.
10. $6\frac{2}{7}$ in.
11. 280
12. $1037\frac{17}{44}$ mi.
13. 57
14. $50.25

Exercise 96, page 180.

1. $402\frac{3}{7}$ sq. in.
2. 704
3. $28\frac{2}{7}$
4. $21\frac{3}{14}$
5. 165 sq. ft.
6. 120 ft.
7. 5.95 ft.
8. $301\frac{5}{7}$
9. 37616.33 gal
10. 209476 gal.

Exercise 97, page 181.

2. 7136
3. $1\frac{31}{33}$
4. 7.48
5. 112
6. 351
7. $67\frac{1}{2}$
8. 96
9. 23760
10. $28.50
11. 2.72
12. 12960 lb.
13. 400
14. 1504
15. (a) $11.14; (b) $60.60; (c) $16.50; (d) $8.76
16. (a) $10.71; (b) $54.24; (c) $382.80; (d) $16.41
17. (1) $2\frac{29}{40}$; (2) $77\frac{1}{40}$
18. 7 in. × 5 in. × 3 in.
19. 576 cu. in.
20. 38 ft.
21. 160 sq. ft.
22. 11 in. × 7 in. × 3 in.
23. 60
24. 224
25. 1440; 120
26. $44\frac{4}{5}$
27. $10\frac{1}{3}$ bu.
28. 100 sq. rd.
29. 46.64
30. $26\frac{1}{4}$ mi.
31. 384
32. $378.30
33. 2957.5 lb.
34. $26\frac{19}{22}$
35. $0.70
36. $1.09
37. 6376.32 lb.
38. 1080741 gal.
39. $0.12
40. 48.63 mi
41. 921
42. 425
43. $0.60
44. $0.10
45. 69.375 mi
46. 2.63 lb.

Exercise 98, page 186.

1. 13 bu. 2 pk. 1 qt.
2. 48 yr. 8 mo. 16 da.
3. 302 rd. 1 yd. 2 ft. 4 in.
4. 1810 yr. 7 mo. 18 da.
5. 84° 54′ 35″
6. 7 A. 119 sq. rd. 29 sq. yd. 2 sq. ft. 36 sq. in.
7. 37 yr. 5 mo. 14 da.
8. 50 ch. 15 li.
9. 24 A. 147 sq. rd. 5 sq. yd. 6 sq. ft. 108 sq. in.
10. 180° 9′ 43″
11. 1785 yr. 9 mo. 24 da.
12. 79° 44′ 30″
13. 11 rd. 4 yd. 1 ft. 2 in.
14. £9 17 *s.* 7 *d.* 2 *far.*
15. Dec. 14, 1799.
16. 61 ch. 6 li.
17. 8 T. 12 cwt. 3 qr. 5 lb.
18. 2 hr. 34 min. 30 sec.
19. 9 hr. 4 min.
20. 8 hr. 43 min.
21. 45 yr.
22. 3 da. 9 hr. 45 min.
23. 134° 44′ 15″
24. 29 da. 13 hr. 37 min.
25. May 24, 1819; [18 yr. 26 da.

Exercise 99, page 188.

1. £26 10 *s.*
2. 27 rd. 3 yd. 6 in.
3. 22 A. 83 sq. rd. 9 sq. yd. 2 sq. ft. 36 sq. in.
4. 5° 38′ $2\frac{1}{4}$″
5. £1 7 *s.* 5 *d.*
6. 6 rd. 2 yd. 1 ft. $9\frac{1}{2}$ in.
7. £31 3 *s.* 9 *d.*
8. 180° 45′ 30″
9. 20 rd. 3 yd.
10. 2 gal. 1 pt. 1 gi.
11. £4 10 *d.* $3\frac{1}{5}$ *far.*
12. 110 sq. rd. 6 sq. yd. 2 sq. ft. 36 sq. in.
13. 27′ $\frac{2}{5}$″
14. 5 pk. 5 qt. $1\frac{1}{2}$ pt.
15. 10
16. 8
17. 3
18. $24.44
19. 28 yd. 1 ft. 6 in.
20. $41.30
21. 97.4
22. 2.9 in.
23. $49.08
24. $61.87
25. $196.95
26. 22.42
27. 28 da. 2 hr. 17 min. [36.1 sec.
28. 23 hr. 15 min. 18.8 sec
29. $22.08

Exercise 100, page 191.

1. 45° N.
2. 65° S.
3. 45° N.
4. 10° S.
5. 55° N.
6. 90° S.
7. 15° E.
8. 65° W.
9. 50° W.
10. 40° E.
11. 240° E.
12. 90° W.
13. 30° 35′ S.
14. 1° 18′
15. 32° 41′ 44″ N.
16. 40° 49′ 31″ N.
17. 71° 57′ 8″
18. 1° 54′ 56″
19. 35° 35′ 11″
20. 17° 10′ 18″ E.
21. 2° 3′ 51″ E.

Exercise 101, page 193.

3. 2 hr.; 6 hr.; 3 min.; 4 min.; 1 sec.; 8 sec.
4. 1 hr. 1 min; 2 min. 3 sec.; 3 hr. 1 min 1 sec.
5. 30°; 90°; 30′; 1°; 30″; 45″
6. 15° 15′; 30′ 45″; 45° 30′ 45″
7. (a) 2 hr. 40 min.; (b) 1 hr. 20 min.; (c) 2 hr. 40 min.
8. (a) 3.20 A.M.; (b) 8.40 P.M., previous day; (c) 8.24 A. M.; (d) 4.12 A.M.
9. (a) 90° W.; (b) 180° W.; (c) 3° 45′ W.; (d) 75° W.
10. (a) 112° 30′ W.; (b) 49° 30′ E.
11. 71° 7′ 45″ W.
12. 75° 9′ 37.5″ W.
13. 77° 3′ 55.5″ W.
14. 4 hr. 56 min. 0.2 sec. W.
15. 9 min. $21\frac{7}{13}$ sec. E.
16. 5 hr. 5 min.
17. 23 hr. 38 min. 59.6 sec.
18. 24 hr. 21 min. 21 sec.
19. 55 min. 23 sec. A.M. Thursday
20. 13° 26′ 13.5″ E.
21. 4 hr. 45 min. 37.5 sec. W.
22. 4 hr 45 min. $49\frac{2}{13}$ sec A.M. Wednesday

Exercise 102, page 198.

6. 2
7. $2
8. $4\frac{1}{2}$ bu.
9. 12 yd.
10. $4\frac{1}{2}$ da.
11. .6 gi.
12. $67.50
13. 6 A.
14. 42 yr.
15. $2594.375
16. 2
17. $9
18. $3\frac{3}{4}$ A.
19. $1\frac{3}{4}$ yd.
20. 1760 ft.
21. 240°
22. $\$417\frac{2}{3}$
23. $95\frac{5}{16}$ gal.
24. 300 sheep.
25. $233.60
26. .15
27. 4
28. $\frac{1}{500}$
29. $\frac{3}{160}$
30. $24.08
31. 1.2 gal.
32. $25.0625
33. 19.2525^{cdm}
34. 2.52^{s}
35. 1.125^{s}
37. 80
38. 1500
39. $100
40. $2\frac{2}{3}$ mi.
41. 20100
42. 500 A.
43. $\frac{5}{8}^{m}$
44. $\frac{8}{15}$
45. 40 da.
46. $1\frac{1}{6}$ da.
47. 64 gal.
48. 96 yd.
49. 40 in.
50. 10.4°
51. $34375
52. $66\frac{2}{3}$ mi.
53. $1560
54. 3
55. 2000^{s}
56. $137.75
64. 80%
65. $33\frac{1}{3}\%$
66. 25%
67. 500%
68. 400%
69. 50%
70. $112\frac{1}{2}\%$
71. 1%
72. .1%
73. 50%
74. 50%
75. $12\frac{1}{2}\%$
76. 500%
77. 400%
78. 5%
79. $\frac{4}{5}\%$
80. 100%
81. $3\frac{1}{8}\%$
82. $3\frac{1}{3}\%$
83. $16\frac{2}{3}\%$
84. $\frac{5}{16}\%$
85. $\frac{1}{50}\%$
86. $\frac{2}{7}\%$
87. $42\frac{6}{7}\%$
88. 4%
89. 1000%
90. $\frac{1}{4}\%$
91. $\frac{1}{5}\%$
92 $416\frac{2}{3}\%$
93. 30%
94. 8.93%
95. 4.78%
96. 469.29%
97. .56%
98. 1.63%
99. 1.05%

Exercise 104, page 204.

2. $55; $220
3. 41.25 gr.; 371.25 gr.
4. 230 sq. mi.; 44985 sq. mi.
5. $68.75
6. 23.22 gr.; 2.58 gr.
7. 30.8 lb.; 5.2 lb.
8. 187.5; 25; 37.5
9. 16.5
10. 226.8 bu.
11. $21.50
12. 85050
13. $\frac{1}{12}$; $\frac{1}{8}$; $\frac{1}{24}$
14. 100; 30; 650
15. 49.84
16. 16.2
17. $304.50
18. $20.25
19. $970.20
20. $\$214\frac{2}{7}$
21. $7\frac{1}{2}$
22. 32
23. 2
24. 90
25. 20 lb.
26. $1\frac{1}{2}$ ft.
28. $2\frac{6}{47}$ lb.
29. 500
30. 1250 sq. mi.; 165 sq. mi.
31. $2\frac{1}{2}$ ct.
32. 4
33. $90\frac{10}{11}$ cu. ft.
34. 26189
35. $6000
36. 28
37. $\$1333\frac{1}{3}$
38. $4060
39. $3000; $5.40
41. 5.4%
42. 1.8%
43. 40%; 60%
44. 95.393%; 4.607%
45. 75%; 25%
46. $37\frac{1}{2}\%$
47. 50%
48. 80%
49. 37.9%
50. 54.4%
51. 23.6%
52. 27.3%
53. 25.1%
54. 17.2%
55. 46.1%
56. 37.8%
57. 14.6%
58. 9.8%
59. 70.15%
60. $1\frac{1}{4}\%$
61. $\frac{1}{5}\%$

Exercise 105, page 209.

2. $195.13
3. $11.25
4. $12\frac{1}{2}$ ct.
5. $36
6. $2025
7. $201.60
8. $171
9. 30.4 ct.
10. $346.50
11. $7
12. $225; $11.25
13. $\$333\frac{1}{3}$
14. 30%
15. $21\frac{7}{8}\%$
16. $17\frac{1}{2}\%$
17. 20%
18. 28%
19. The former by 10%
22. $17.57
23. $8.46

Exercise 106, page 212.

4. $40; $240
5. $4; $36
6. 40 ct.; $1
7. $3; $803
8. 60 ct.; 90 ct.
9. 1 ct.; 2 ct
10. 12 ct.
11 $93\frac{3}{4}$ ct.
12 $35
13. $\$\frac{3}{4}$
14. $11
15. 30 ct.; $2.10
16. $4; $20

17. 5 ct.; 70 ct.
18. 30 ct. l.; $39.70 s. p.
19. 1 ct.; 1 ct.
20. $1; 0
21. $8; $1992
22. $40; $10
23. $64; $8
24. 2 ct.; 4 ct.
25. 50 ct; 10 ct.
26. $3; $1.50
27. $1; 10 ct.
28. $1.50; 50 ct.
29. $220
30. 15 ct.
31. $3.13
32. $2000
33. 16 ct.
34. $99
35. $33\frac{1}{3}\%$ g.
36. 20% l.
37. 200% g.
38. $66\frac{2}{3}\%$ l.
39. 100% g.
40. $\frac{1}{2}\%$ g.
41. 48 ct.
42. $2\frac{1}{32}$ ct.
43. $2.50
44. $3.60
45. 50% g.
46. $33\frac{1}{3}\%$ g.
47. $33\frac{1}{3}\%$ l.
48. $6\frac{2}{3}\%$ g.
49. $15\frac{5}{13}\%$
50. $1.68
51. 100%
52. 50%
53. $2
54. $16\frac{2}{3}\%$ l.
55. $2
56. 15 ct.
57. 50%
58. 5% g.
59. $3.84
60. 100%
61. 25%
62. 52%
63. $6\frac{1}{4}\%$ l.
64. $6\frac{2}{3}$
65. $7\frac{1}{2}$ ct.
66. $5.64
67. $5
68. $100; 10%
69. 125%
70. 18 ct.
71. 15 ct.
72. $56\frac{4}{5}\%$

Exercise 107, page 217.

3. $52.50
4. $74.10
5. $136.74
6. $3592.40
7. $26
8. $374.40
9. $4164.50
10. 2000 bu.
11. $576
12. $106.75
13. 476; $0.80
14. $3369.40
15. $449
16. $200
17. $12.50
18. 576; $0.24
19. $150
20. 10%
21. 10%
22. 5%
23. 10%

Exercise 108, page 221.

3. $7.20
4. $6
5. $5.50
6. $60.94
7. $10.25
8. $122.88
9. $2.96
10. $9.39
11. $13.84
12. $3.59
13. $11.35
14. $9.23
15. $55.35
16. $16.96
17. $173.78
18. $46.23
19. $2.98
20. $44.05
21. $30.10
22. $27.30
23. $37.99
24. $2.40; $242.40
25. $7.50; $257.50
26. $1.24; $83.74
27. $0.61; $49.51
28. $44.77; $370.37
29. $15.46; $942.96
30. $7.41; $285.41
31. $1.49; $41.09
32. $0.87; $83.81
33. $2.22; $102.22
34. $3.84; $101.84
35. $5.78; $213.63
36. $2.63
37. $17.55
38. $14.56
39. $1.17
40. $8.90
41. $132.10

Exercise 109, page 224.

2. $200
3. $100
4. $359.56
5. $344.44
6. $800.44
7. $650
8. $880.80
9. $360
10. $965.86
11. $240; $22.40
12. $450; $31.50
13. $1054.02; $116.48
14. $1842.47; $157.53

Exercise 110, page 226.

2. $4\frac{1}{2}\%$
3. $4\frac{4}{5}\%$
4. 6%
5. 5%
6. 5%
7. 4%
8. 20%
9. 4.8%
10. 4%
11. 3%
13. $6\frac{2}{3}\%$
14. 4%
15. 6%

Exercise 111, page 227.

2. 3 yr. 72 da.
3 2 yr. 315 da.
4. 1 yr.
5. 1 yr. 90 da.
6. 3 yr. 120 da
7. 11 yr. 40 da.
8. May 12, 1901
9. Nov. 13
10. Oct. 28, 1902
11. July 25
13. $28\frac{4}{7}$ yr.
14. $33\frac{1}{3}$ yr.
15. $12\frac{1}{2}$ yr.

Exercise 112, page 229.

6. $1365
7. $2379.38
8. $2464.06
9. $695
10. $456.50
11. $165.19
12. $5595
13. $11675
14. $679.38
15. $1158.75
16. $6697.50
17. 10
18. 36; $44.75
19. 36; $65.50
20. 87; $27.625
21. $2000
22. 8
23. 20; $102.50
24. $240
25. $30
26. $50
27. $1200
28. $120
29. $100
30. $25
31. $550
32. $205
33. $52.50
34. 248
35. 20
36. $2727
37. $5960.63
38. $2970
39. $32
40. $275
41. $200.
42. 4.80 %
43. 4.17 %
44. 3.48 %
45. 3.55 %
46. 3.37 %
47. $120

Exercise 113, page 235.

2. $546.36; $46.36
3. $378.74; $78.74
4. $456.47; $56.47
5. $415.77; $40.77
6. $2262.82; $262.82
7. $551.91; $51.91
8 $347.78; $47.78
9. $525.47; $25.47
10. $522.84; $22.84
11. $1104.08; $104.08
12. $3746.59; $746.59
13. $1049.04; $209.04
14. $2392.29; $392.29
15. $10000
16. $3000
17. $4000
18. $25000
19. $42000000

Exercise 114, page 240.

11. $222.48
12. $5.49
13. $203

Exercise 116, page 249.

2. $495
3. $594
4. $432.08
5. $199
6. $237.96
7. $195.87
8. $242.87
9. $497.96
10. $149.84
11. $347.92
13. $101.03
14. $345.18
15. $60.92
16. $404.04
17. $597.99
18. $242.42

Exercise 117, page 252.

1. $126.56
2. $499.38
3. $251.25
4. $498.75
5. $137.53
6. $356.85
7. $158.20
8. $101.25
9. $196.95
10. $497.50
11. $197.97
12. $493.96
13. $597.01
14. $252.41
15. $592.59
16. $595.53

Exercise 118, page 255.

2. $1164.60
3. $122.52
4. $138.27
5. $631.68
6. $67.80
7. $1118.45
8. $96.76
9. $557.94
10. $1134
11. $1135.25
12. $114.10
13. $1096
14. $314.93
15. £193 16 *s.* 7 *d.*
16. £150
17. 2057.5 francs
18. 1284.69 francs
19. 320 marks
20. 480 marks
21. 280 guilders
22. 500 guilders
23. $1061.22
24. 4193.97 marks

Exercise 119, page 259.

1. $124.61
2. $237.04
3. $356.31
4. $554.86
5. $168.34
6. $65.73
7. $119.87
8. $238.04
9. $357.57

Exercise 120, page 262.

2. 10 : 13
3. 8 : 9
4. 15 : 16
5. 2 : 3
6. 7 : 30
7. 11 : 3
8. 5 : 24
9. 128 : 25
10. 5 : 9
11. $\frac{16}{25}$
12. $3\frac{1}{8}$ bu.
13. $1\frac{1}{3}$
14. $1\frac{1}{5}$
15. $1\frac{1}{2}$; 8
16. $\frac{8}{9}$; $\frac{64}{81}$
17. .92
18. $654\frac{3}{8}$ lb.
19. $\frac{144}{175}$
20. $1\frac{17}{175}$
21. 300

Exercise 121, page 264.

2. $x : 6 = 1 : 3$
3. $2x : 11 = 2 : 3$
4. $3x : 10 = 8 : 9$
5. $x : 10 = 1 : 2$
7. $2\frac{1}{3}$
8. $2\frac{11}{12}$
9. $\frac{39}{40}$
10. 15
11. 13
12. $1\frac{1}{14}$

Exercise 122, page 266.

4. $33.75
5. $2.30
6. $4 03
7. $11.25
8. $$28\frac{4}{5}$ ft.
9. 75
10. $3\frac{1}{3}$ in.
11. 32 ct.
12. $50\frac{10}{13}$ mi. per hr.
13. 18.253125
14. $40\frac{40}{63}$
15. 25 ct.
16. $1.875
17. 24897.60
18. $266\frac{2}{3}$
19. 5.56 in.; 5.21 in.; 4.34 in.
20. 1030
21. 240; 336
22. 2240
23. $197\frac{17}{35}$
24. $43\frac{1}{3}$ mi.
25. 7290
26. 3240
27. 225
28. 192 lb.

Exercise 123, page 269.

2. 18
3. 216
4. 77 yd.
5. $28.13
6. 3 da.
7. 15 ft.
8. 880
9. $5\frac{1}{3}$ ft.
10. $1596\frac{7}{8}$
11. $1\frac{1}{4}$
12. $26\frac{1}{4}$
13. $\frac{1}{6}$

Exercise 124, page 271.

3. $60; $40
4. $50; $200
5. $55\frac{5}{9}$ lb.; $44\frac{4}{9}$ lb.
6. 9; 15
7. 60 lb.; 32 lb.
8. 30; 45
9. $375
10. 165; 110
11. 6; 4
12. $\frac{20}{57}$; $\frac{6}{19}$
13. 1.65; 1.1
14. $\frac{7}{12}$; $\frac{1}{2}$
15. 4 lb. black; 6 lb. green
16. $\frac{4}{5}$ lb. tin; $1\frac{1}{5}$ lb. lead
17. $18\frac{2}{11}$ lb.; $81\frac{9}{11}$ lb.
18. 16 mi.; 18 mi.
19. $14.40; $21.60
20. $800; $1200; $2400
21. 20 bu. oats; 30 bu. corn; 50 bu. rye

Exercise 125, page 273.

2. $300; $225; $195
3. $150; $90
4. M, $192.75; N, $128.50
5. R, $2400; S, $2880; T, $720
6. M, $6000; N, $4000; P, $5000
7. A, $1203.65; B, $752.28; C, $601.82
8. C's capital, $9900; A's gain, $100; B's gain, $80
9. A, $3424; B, $2996
10. A, $700; B, $$933\frac{1}{3}$; C, $$1166\frac{2}{3}$
12. $270; $240
13. M, $1200; N, $1350; P, $120
14. M, $900; N, $1000; P, $1200
15. R, $840; S, $510
16. A, $108; B, $100; C, $147
17. R, $10000; S, $4000; T, $5000

Exercise 126, page 276.

3. 36 ct.
4. $1.34
5. 5.15 ct.
6. 18.67 ct.
7. 9.56 ct.
8. $$56\frac{2}{33}$
9. 26
11. $7\frac{15}{19}$ mo.
12. Feb. 4
13. Nov. 1 of the following year
14. July 1

Exercise 127, page 279.

7. 324; 5832
8. 1225; 42875
9. 2401; 117649
10. 15625; 1953125
11. 163216; 65939264
12. 708964; 596947688
19. .0625; .015625
20. .003136; .000175616
21. 6.5536; 16.777216

Exercise 129, page 282.

2. 32
3. 46
4. 59
5. 68
6. 76
7. 86
8. 95
9. 119
10. 225
11. 327
12. 466
13. 508
14. 564
15. 603
16. 671
17. 706
18. 760
19. 802
20. 860
21. 908
23. 7.3
24. 2.78
25. 76.5
26. 8.08
27. 5.66
28. 6.71
29. 17.92
30. 22.54
31. 1.85
32. 1.19
33. .15
34. .05
35. $\frac{11}{12}$
36. $\frac{20}{23}$
37. $\frac{16}{27}$
38. $4\frac{3}{8}$
39. $2\frac{2}{3}$
40. $2\frac{9}{17}$
41. .94
42. 2.31

Exercise 130, page 285.

2. 15
3. 26
4. 37
5. 58
6. 75
7. 88
8. 113
9. 256
10. 304
11. 356
12. 503
13. 560
15. 2.2
16. 1.66
17. 25.5
18. .19
19. .34
20. .47
21. .05
22. 79.2
23. 2.92
24. 5.01
25. 1.65
26. 1.55
27. 6.69
28. .17
29. .93
30. .65
31. $\frac{4}{5}$
32. $\frac{9}{10}$
33. $\frac{5}{12}$
34. $\frac{21}{22}$
35. $2\frac{1}{3}$
36. $1\frac{4}{5}$
37. .84
38. .66

Exercise 131, page 292.

1. 85°
2. 80°
3. 65°
4. 18 ft.
5. 47° 45′
6. 67° 30′; 22° 30′
7. $98\frac{2}{11}^\circ$; $49\frac{1}{11}^\circ$; $32\frac{8}{11}^\circ$
8. 60 ft.
9. 28 ft.
10. 65 ft.
11. 26.83 ft.; 13.42 ft.
12. 69.394 ft.
14. 8
15. 84 sq. rd.
16. 692.82 sq. ft.
17. 3990.64 sq. ft.
18. 10.83
19. 15 ft.
20. $48\frac{4}{9}$ ft.
21. $302\frac{1}{2}$ yd.
22. $6\frac{3}{4}$ ft.
23. 60 A.
24. 54 sq. yd.

Exercise 132, page 298.

1. 48°; 132°; 132°
2. 60 ft.
3. 39 ft.; 2028 sq. ft.
4. 20 yd.; 384 sq. yd.
5. 32 53 ft.
6. 25.46 ft.; 648 sq. ft.
7. 100.8 A.
8. 3 rd.
9. 48 rd.
10. 768 sq. ft.; [40 ft.
11. 1152 sq. ft.
12. 18 rd.
13. 69 57 yd.
14. 189 sq. ft.
15. 40 sq. ft.
16. 30 A. 2.73 sq. ch.
17. $21\frac{1}{3}$
18. 280
19. 7 05 sq. yd.
20. 239
21. .87 sq. ft.
22. $21\frac{1}{3}$ ft.
23. 10 ft.
24. $98
25. 252 sq. rd.
26. 27.216
27. 3 A.
28. 4 sq. in.
29. 20 ft.
30. 15.36^a

Exercise 133, page 302.

1. 64.95 sq. ft.
2. 93.53 sq. yd.
3. 221.70 sq. rd.
4. 237.85 sq. ft.
5. 70 78 sq. yd.
6. 6 ft.
7. 11.31 in.
8. 5.04
9. 73.14
10. 32 ft.; 64 sq. ft.
11. $10\frac{2}{7}$ ft.
12. $21\frac{3}{7}$ sq. ft.
13. 200 sq. in.
14. $176\frac{1}{14}$ sq. in.
15. 16.92 rd.
16. $1257\frac{1}{7}$ sq. yd.
17. 9 ft.
18. 10 ft.
19. 1680
20. $3\frac{1}{7}$ sq. ft.
21. 77.2544 sq. ft.

Exercise 134, page 305.

3. 48 sq. ft.
4. 18 cu. ft.
5. 130 sq. ft.
6. 180 sq. ft.
7. 6 ft.
8 5 ft.
9. 7293 lb.
10. $6\frac{17}{18}$ ft.
11. 24.41 ft.
12. 10.39 ft.
13. 128 sq. ft.
14. $43\frac{41}{63}$

Exercise 135, page 307.

3. 195 sq. in.
4. 252 cu. in.
5. 8.54
6. 540 sq. in.
7. $2\frac{1}{2}$ sq. ft.
8. 8 in.
9. $1357\frac{5}{7}$ cu. in.
10. $1885\frac{5}{7}$ sq. in.
11. 16 in.
12. 21
13. 10 in.
14. 3 ft.
15. $4\frac{5}{7}$ cu. in.
16. $47\frac{1}{7}$
17. $37\frac{5}{7}^{ccm}$
18. 48^{ccm}

Exercise 136, page 310.

3. 360 sq. in.
4. $2702\frac{6}{7}$ cu. in.
5. 105 sq. ft.
6. $179\frac{1}{7}$ cu. ft.
7. 12.96 qt.
8. 1975 gal.

Exercise 137, page 311.

1. $452\frac{4}{7}$ sq. in.
2. $268\frac{4}{21}$ cu. in.
3. $19\frac{9}{14}$
4. $179\frac{2}{3}$ cu. ft.
5. $3\frac{1}{2}$ ft.
6. 14 in.
7. 88 in.
8. 21 in.
9. 5544 sq. in.; 38808 cu. in.
10. 176 in.
11. $18\frac{6}{7}$ ft.
12. $61625\frac{3}{8}$cu. in.
13. 154 sq. ft.
14. 2 in.
15. 16 in.
16. $113\frac{1}{7}$ sq. in.
17. 329.27 cu. ft.
18. $7\frac{1}{14}$ cu. in.

Exercise 138, page 314.

1. $4\frac{1}{2}$ ft.
2. 154 sq. ft.
3. 60.84 sq. ft.
4. $268\frac{4}{21}$
5. 21 ft.
6. $\frac{5}{2}\sqrt{3}$
7. 4 in.
8. $\frac{21}{44}$ ft.
9. 9.52 ft.
10. $711\frac{1}{9}$ sq. in.
11. 12 rd.
12. $34\frac{11}{64}$ gal.
13. 4
14. $149\frac{1}{3}$
15. 32 hr.
16. $55\frac{1}{2}$ in.; 24 in.
17. 24 in.
18. 24 ft.

Exercise 139, page 316.

1. 800
2. $3\frac{1}{8}$; 47.17 rd.
3. 16.2
4. 4 A. 7 sq. rd.
5. 4704
6. $18\frac{3}{4}$ ft.
7. 184
8. 8 in.
9. 5184 sq. ft.
10. 816
11. 32
12. 42
13. $247\frac{1}{2}$
14. $67\frac{1}{21}$
15. $380\frac{2}{7}$ sq. ft.
16. $2828\frac{4}{7}$ sq. ft.
17. $28\frac{2}{7}$
18. 100
19. 792
20. 1
21. $13\frac{5}{7}$
22. 134
23. $1244\frac{4}{7}$
24. 14 43
25. $74\frac{1}{2}$ nearly
26. $69\frac{5}{9}$
27. $28\frac{26}{27}$
28. $414\frac{18}{25}$
29. $3127\frac{1}{2}$ lb.
30. 2750^{Kg}
31. $49\frac{8}{27}$
32. $\frac{1}{2}$ in.
33. $170\frac{6}{25}$
34. 19 ft. 3 in.
35. $46\frac{7}{8}$ lb.
36. $86.40
37. 9 ft.
38. $3
39. 8835 lb
40. 15.52
41. $5.32

Exercise 140, page 321.

1. 9.
2. $128\frac{1}{3}$.
3. 81.
4. $91\frac{1}{8}$.
5. 162 ft.
6. 80 ft.
7. $340\frac{5}{18}$.
8. 965 ft.
9. 320 ft.
10. 900 ft.
11. 1920 ft.
12. 1296 ft.
13. 1800 ft.
14. 6272 ft.
15. 1372 ft.
16. 243 ft.
17. 96 ft.
18. 600 ft.
19. 48 ft.
20. 300 ft.

Miscellaneous Problems, page 320.

7. 457.047
8. The latter by
9. 77 ct. [$9
11. $4
12. 1067 ; 961
13. $684
14. $140
15. 130 lb.
16. 14 bu.
17. 492 bu.
18. $960
20. The former by 25 ct.
21. 523 ; 461
22. 668 ; 334
23. 9
24. 112200
28. There are 9
29. 43
30. 1061
31. 12
32. Mar. 29
33. $100

34. $84
35. 7
36. $22\frac{5}{6}$
37. $\frac{66}{1175}$
38. 5 ft.
39. $1\frac{1}{3}$ da.
40. $\frac{3}{5}$ da.
41. $\frac{6}{17}$ da.
42. Ten and one hundredth
43. $13333333\frac{1}{3}$
44. $1\frac{19}{25}$
45. $275
46. 20 mi.
47. $1\frac{5}{13}$
48. 12
49. $21\frac{1}{4}$
50. $\frac{11}{12}$; $13\frac{3}{4}$
51. $1\frac{1}{11}$ min ; $5\frac{5}{11}$ min.
52. 40 ; $43\frac{7}{11}$
53. $27\frac{3}{11}$ min.
54. 192 rd. ; 128 rd.

55. 2640
56. 4
57. 1 pwt. $2\frac{2}{3}$ gr.
58. $3515\frac{155}{264}$
59. 444
60. 2 pk.
61. $118\frac{3}{4}$ lb.
62. 15 ; 191.25
63. $743\frac{2}{11}$ mi.
64. $4.875
65. 176
66. 70 ct. ; 80 ct.
67. $30.74
68. 36 min.
69. $2 05
70. $86 80
71. $6.27
72. 8 ft.
73. 1144 yd.

74. 226.76 lb.
75. $12\frac{9}{160}$
76. 50 ct.
77. 60%
78. $17\frac{1}{3}$%
79. 25%
80. $9
81. 32%
82. $16\frac{2}{3}$% loss
83. 34
84. 50 bu.
85. 50%
86. 20%
87. 60 ct.
88. $12\frac{1}{2}$%
89. 10 ct.
90. $48\frac{8}{9}$%
91. 20%
92. $30; $45 ;
93. 4% loss [$5\frac{2}{3}$%
94. $9684
95. $1107

96. 2%
97. 10%
98. $7.50
99. $18.40
100. $210.50
101. $5\frac{2}{5}$%
102. $47.25
103. $85
104. $4978.13
105. 72 ; $82.50
106. 14 ; $113.25
107. 43 ; $2.625
108. $37.50
109. $$119\frac{7}{8}$
110. $$59\frac{7}{8}$

111. Nov. 28
112. 7 ct.
113. $4
114. $200
115. $103.03
118. $400 ; $800
119. $30 ; $18
120. $240 ; $180 ; $60
121. $750 ; $1000 ;
122. 1100 ft. [$1250
123. $62\frac{1}{2}$ ft.
124. $3200 ; $4000 ;
125. $1383\frac{2}{11}$ [$4800
126. $3\frac{1}{3}$
127. 105

128. 3
129. $3\sqrt{3}$
130. $3\frac{3}{8}$ lb.
131. 8 rd.
132. 10 ft.
134. $\frac{3}{16}$ gal.
135. $8.75
136. 1036800
137. 242
138. 576 ; 300
139. 1 sq. ft. 18 sq. in.
140. 54000
141 $737.28

142. $31.20
143. $188.12
144. $140\frac{5}{8}$
145. $2\frac{2}{3}$ ft. ; 1024
146. $5\frac{5}{8}$ in.
147. $37\frac{5}{7}$
148. $54.72
149. $82\frac{10}{11}$
150. 128 ; 11.12
151. 84 [T.
152. $3\frac{1}{2}$ ft.
153. $12

154. 20°
155. 7928
156. 50 yd.
157. 32
158. $379.47
159. $\frac{11}{64}$ ct.
160. $1\frac{1}{3}$
161. 20
162. 32 yr.
163. $10
164. 25 hr. ; $16\frac{2}{3}$
165. $42\frac{6}{7}$ [hr.

166. $3.24
167. $22\frac{1}{2}$
168. $1.28
169. $6000
170. B owes A
171. 10 [10 ct.
172. $80\frac{2}{3}$%
173. 6
174. 2380 ; 80 ct.
175. 6 ft. ; 10 ft.
176. $2\frac{2}{3}$ da.
177. $2\frac{2}{3}$ ft.

178. 13.598
179. $59\frac{43}{48}$
180. 9 ft.
181. 132
182. $8000
183. $1050
184. $250
185. $9.38
186. $1440
187. 440 sq. ft
188. 25
189. $500

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